HANDBOOK OF

MACHINE FOUNDATIONS

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STRUCTURAL ENGINEERING RESEARCH CENTRE
Roorkee — Madras

HANDBOOK OF

MACHINE FOUNDATIONS

P. SRINIVASULU
C. V. VAIDYANATHAN

Structural Engineering Research Centre
Madras

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FOREWORD

Machine foundations form a vital and expensive part of any industrial complex. With the rapid pace of industrial growth of the country—which is the goal of our successive Five Years Plans—a large number of machine foundations are being built in the various industrial establishments. The subject of machine foundations thus assumes a great importance in the context of our national economy.

Till recently, it has been the practice to design machine foundations on the basis of rules of thumb and empirical formulae. However, with the recent advances in the fields of structural and soil dynamics, the design principles have gradually been established for typical groups of machine foundations. It has thus become imperative for the designers to know the various aspects in the analysis, design and construction of machine foundations in order to produce efficient and economical designs.

It is gratifying to note that the Structural Engineering Research Centre, has prepared a very useful reference manual on this subject and I have great pleasure in writing a foreword to this timely venture. I do hope that this handbook will prove popular with design engineers engaged in this discipline of work.

Y. NAYUDAMMA

Director-General
Council of Scientific and Industrial Research

New Delhi

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Major, A. (1962): *Vibration Analysis and Design of Foundations for Machines and Turbines,* Akademiai Kiado, Budapest.
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Richart, F. E., Jr., Hall, J. R., Jr. and Woods, R. D. (1970): *Vibrations of Soils and Foundations*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, USA.
Rausch, E. (1959): *Maschinen Fundamente und andere Dynamisch Beanspruchte Baukonstruktionen,* V D I, Verlag, GMBH, Dusseldorf, W. Germany.
Pauw, A.)1935): *A Dynamic Analogy for Foundation Soil System,* ASTM, Spec. Tech. Pub. No. 156.
Indian Standard Institution: *IS 2974* (Parts I-V) and *IS 5249*

Reference has been made to the works of the following authors at appropriate portions in the text:

Tschebotarioff (1953), Sung (1953), Arnold Bycroft and Warburton (1953), Newcomb (1957), Lindley and Gent (1959), Balakrishna Rao and Nagaraj (1960), Alpan (1961), Hsieh (1962), Richart (1962), Ford and Haddow (1966), Hall (1967), Lysmer (1967) and Lindley (1970).

These sources are sincerely acknowledged.

Thanks are also due to the various industrial establishments in India and abroad for furnishing data on machine foundations designed or constructed by them.

Besides the Indian Standard Codes of Practice, reference has been made at appropriate places in the text to the German Codes (DIN 4024 and DIN 4025), the specifications of the USSR (CH-18-58) and the Hungarian Standard (MSZ-15009-64) for purpose of illustration. These sources are sincerely acknowledged.

Information concerning references which could not be mentioned above for reasons of the authors not being aware of original sources of certain data used herein will be gratefully received and necessary corrections made in future editions.

PREFACE

Rapid industrialization of the country under the successive Five Year Plans involves the installation of machines of various types in industrial establishments. The design of foundations for such machines calls for specialized knowledge. Codes relating to machine foundations provide only very general guidance. There are hardly any works of reference which provide detailed information on different types of machine foundations. In the absence of such a manual, widely varying practices are being followed.

The effort of the authors of this book has been to present the principles of analysis, design and construction of machine foundations of different types in sufficient detail. To make the book self-contained, elements of structural dynamics are presented in Chapter 2.

A feature of the book which the designers would specially welcome is the inclusion of numerical examples.

A chapter each is devoted to the design of block and framed foundations. Vibration isola-tion and construction details have also received adequate attention.

Readers of this handbook who are intimately concerned with machine foundations may perhaps like to write to the authors about case histories of "sick" foundations and corrective measures which in their experience have proved effective. Although a section on case histories has been included in Chapter 7, a separate chapter on such documented case histories would certainly enhance the value of this work.

G. S. RAMASWAMY

Director
Structural Engineering Research Centre

Madras

AUTHORS' NOTE

A handbook giving the various aspects of analysis, design and construction of machine foundations is long overdue. Machine foundations form an important part of any industrial complex. To implement the programme of rapid industrialization, numerous machine foundations are being constructed in the various industrial establishments. The Structural Engineering Research Centre has received queries from many industrial units in public and private sectors for advice on design and performance of their machine foundations. It is realized that the literature available in the field is very meagre, and design engineers are not well acquainted with the theory of structural vibrations. It is, therefore, hoped that this handbook will fill the lacunae by serving as a useful reference manual for designers of machine foundations.

The scope of this handbook includes explanation of principles of planning, design and construction of machine foundations illustrated with practical examples. Chapter 1 presents the general background to the subject and the basic concepts. Chapter 2 deals with the vibration theory applied to single- and two-degree freedom systems subjected to free and forced vibrations. A general theoretical treatment of multiple-degree freedom systems is also included for the benefit of the interested readers. Chapter 3 describes the evaluation of design parameters by computation as well as by field testing. Chapter 4 groups the analysis and design of block foundations for machines subjected to impact type forces (e.g., hammers) and periodical forces (e.g., reciprocating machines). Chapter 5 deals with the analysis and design of framed foundations for high speed machinery such as turbo-generator sets. Chapter 6 gives the general principles of design of block foundations for other miscellaneous machines which cannot be distinctly classified. Chapter 7 deals with the principles of and methods for structural isolation of machine foundations. Chapter 8 gives the constructional details of machine foundations with explanatory sketches. Numerical examples illustrating the design principles are included in each chapter. Useful data for designers' ready reference and a select bibliography are appended at the end of the book.

The authors express their sincere gratitude to Prof. G. S. Ramaswamy, Director, Structural Engineering Research Centre, for giving constant encouragement during the preparation of this manual.

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The authors sincerely thank their colleague Mr. R. Narayanan for his assistance in checking the numerical examples.

The authors thank Mr. S. Azeez Ahmad for editing the text and Mr. N. Jayaraman for help in the production of the *Handbook.*

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Particular acknowledgement is due to the Publishers for their cooperation in various aspects of the editorial and production work. Special mention is made in this context to the cooperation received from Mr. Y. N. Arjuna, Associate Editor.

Needless to add, any comments or suggestions from the readers for the improvement of the book will be gratefully received.

Madras

P. Srinivasulu
C. V. Vaidyanathan

SYMBOLS & ABBREVIATIONS

a	Amplitude
a_1, a_2	Non-dimensional amplitude factors
a_a	Amplitude of anvil
a_f	Amplitude of foundation
A_f	Base area of a block foundation
A_a	Area of anvil base
A_b	Base area of concrete test block
A_b	Area of beam section
A_c	Area of column section
A_{st}	Area of steel reinforcement
b	Mass ratio
B	Width of foundation
B_i	Modified mass ratio for mode i
C	Damping coefficient
C_o	Critical damping coefficient
C_m	Factor for apparent soil mass
C_i	Factor for mass moment of inertia of soil
C_z	Coefficient of elastic uniform compression
C_τ	Coefficient of elastic uniform shear
C_θ	Coefficient of elastic non-uniform compression
C_ψ	Coefficient of elastic non-uniform shear
C_b	Centrifugal force on cross beam
C_c	Centrifugal force on column
d	Diameter of spring wire
d_s	Diameter of pressure bulb of soil
D	Diameter of spring coil
D_i	Distance of frame i from the first frame in framed foundations
e	Eccentricity of rotating mass
e	Eccentricity of centroid of base area from centre of gravity in block foundations
E	Modulus of elasticity
E_o	Impact energy
f_n	Natural frequency
f_{nr}	Reduced natural frequency
f_{n1}, f_{n2}	Coupled natural frequencies in a two-degree system
f_m	Operating speed of machine
F_d	Dynamic force
F_m	Inertial force
F_s	Short-circuit force
F_1, F_2	Functions used by Hsieh in block foundations
F_h	Horizontal dynamic force
F_t	Dynamic force transmitted
g	Acceleration due to gravity
G	Shear modulus
h	Height of spring coil
h	Height of equivalent surcharge of soil
h_o	Height of fall of hammer head
h_i	Height of column of frame i in framed foundation
H	Height of block foundation
i	Suffix to designate the mode of vibration or frame number
i	Imaginary unit $\sqrt{-1}$
I_o	Moment of inertia of base area of a block foundation
I_b	Moment of inertia of cross section of beam
I_c	Moment of inertia of cross section of column
I_x,I_y,I_z	Moment of inertia about x, y and z axes
I'	Moment of inertia of a group of isolated supports
J_1, J_2	Elastic layers used in the hammer foundation system
k	Coefficient of impact
K	Stiffness coefficient
K	Ratio of stiffness of beam and column in framed foundations
K_z	Stiffness of soil against translation in vertical (z) direction
$K_{\theta x}$,$K_{\theta y}$	Stiffness of soil against rotation about x and y axes
K_ψ	Stiffness against torsional oscillations
K_s	Spring stiffness
K_b	Lateral stiffness of cross frame in framed foundations
l	Effective span of frame beam
L	Length of foundation
m	Spring supported mass
m	Mass of machine and foundation
m_e	Eccentric mass

m_{reo} Reciprocating mass
m_{rot} Rotating mass
m_f Mass of foundation
m_s Apparent soil mass for translatory modes
m_t Mass of hammer head (tup)
m_{st} Mass of hammer frame
m_a Mass of anvil
M_0 Amplitude of rocking moment
M_d Dynamic moment
n Number of windings in a spring coil
n_1 Number of spring casings
n_2 Number of spring coils in each casing
N Total number of elastic supports
N Load on the frame columns
p Mean pressure on piston
P_0 Amplitude of exciting force
P Concentrated machine load
q Intensity of distributed load
Q Shear force
Q Total distributed load on cross beam ($=q_l$) in framed foundations
r Radius of crank
r Ratio of length of sides of a rectangular foundation
r_0 Radius of equivalent circular base (of foundation) for translatory modes
r_θ Radius of equivalent circular base for rotatory modes
r_ψ Radius of equivalent circular base for twisting mode
R_b Rotating weights on frame beams
R_c Rotating weights on frame columns
s Lever arm for frequency-dependent twisting moment
S Height of centre of gravity above the base of foundation
t Thickness of elastic layer
T Transmissibility
T_0 Amplitude of twisting moment
T^o Differential temperature in degrees
v Initial velocity of hammer head
V Velocity of the vibrating system after impact
V_s Velocity of shear wave
W Weight of machine and foundation
W_u Weight of upper tup in counter-blow hammers
W_l Weight of lower tup in counter-blow hammers
x, y, z Suffixes used to denote linear or rotatory motion in (or about) x, y and z axes respectively.
X_c Distance of centre of inertia from first frame in framed foundations
X_H Distance of centre of rigidity from first frame in framed foundations
X_{0G}, Y_{0G}, Z_{0G} Coordinates of centroid of base area
X_R Distance of centre of gravity of rotating loads from the first frame in framed foundations.
$\bar{X}, \bar{Y}, \bar{Z}$ Coordinates of centre of gravity of machine and foundation
Z Vertical height of the horizontal oscillating force above the common centre of gravity.

Greek letters

α A factor defined each time it appears in the text
α_1, α_2 Distance of centres of rotation in a block foundation measured from the base
α_0 A constant used by Ford and Haddow
$\alpha_z, \alpha_x, \alpha_\theta$ Stiffness factors of rectangular foundations for vertical, horizontal and rocking modes respectively
θ Angle of rotation
γ_z Factor for the vertical spring constant of soil (used in Pauw's method, chapter four)
$\gamma_{\theta x}$, $\gamma_{\theta y}$ Soil stiffness factors for rotation about x and y axes respectively (used in Pauw's method, chapter four)
β Rate of change of modulus of elasticity (E) of soil with depth
β Efficiency of spring absorbers
β_0 Decay factor used by Ford and Haddow
β' A factor used by Pauw
ν Poisson ratio
ζ Damping ratio (C/C_c)
ρ Mass density of soil
ϵ Thermal coefficient of expansion
δ_v Vertical displacement
δ_h Horizontal displacement
η Frequency ratio in a single-degree system
η_1, η_2 Frequency ratios in a two-degree system
σ_p Permissible soil stress
σ_{st} Stress due to static loads
σ_d Stress due to dynamic loads
τ Shear stress
ω_n Natural frequency (sec^{-1})
ω_m Operating frequency of machine (sec^{-1})
ω_{n1}, ω_{n2} Circular natural frequencies in a two-degree system
ω_a, ω_z Limiting frequencies in hammer foundation–soil systems
$\bar{\omega}_{n1}$, $\bar{\omega}_{n2}$ Limiting circular frequencies in a two-degree system for vertical vibrations

ω_{z}, ω_{s}	Limiting circular frequencies in coupled sliding and rocking system
μ	Dynamic factor
ξ	Fatigue factor
φ	Mass moment of inertia about the axis passing through the common centre of gravity of machine foundation and perpendicular to the plane of vibration
ϕ	Diameter of reinforcing bar (used in figures)
φ_{0}	Mass moment of inertia about a parallel axis passing through centroid of base area of foundation
Δ	Logarithmic decrement

Abbreviations used in the text

cm	Centimetre
cps	Number of cycles per second
cpm	Number of cycles per minute
m	Metre
min	Minimum
max	Maximum
mm	Millimetre
rpm	Revolutions per minute

CONTENTS

Foreword .. v
Acknowledgements .. vi
Preface .. vii
Authors' Note .. ix
Symbols and Abbreviations .. xi

CHAPTER ONE **INTRODUCTION** 1

1.1 Types of Machine Foundations 1
1.2 General Requirements of Machine Foundations 2
1.3 Dimensional Criteria 3
1.4 Design Data 3
1.5 Dynamic Loads Induced in Simple Crank Mechanisms 4
1.6 Permissible Amplitudes 6
1.7 Permissible Bearing Pressures 7

CHAPTER TWO **GENERAL THEORY** 8

2.1 Resonance and its Effect 8
2.2 Theory of a Single-Degree Freedom System 8
2.3 Theory of a Two-Degree Freedom System 13
2.4 Multiple-Degree Freedom System 21
2.5 Transient Response 27

CHAPTER THREE **EVALUATION OF DESIGN PARAMETERS** 30

3.1 Importance of Design Parameters 30
3.2 Geometrical Properties of Machine Foundations 31
3.3 Physical Properties of the Elastic Base and their Experimental Evaluation 33

CHAPTER FOUR **ANALYSIS AND DESIGN OF BLOCK-TYPE MACHINE FOUNDATIONS** 51

4.1 Modes of Vibration of a Block Foundation 51
4.2 Review of Methods for Dynamic Analysis 52
4.3 Recommended Method of Analysis for Block Foundations 69
4.4 Foundations for Machines Inducing Periodical Forces (Example : Reciprocating Machinery) 74
4.5 Foundations Subject to Impact-Type Forces (Example : Hammers) 103

CHAPTER FIVE **ANALYSIS AND DESIGN OF FRAMED FOUNDATIONS FOR HIGH-SPEED MACHINERY** 135

5.1 Design Data 136

5.2 Special Considerations in Planning 136
5.3 Principal Design Criteria 138
5.4 Dynamic Analysis 138
5.5 Structural Design 149
5.6 Numerical Example for Design of Framed Foundation for a 200 MW Turbo-Generator 153

CHAPTER SIX FOUNDATIONS FOR MISCELLANEOUS MACHINES 175

6.1 Rotary-Type Machines with Low Frequency 175
6.2 Machine Tools 180
6.3 Impact-Type Machines—other than Hammers 181
6.4 Fans and Blowers 183
6.5 Looms 184
6.6 Testing Machine with Pulsator 185
6.7 Machines Installed on Building Floors 185
6.8 Numerical Examples 186

CHAPTER SEVEN VIBRATION ISOLATION 192

7.1 Active and Passive Types of Isolation—Transmissibility 193
7.2 Methods of Isolation in Machine Foundations 195
7.3 Isolation in Existing Machine Foundations 195
7.4 Case Histories 202
7.5 Properties of Isolating Materials 205
7.6 Numerical Examples 207

CHAPTER EIGHT CONSTRUCTIONAL DETAILS OF MACHINE FOUNDATIONS 211

8.1 Concreting 211
8.2 Reinforcement 211
8.3 Expansion Joints 213
8.4 Coonecting Elements 213
8.5 Methods of Laying Spring Absorbers 215
8.6 Provision for Tuning 217

APPENDIX A Useful Data for Ready Reference 218
APPENDIX B Terminology 226
APPENDIX C Selected Bibliography 229

Index 235
237

CHAPTER ONE

Introduction

THE DESIGN of a machine foundation is more complex than that of a foundation which supports only static loads. In machine foundations, the designer must consider, in addition to the static loads, the dynamic forces caused by the working of the machine. These dynamic forces are, in turn, transmitted to the foundation supporting the machine. The designer should, therefore, be well conversant with the method of load transmission from the machine as well as with the problems concerning the dynamic behaviour of the foundation and the soil underneath the foundation.

That the knowledge in this field has lagged behind other branches of technology is partly due to the fact that the responsibility for satisfactory performance of a machine is divided between the machine designer, who is usually a mechanical engineer, and the foundation designer, whose task is to design a suitable foundation consistent with the mechanical requirements and satisfying the required tolerances. It is, therefore, desirable that the mechanical and civil engineers work in close coordination from the planning stage until the machinery is installed on the foundation.

Until recently, the practice in design offices for the design of machine foundations has been almost entirely based on empirical rules, since very little was known about the behaviour of foundations subjected to dynamic loads. With the developments in the fields of soil and structural dynamics, the design principles were gradually established without dependence on mere empirical methods. The object of this manual is to present these design criteria in such a manner that the designer may find them convenient for application to practical problems.

1.1 Types of Machine Foundations

Based on the design criteria of their foundations, machines may be classified as follows:

a. Those producing impact forces, e.g., forge hammers, presses.
b. Those producing periodical forces, e.g., reciprocating engines such as compressors.
c. High speed machinery such as turbines and rotary compressors.
d. Other miscellaneous machines.

Considering tbeir structural form, machine foundations are generally classified as follows:

a. Block-type foundations (Fig. 1.1a) consisting of a pedestal of concrete on which the machine rests.
b. Box or caisson-type foundations (Fig. 1.1b) consisting of a bollow concrete block supporting the macbinery on its top.
c. Wall-type foundations (Fig. 1.1c) consisting of a pair of walls which support the macbinery on their top.
d. Framed-type foundations (Fig. 1.1d) consisting of vertical columns supporting on their top a horizontal frame-work which forms tbe seat of essential machinery.

Machines producing impulsive and periodical forces at low speeds are generally mounted on block-type foundations, while those working at high speeds and the rotating type of machinery are generally mounted on framed foundations. However, owing to certain local conditions, this may not always be possible, in which case alternative types may be adopted as found suitable.

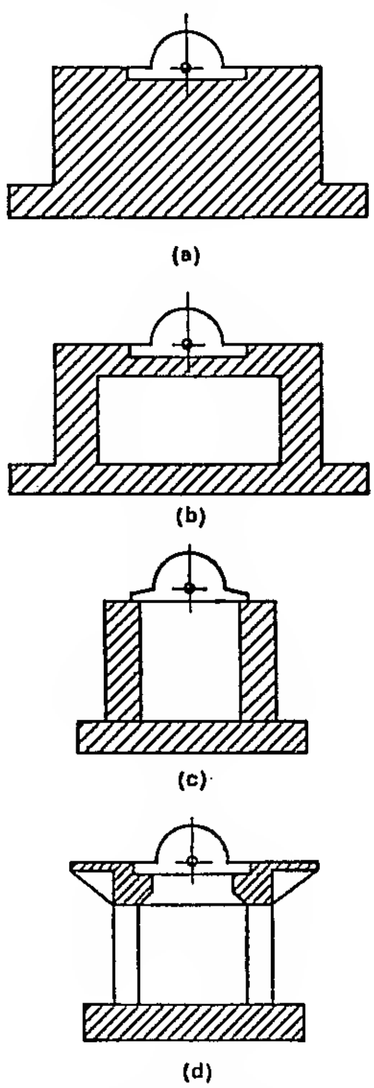


Fig. 1.1: Types of Machine Foundations—(a) Block-Type, (b) Box-Type, (c) Wall-Type, (d) Framed-Type.

Certain machines such as lathes, which induce very little dynamic force, may be bolted directly to the floor without special foundations.

Based on their operating frequeny, machines may be dividcd into thrce categories:

i.	Low to medium frequencies:	0–500 rpm
ii.	Medium to high frequencies:	300–1000 rpm
iii.	Very high frequencies:	Greater than 1000 rpm

Group i comprises of large reciprocating engines, compressors and large blowers. Reciprocating engines generally operatc at frequencies ranging within 50–250 rpm. For this group, foundations of block type with large contact area with the soil are generally adopted.

Group ii consists of foundations of medium-sized reciprocating engines such as diesel and gas engines. Block foundations resting on springs or suitable elastic pads are generally suggested for this group in order to maintain the natural frequencies of the foundation considerably below the operating frequency.

Group iii includes high-speed internal combustion engines, electric motors and turbogenerator sets. Where massive block foundations are used, small contact surfaces and suitable isolation pads are desirable to lower the natural frequencies. Turbo-machinery requires framed-type foundations which accommodate tbe necessary auxiliary equipment between the columns.

1.2 General Requirements of Machine Foundations

The following requirements should be satisfied from tbe design point of view:

a. The foundation should be able to carry the superimposed loads without causing shear or crushing failure.

b. The settlements should be within the permissible limits.
c. The combined centre of gravity of machine and foundation should as far as possible be in the same vertical line as the centre of gravity of the base plane.
d. No resonance should occur, hence the natural frequency of foundation-soil system should be either too large or too small compared to the operating frequency of the machine. For low-speed machines, the natural frequency should be high, and vice-versa.
e. The amplitudes under service conditions should be within permissible limits. The permissible limits are generally prescribed by the machine manufacturers.
f. All rotating and reciprocating parts of a machine should be so well balanced as to minimize the unbalanced forces or moments. This is generally the responsibility of the mechanical engineers.
g. Where possible, the foundation should be planned in such a manner as to permit a subsequent alteration of natural frequency by changing base area or mass of the foundation as may subsequently be found necessary.

From the practical point of view, the following requirements should be fulfilled:

a. The ground-water table should be as low as possible and ground-water level deeper by at least one-fourth of the width of foundation below the base plane. This limits the vibration propagation, ground-water being a good conductor of vibration waves.

V. Imp.

b. Machine foundations should be separated from adjacent building components by means of expansion joints.
c. Any steam or hot air pipes, embedded in the foundation must be properly isolated.

V. imp

d. The foundation must be protected from machine oil by means of acid-resisting coating or suitable chemical treatment.
e. Machine foundations should be taken to a level lower than the level of the foundations of adjoining buildings.

1.3 Dimensional Criteria

The dimensions of machine foundations are fixed according to the operational requirements of the machine. The outline dimensions of the foundation are generally provided by the machine manufacturers. If the choice of dimensions is assigned to the designer, the minimum possible dimensions of the foundation satisfying the design criteria should be selected.

Given the dimensions of the foundation and the particular site conditions, the designer must ascertain the natural frequency of the foundation-soil system and the amplitudes of its motion under operating conditions. For satisfactory design, the requirements explained under Section 1.2 should be satisfied. If the design requirements are not satisfied, the designer may suggest alterations in the dimensions of the foundation suggested by the machine suppliers. Any such alterations should, however, be approved by the mechanical engineers concerned.

1.4 Design Data

The specific data required for design vary depending upon the type of machine. The general requirements of data for the design of machine foundations are, however, as follows:

a. Loading diagram showing the magnitude and positions of static and dynamic loads exerted by the machine on its foundation.
b. Power of engine and the operating speed.
c. Diagram showing the embedded parts, openings, grooves for foundation bolts, etc.
d. Nature of soil and its static and dynamic properties as required in design calculations.

1.5 Dynamic Loads Induced in Simple Crank Mechanisms

Before a satisfactory design can be made for a machine foundation, it is necessary to obtain complete information about the magnitude and characteristics of dynamic loads involved.

Dynamic loads exerted on a machine foundation can be generally classified into two categories:

a. Shock or impulsive type of loads which occur at regular intervals (e.g., hammers and presses).
b. Steady-state loads which vary with time according to sine or cosine law (e.g., reciprocating and rotating machines).

The machinc manufacturers gcncrally furnish data concerning the unbalanced forces. For certain basic types of equipmcnt, however, the unbalanced forces can bc calculated. A bricf discussion on how the unbalanced forccs arc calculated for simplc mechanisms is outlined bclow:

i. Internal combustion engines, piston-type compressors, pumps, steam engines, etc., produce rcciprocating forces. The crank mechanism convcrts the reciprocating motion to a rotary motion, and vicc-versa. A simple crank mechanism for a single-cylinder engine is shown in Fig. 1.2. It consists of a piston which moves insidc a cylindcr, a crank of length r which rotates about a point O and a connecting rod of length l which is attached to the piston at point P (known as "wrist pin") and to the crank shaft at point C (known as "crank pin"). The crank pin C follows a circular path while the wrist pin P oscillates along a linear path. Points on the connecting rod between C and P follow an elliptical path.

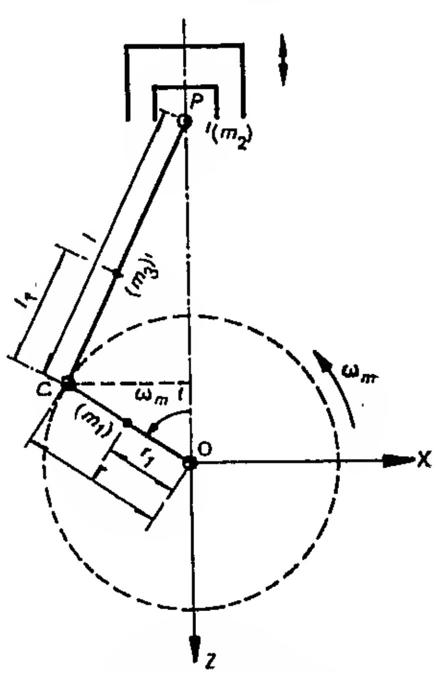


Fig. 1.2: Simple Crank Mechanism.

Designating the total reciprocating mass which moves with the piston as m_{rec} and the rotating mass moving with the crank as m_{rot}, the unbalanced inertial forces P_z (along the direction of piston) and P_x (along the perpendicular direction) can be written as

$$P_z = (m_{rec} + m_{rot})\, r\, \omega_m^2 \cos \omega_m t + m_{rec} \frac{r^2 \omega_m^2}{l} \cos 2\, \omega_m t \tag{1.1}$$

and

$$P_x = m_{rot}\, r \omega_m^2 \sin \omega_m t \tag{1.2}$$

Where ω_m is the angular speed of rotation and r is the radius of crank. For the simple crank mechanism considered (Fig. 1.2) the reciprocating and rotating masses are given by the following relations:

$$m_{rec} = m_2 + m_3 \left(\frac{l_1}{l} \right) \tag{1.3}$$

$$m_{rot} = \frac{r_1 m_1}{r} + \left(1 - \frac{l_1}{l}\right) m_3 \qquad (1.4)$$

where

m_1 — mass of crank

m_2 — mass of reciprocating parts, viz., piston, piston rod and crank head

m_3 — mass of connecting rod

l — length of connecting rod

l_1 — distance between the centre of gravity of connecting rod and the crank pin C

r_1 — distance between the centre of gravity of crank shaft and the centre of rotation.

The expression for the inertial force along the axis of the piston (Eq. 1.1) has, besides a primary component working at the frequency of rotation (ω_m), a secondary component working at twice that frequency ($2\omega_m$).

The inertial force due to rotating masses can be eliminated completely by an operation known as "counter-balancing." Imbalance due to reciprocating mass cannot, however, be avoided. In multi-cylinder engines, it is possible to arrange the cylinders in a manner such that the unbalanced forces are minimized.

For a particular machine, the unbalanced primary and secondary forces are supplied by the machine manufacturer. Alternatively, the designer should be provided with all the data (e.g. weights of reciprocating and rotating parts) necessary for their calculation.

ii. Turbines, centrifugal pumps and turbo-generator sets are examples of rotating machinery. Although rotating machinery is balanced before erection, in actual operation some imbalance always exists. Here, imbalance in a rotating machine means the axis of rotation not passing through the principal axis of inertia of the whole unit. In the case of high-speed machines such as turbines, even a small eccentricity can produce large unbalanced forces.

Fig. 1.3a shows a typical rotating mass type oscillator in which a single mass m_e is placed on a rotating shaft at an eccentricity e from the axis of rotation. Such an arrangement causes an unbalanced vertical force given by

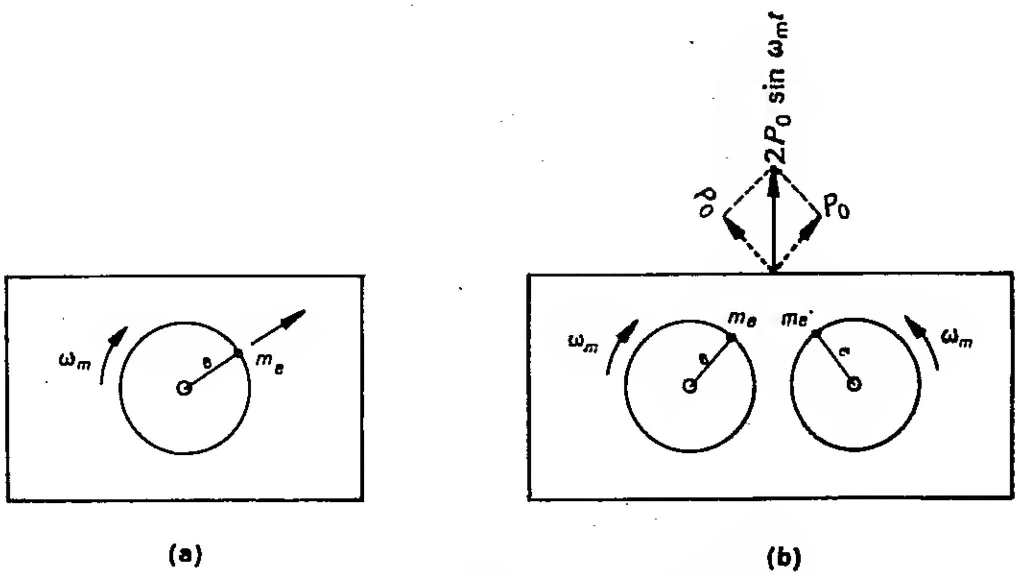


Fig. 1.3: Rotating Mass Type Oscillator with: (a) Single Shaft, (b) Two Shafts.

$$P = m_e e\, \omega_m^2 \sin \omega_m t \tag{1.5}$$

Fig. 1.3b shows two equal masses mounted on two parallel shafts at the same eccentricity, the shafts rotating in opposite directions with the same angular velocity. Such an arrangement produces an oscillating force with a controlled direction.

For the arrangement shown in this figure horizontal force components cancel and the vertical components are added up. If each mass m_e has an eccentricity e, then the amplitude of vertical oscillating force P_0 produced is $2\, m_e e\, \omega_m^2$. This principle is applied in the mechanical oscillators employed for dynamic soil testing which will be explained in Section 3.3.

1.6 Permissible Amplitudes

The permissible amplitudes are generally specified by the manufacturers of machinery. The permissible amplitude of a machine foundation is governed by the relative importance of the machine and the sensitivity of neighbouring structures to vibration.

Where manufacturer's data does not contain the permissible amplitudes, the values shown in Fig. 1.4 suggested by Richart may be adopted for preliminary designs. The envelope described by the shaded line in Fig. 1.4 indicates only a limit for "safety" and not

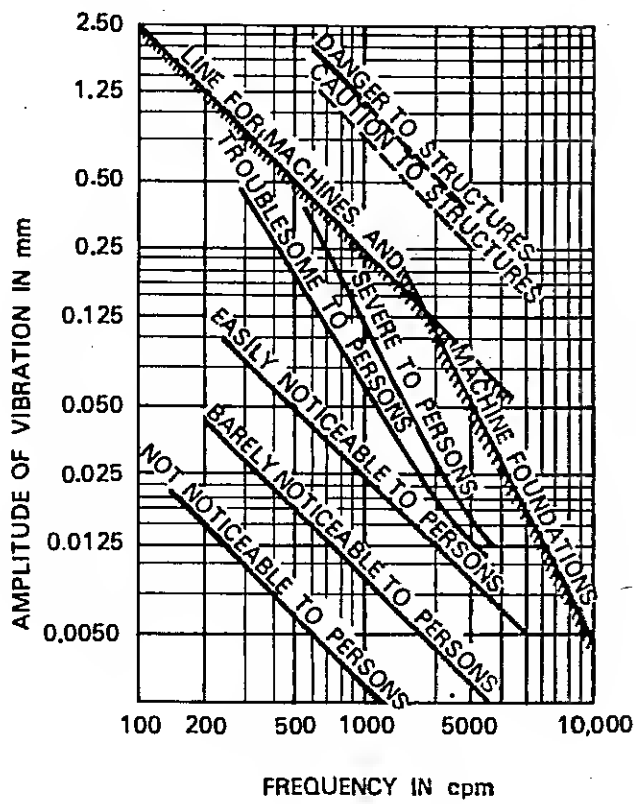


Fig. 1.4: Allowable Limits for Vertical Vibration Amplitudes (After Richart, F. E., Jr. *et al.*, *Vibration of Soils and Foundations*, Prentice-Hall Inc., New Jersey, USA, 1970; with permission).

a limit for satisfactory operation of machines. The latter can be furnished only by the suppliers of the machinery. Barkan[C1.1] has proposed the following values from his observations of machine performance:

For foundations of sensitive equipment such as calibration test stands and precision machines, the design criteria should be established either by the user or equipment manu-

Type	Permissible amplitudes (cm)
1. Low-speed machinery (500 rpm)	0.02 to 0.025
2. Hammer foundations	0.1 to 0.12
3. High-speed machinery:	
a. 3000 rpm	
i. Vertical vibrations	0.002 to 0.003
ii. Horizontal vibrations	0.004 to 0.005
b. 1500 rpm	
i. Vertical vibrations	0.004 to 0.006
ii. Horizontal vibrations	0.007 to 0.009

facturer. For such installations in which the equipment itself is not the source of vibration, it is necessary to evaluate the ambient vibrations at the site and provide suitable isolation in order to contain the amplitudes of movement within acceptable limits.

1.7 Permissible Bearing Pressures

a. Soil

The bearing pressure on soil should be evaluated by adequate sub-soil exploration and testing in accordance with IS:1892 and IS: 1904 or equivalent specification.

b. Timber

The permissible compressive stress on timber (generally used under the anvil of hammer foundations) may be taken from IS: 883-1966 (Indian Standard Code of Practice for Use of Structural Timber in Buildings) or equivalent specification.

c. Other Materials

The permissible bearing pressures on other elastic materials such as felt, cork and rubber are generally given by the firms manufacturing these materials. No specific values are suggested here since they vary in wide limits.

CHAPTER TWO

General Theory

2.1 Resonance and its Effect

ANY PHYSICAL system has a characteristic frequency of its own known as "natural frequency." This is defined as the frequency at which the system would vibrate when subjected to free vibrations. As the operating frequency of a machine approaches the natural frequency of its foundation, the amplitudes tend to become large. The system is said to be in "resonance" when the two frequencies become equal. At resonance, it is found that in addition to excessive amplitudes, large settlements also occur.

In the design of machine foundations, an important criterion is, therefore, to avoid resonance in order that the amplitudes of vibration may not be excessive. The occurrence of resonance can be mathematically explained by considering a simple case of a single-degree freedom system.

2.2 Theory of a Single-Degree Freedom System

Consider a single-degree freedom system (Fig. 2.1) formed by a rigid mass m resting on a spring of stiffness K and a viscous damper having damping coefficient C. A system is said to be of a single degree when its motion is constrained to one direction only.

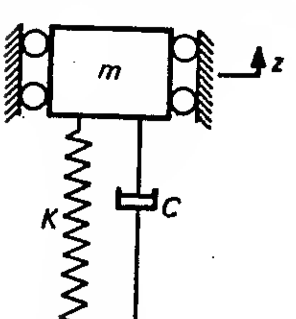


Fig. 2.1: Single-Degree Freedom System.

a. Free Vibrations

Let the system be set in motion by giving an initial velocity V to the mass. The equation of motion for the free vibration of the system is

$$\underbrace{m\ddot{z}}_{\text{inertial force}} + \underbrace{C\dot{z}}_{\text{damping force}} + \underbrace{Kz}_{\text{spring force}} = 0 \qquad (2.1)$$

In Eq. 2.1, z denotes the displacement, $\dot{z}$ the velocity and $\ddot{z}$ the acceleration of the mass. The right-hand side is zero since there

is no external force on the system during vibration.

The solution of Eq. 2.1 can be written as

$$z = a_d\, e^{-Ct/2m} \sin \sqrt{\frac{K}{m} - \frac{C^2}{4m^2}}\, t \tag{2.2}$$

where a_d is a constant which represents the maximum displacement and known as "free amplitude" of the damped system. The frequency of oscillation (ω_{nd}) is given by

$$\omega_{nd} = \sqrt{\frac{K}{m} - \frac{C^2}{4m^2}} \tag{2.3}$$

where ω_{nd} denotes the natural frequency of a damped single-degree freedom system.

To obtain the free amplitude a_d, the initial conditions at the time when the motion was set in, should be considered, i.e. when $t = 0$, $z = 0$ and $\dot{z} = V$.
Substituting

$$a_d = V \Big/ \sqrt{\frac{K}{m} - \frac{C^2}{4m^2}} \tag{2.4}$$

Substituting $C_c = 2\sqrt{km}$ where C_c is called the "critical damping" and $C/C_c = \zeta$, where ζ is called the "damping ratio" Eqs. 2.3 and 2.4 give

$$\omega_{nd} = \sqrt{\frac{K}{m}}\ \sqrt{1-\zeta^2} \tag{2.5}$$

and

$$a_d = V \Big/ \sqrt{\frac{K}{m}(1-\zeta^2)} \tag{2.6}$$

Eqs. 2.5 and 2.6 give the "damped natural frequency" and the "damped free amplitude" of a single-degree system.

Corollary: If damping is neglected, $C = 0$, or $\zeta = 0$. Dropping the suffix "d" to denote the undamped case, Eqs. 2.5 and 2.6 reduce to

$$\omega_n = \sqrt{\frac{K}{m}} \tag{2.7}$$

and

$$a = V\sqrt{\frac{m}{K}} \tag{2.8}$$

Application: The relations given above for the undamped single-degree system undergoing free vibrations will be used in the analysis of hammer foundations (see examples given under Section 4.5.7).

b. Forced Vibrations

Let the system shown in Fig. 2.1 be subjected to a harmonic exciting force $P_0 \sin \omega_m t$. Depending on the type of excitation, two cases can be considered—one case in which the amplitude of excitation is constant and the other in which the amplitude is proportional

to the square of the circular operating frequency ω_m. As discussed earlier under Section 1.5, the latter case arises in reciprocating or unbalanced rotating mechanisms. This case is of main interest in machine foundations.

i. *Constant Force Excitation*

The amplitude of exciting force (P_0) is constant, i.e., independent of forcing frequency in this case.

The equation of motion of a damped single-degree freedom system subjected to forced excitation can be written as

$$\underset{\substack{\text{inertial}\\ \text{force}}}{m\ddot{z}} + \underset{\substack{\text{damping}\\ \text{force}}}{C\dot{z}} + \underset{\substack{\text{spring}\\ \text{force}}}{Kz} = \underset{\substack{\text{exciting}\\ \text{force}}}{P_0 \sin \omega_m t} \tag{2.9}$$

where P_0 is the amplitude of exciting force.

Under steady-state forced excitation, the system has a tendency to vibrate at the operating frequency ω_m. The solution of Eq. 2.9 under steady-state conditions (neglecting the transient part corresponding to free vibrations) may, therefore, be expressed as

$$z = a_d \sin (\omega_m t + \alpha) \tag{2.10}$$

where a_d is amplitude and α is the phase difference between the exciting force and displacement.

Substituting Eq. 2.10 in Eq. 2.9 and solving the following expressions for a_d and α can be obtained:

$$a_d = \frac{P_0}{\sqrt{(K - m\omega_m^2)^2 + C^2 \omega_m^2}} \tag{2.11a}$$

$$\tan \alpha = \frac{C\omega_m}{K - m\omega_m^2} \tag{2.11b}$$

Substituting

$$\omega_n^2 = K/m, \ \zeta = C/(2\sqrt{Km}) \text{ and } \eta = \omega_m/\omega_n$$

Eqs. 2.11a and 2.11b can be reduced to:

$$a_d = \frac{P_0}{K\sqrt{(1-\eta^2)^2 + (2\eta\zeta)^2}} \tag{2.12a}$$

$$\tan \alpha = \frac{2\eta\zeta}{1-\eta^2} \tag{2.12b}$$

Substituting $P/K = z_{st}$, the static displacement Eq. 2.12a can be written as

$$z = z_{st}\,\mu \tag{2.13a}$$

where

$$\mu = \frac{1}{\sqrt{(1-\eta^2)^2 + (2\eta\zeta)^2}} \tag{2.13b}$$

Here μ is called the "dynamic magnification factor." Fig. 2.2a shows the variation of μ with η (Eq. 2.13b) for various values of ζ.

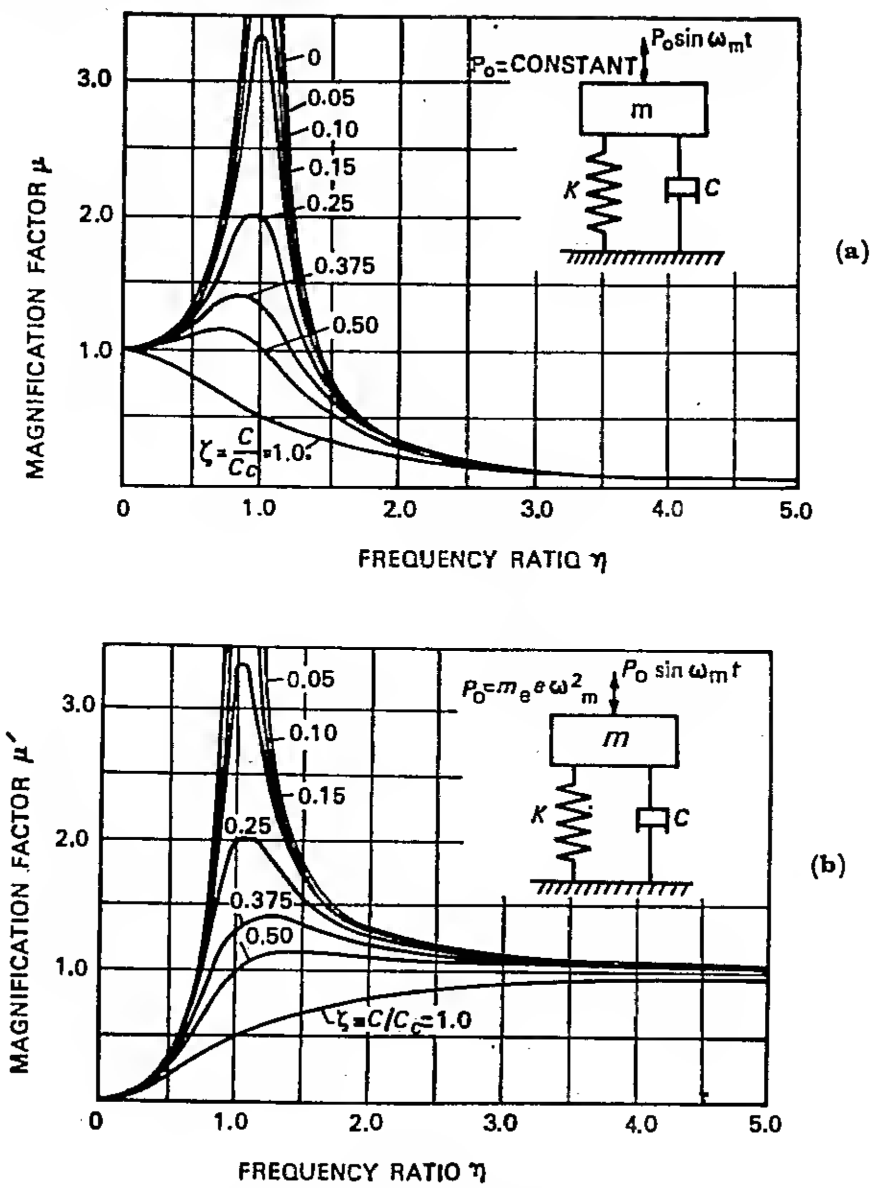


Fig. 2.2: Response of a Single-Degree Damped System under—(a) Constant Force Excitation (Eq. 2.13b), (b) Rotating Mass Type Excitation (Eq. 2.18).

ii. *Rotating Mass Type Excitation*

As seen under Section 1.5, the exciting force P in the case of reciprocating or unbalanced rotating mass type excitation is of the form

$$P = (m_e e \omega_m^2) \sin \omega_m t \tag{2.14}$$

where m_e is the reciprocating or unbalanced rotating mass, e denotes the displacement in the case of reciprocating type and eccentricity of unbalanced mass in the case of rotating-

type mechanisms, and ω_m is frequency of motion. The amplitude of exciting force P_0 $(= m_e e\omega_m^2)$ in this case is directly proportional to the square of operating frequency (ω_m).

The equation of motion for a single degree of freedom system subjected to this type of forced excitation is written as

$$m\ddot{z} + C\dot{z} + Kz = (m_e e\omega_m^2) \sin \omega_m t \tag{2.15}$$

Substituting for $P_0 = m_e e\omega_m^2$ in Eq. 2.11a, the solution becomes

$$a_d = \frac{m_e e\omega_m^2}{\sqrt{(K-m\omega_m^2)^2+C^2\,\omega_m^2}} \tag{2.16}$$

Substituting $\omega_n^2 = K/m$; $\zeta = \dfrac{C}{2\sqrt{Km}}$ and $\eta = \omega_m/\omega_n$

Eq. 2.16 gives

$$\frac{a_d}{m_e e/m} = \eta^2 \frac{1}{\sqrt{(1-\eta^2)^2 + (2\eta\zeta)^2}} \tag{2.17}$$

or

$$\mu' = \eta^2\mu \tag{2.18}$$

where μ' is the "magnification factor" defined by the left-hand side of Eq. 2.17; μ is the magnification factor for the corresponding case of constant force excitation (Eq. 2.13b). Fig. 2.2b shows the variation of μ' with η (Eq. 2.18) for various values of ζ.

The expression for α is the same as that given in Eq. 2.12b.

Corollary: When damping in the system is neglected, i.e., $C = 0$ or $\zeta = 0$, then

$$\mu = \frac{1}{1-\eta^2} \text{ for constant force excitation} \tag{2.19a}$$

and

$$\mu' = \frac{\eta^2}{1-\eta^2} \text{ for rotating-mass type excitation} \tag{2.19b}$$

Further when $\eta = 1$, both μ and μ' become infinity. This marks the stage of "resonance."

In practice, the amplitude at resonance will be finite because of damping which is inherently present in any physical system. It is, however, desirable to ensure in the design of any dynamically loaded structure that the value of frequency ratio η, is far from unity. According to IS: 2974 (Pt. I), the working range for the frequency ratio η is given by the inequality

$$1.4 < \eta < 0.5 \tag{2.20}$$

Figs. 2.2a, b show the amplitude–frequency relations for damped-forced vibration of a mass-spring system under the action of constant force type and rotating mass type excitations.

As can be seen from the diagrams, the curves for the two sets of cases are similar in appearance. It may be noticed, however, that the resonant peaks for increasing values of damping gradually shift away from the ordinate at $\eta = 1$. The peaks occur at values of η less than one in the case of constant force excitation and at values of η greater than one in the case of rotating mass type excitation.

The expressions for resonant frequency and amplitudes for a viscously damped single-degree freedom system for the two cases are given in Table 2.1.

Table 2.1

RELATIONS FOR A SINGLE-DEGREE OF FREEDOM SYSTEM

	Constant force excitation (P_0=constant)	Rotating mass type excitation ($P_0 = m_e e \omega_m^2$)
Resonant frequency	$f_n \sqrt{1-2\zeta^2}$	$f_n \dfrac{1}{\sqrt{1-2\zeta^2}}$
Amplitude at frequency f	$\dfrac{P_0}{K}\left[\dfrac{1}{(1-\eta^2)^2+(2\eta\zeta)^2}\right]^{\frac{1}{2}}$	$\dfrac{m_e e}{m}\eta^2\left[\dfrac{1}{(1-\eta^2)^2+(2\eta\zeta)^2}\right]^{\frac{1}{2}}$
Maximum amplitude of vibration	$\dfrac{P_0}{K}\,\dfrac{1}{2\zeta}\,\dfrac{1}{\sqrt{(1-\zeta^2)}}$	$\dfrac{m_e e}{m}\,\dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$

where

Undamped natural frequency $(f_n) = \dfrac{1}{2\pi}\sqrt{\dfrac{K}{m}}$

Damping ratio $(\zeta) = C/C_c$

Critical damping $(C_c) = 2\sqrt{Km}$

Application: The theory of a single mass spring system under forced vibrations is used in the analysis of block foundations for reciprocating or rotating type of machinery (Section 4.4).

2.3 Theory of a Two-Degree Freedom System

2.3.1 Undamped Case

a. Free Vibrations

Fig. 2.3a shows a two-degree freedom system consisting of masses m_1 and m_2 and springs having stiffnesses K_1 and K_2. Free vibrations are induced in the system by giving an initial velocity or displacement to one of the masses. The differential equations characterizing the motion of the masses m_1 and m_2 are given by

$$m_1 \ddot{z}_1 + K_1 z_1 + K_2 (z_1 - z_2) = 0 \tag{2.21a}$$

$$m_2 \ddot{z}_2 + K_2 (z_2 - z_1) = 0 \tag{2.21b}$$

Let ω_{n_1} and ω_{n_2} be the circular natural frequencies of the system. It can be derived that ω_{n_1} and ω_{n_2} are the roots of the following fourth-order equation (quadratic in ω_n^2)

$$f(\omega_n^2)=\omega_n^4-(\bar{\omega}_{n1}^2+\bar{\omega}_{n2}^2)\,(1+\alpha)\,\omega_n^2+(1+\alpha)\,\bar{\omega}_{n1}^2\,\bar{\omega}_{n2}^2=0 \tag{2.22}$$

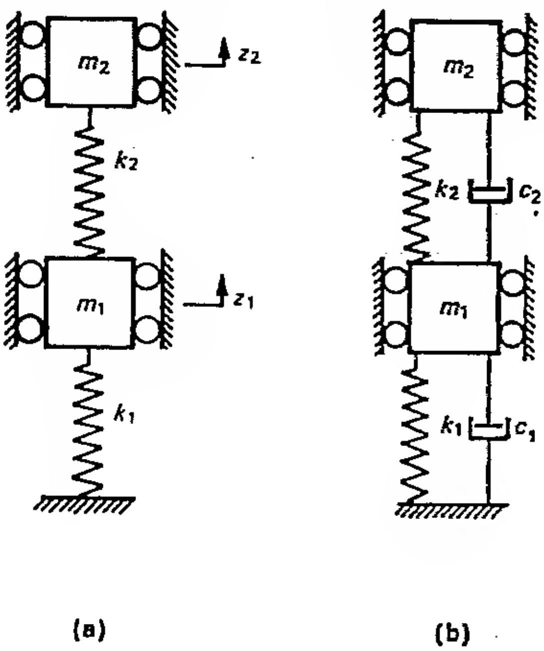


Fig. 2.3: Two-Degree Freedom System—**(a)** Without Damping, **(b)** With Damping.

where $\bar{\omega}_{n1}$ and $\bar{\omega}_{n2}$ are the "limiting frequencies" which are defined as follows:

$$\bar{\omega}_{n2}^2=\frac{K_2}{m_2} \tag{2.23a}$$

$$\bar{\omega}_{n1}^2=\frac{K_1}{m_1+m_2} \tag{2.23b}$$

and

$$\alpha=\frac{m_2}{m_1} \tag{2.24}$$

$\bar{\omega}_{n2}$ is the frequency of the system when stiffness K_1 is assumed to be infinity (bottom spring is rigid) and $\bar{\omega}_{n1}$ is the frequency when K_2 is assumed to be infinity.
Eq. 2.22 may be rewritten as

$$\left(\frac{\omega_n}{\bar{\omega}_{n2}}\right)^4-\left(\frac{\omega_n}{\bar{\omega}_{n2}}\right)^2\left(\frac{\alpha+\beta+\alpha\beta}{\beta}\right)+\frac{\alpha}{\beta}=0 \tag{2.25}$$

where $\beta=K_2/K_1$

Eq. 2.25 is a quadratic in $\left(\dfrac{\omega_n}{\bar{\omega}_{n2}}\right)^2$ and gives two real roots for ω_n which are the two circular natural frequencies of the system. Fig. 2.4 shows the variation of $\dfrac{\omega_n}{\omega_{n2}}$ with the mass ratio α for the particular case where $\beta=\alpha$ or $\dfrac{K_2}{K_1}=\dfrac{m_2}{m_1}$. The practical significance of this particular case will be explained in Sec. 7.3c when dealing with vibration isolation

in existing machine foundations. It may be observed from Fig. 2.4 that the smaller the mass ratio α the closer are the two natural frequencies.

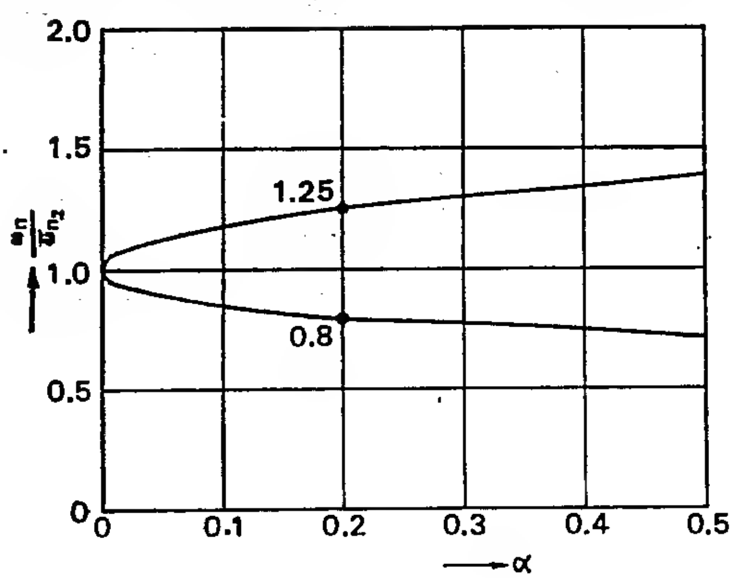


Fig. 2.4: Variation of Frequency Ratio η with Mass Ratio α for the Case when $\frac{K_2}{K_1} = \frac{m_2}{m_1}$.

The free motion of the two masses can be expressed as

$$z_1 = A_1 \sin \omega_{n_1} t + A_2 \sin \omega_{n_2} t \tag{2.26a}$$

and

$$z_2 = B_1 \sin \omega_{n_1} t + B_2 \sin \omega_{n_2} t \tag{2.26b}$$

To obtain the free amplitudes of the two masses, the initial conditions should be used. Let it be assumed that the free vibrations are set in when the upper mass is given an initial velocity V. This case will be illustrated in the analysis of hammer foundations.

The initial conditions to be used are:

At time $t=0$

$$z_1 = z_2 = 0 \tag{2.27a}$$

$$\dot{z}_2 = V \text{ and } \dot{z}_1 = 0 \tag{2.27b}$$

Using these initial conditions, the displacements z_1 and z_2 of masses m_1 and m_2 can be expressed as

$$z_1 = \frac{(\bar{\omega}_{n2}^2 - \omega_{n2}^2)\,(\bar{\omega}_{n2}^2 - \omega_{n1}^2)}{\bar{\omega}_{n2}^2\,(\omega_{n1}^2 - \omega_{n2}^2)}\, V \left\{ \frac{\sin \omega_{n1} t}{\omega_{n1}} - \frac{\sin \omega_{n2} t}{\omega_{n2}} \right\} \tag{2.28a}$$

$$z_2 = \frac{V}{(\omega_{n1}^2 - \omega_{n2}^2)} \left\{ \frac{(\bar{\omega}_{n2}^2 - \omega_{n2}^2)}{\omega_{n1}} \sin \omega_{n1} t - \frac{(\bar{\omega}_{n2}^2 - \omega_{n1}^2)}{\omega_{n2}} \sin \omega_{n2} t \right\} \tag{2.28b}$$

Usually the amplitudes associated with the higher of the two frequencies ω_{n1} and ω_{n2} will be small. If $\omega_{n1} > \omega_{n2}$, neglecting the part contributed by the higher natural frequency, the amplitudes a_1 and a_2 can be written as

$$a_1 = -\frac{(\bar{\omega}_{n2}^2 - \omega_{n2}^2)\,(\bar{\omega}_{n2}^2 - \omega_{n1}^2)}{\bar{\omega}_{n2}^2\,(\omega_{n1}^2 - \omega_{n2}^2)}\, \frac{V}{\omega_{n2}} \tag{2.29a}$$

and
$$a_2 = -\frac{(\bar{\omega}_{n2}^2 - \omega_{n1}^2)}{(\omega_{n1}^2 - \omega_{n2}^2)} \frac{V}{\omega_{n2}} \tag{2.29b}$$

Application: The dynamic analysis of hammer foundations, which will be explained in Section 4.5 is based on a two-degree system undergoing free vibrations. Eqs. 2.25, 2.29a and 2.29b derived in this section will be used for the computation of natural frequencies and amplitudes respectively of hammer foundations.

b. Forced Vibrations

CASE 1: When the exciting force acts only on mass m_2.

Consider tbe two-degree system shown in Fig. 2.3a. The mass m_2 is subjected to the action of an oscillating force $P_0 \sin \omega_m t$, where P_0 is the peak force and ω_m is the operating frequency. The differential equations of motion for forced oscillation of the system are given by

$$m_1 \ddot{z}_1 + K_1 z_1 + K_2 (z_1 - z_2) = 0 \tag{2.30a}$$

$$m_2 \ddot{z}_2 + K_2 (z_2 - z_1) = P_0 \sin \omega_m t \tag{2.30b}$$

Solving, the amplitudes a_1 and a_2 of thc two masses m_1 and m_2 are obtained as under:

$$a_1 = \frac{\bar{\omega}_{n2}^2}{m_1 f(\omega_m^2)} P_0 \tag{2.31a}$$

and
$$a_2 = \frac{[(1+\alpha)\,\bar{\omega}_{n1}^2 + \alpha\,\bar{\omega}_{n2}^2 - \omega_m^2]}{m_2 f(\omega_m^2)} P_0 \tag{2.31b}$$

where $\bar{\omega}_{n2}$, $\bar{\omega}_{n1}$ and α are defined by Eqs. 2.23a and 2.23b and 2.24 respectively and

$$f(\omega_m^2) = \omega_m^4 - (1+\alpha)(\bar{\omega}_{n1}^2 + \bar{\omega}_{n2}^2)\,\omega_m^2 + (1+\alpha)\,\bar{\omega}_{n1}^2\,\bar{\omega}_{n2}^2 \tag{2.32}$$

It may be noticed that Eq. 2.32 is the same as Eq. 2.22 with ω_m substituted for ω_n.

Application: This case will be illustrated in the analysis of block foundations resting on absorbers for vertical reciprocating engines (Section 4.4) and for the analysis of a cross-frame of a framed foundation by the amplitude method (Chapter 5).

CASE 2: When the exciting force acts only on mass (m_1).

Consider again the same system shown in Fig. 2.3a. The oscillating force $P_0 \sin \omega_m t$ now acts on mass m_1. The differential equations characterizing the motion of the system are

$$m_1 \ddot{z}_1 + K_1 z_1 + K_2 (z_1 - z_2) = P_0 \sin \omega_m t \tag{2.33a}$$

$$m_2 \ddot{z}_2 + K_2 (z_2 - z_1) = 0 \tag{2.33b}$$

Solving the amplitudes a_1 and a_2 are obtained thus:

$$a_1 = \frac{P_0}{m_1 f(\omega_m^2)} (\bar{\omega}_{n2}^2 - \omega_m^2) \tag{2.34a}$$

$$a_2 = \frac{P_0}{m_1 f(\omega_m^2)} [\bar{\omega}_{n2}^2] \tag{2.34b}$$

Eqs. 2.34a and 2.34b may be written in the form of expressions for dynamic factors μ_1 and μ_2 as follows with appropriate substitutions

$$\mu_1 = \frac{|a_1|}{a_{st}} = \frac{\eta_2^2(\eta_1^2 - \eta_1^2\eta_2^2)}{(\eta_1^2 - \eta_1^2\eta_2^2)(\eta_2^2 + \eta_1^2\alpha - \eta_1^2\eta_2^2) - \eta_1^4\alpha} \tag{2.34c}$$

and

$$\mu_2 = \frac{|a_2|}{a_{st}} = \frac{\eta_1^2\eta_2^2}{(\eta_1^2 - \eta_1^2\eta_2^2)(\eta_2^2 + \eta_1^2\alpha - \eta_1^2\eta_2^2) - \eta_1^4\alpha} \tag{2.34d}$$

where

$$a_{st} = \frac{P_0}{K_1}$$

$$\eta_1 = \frac{\omega_m}{\sqrt{K_1/m_1}}$$

and

$$\eta_2 = \frac{\omega_m}{\sqrt{K_2/m_2}}$$

For the particular case when $\frac{K_2}{m_2} = \frac{K_1}{m_1}$ (considered in Section 2.3a), that is, $\eta_2 = \eta_1$ ($= \eta$ say) the Eqs. 2.34c and 2.34d may be further simplified as

$$\mu_1 = \frac{1 - \eta^2}{(1 - \eta^2)(1 + \alpha - \eta^2) - \alpha} \tag{2.34e}$$

and

$$\mu_2 = \frac{1}{(1 - \eta^2)(1 + \alpha - \eta^2) - \alpha} \tag{2.34f}$$

Fig. 2.5 shows the variation of μ_1 and μ_2 (given by Eqs. 2.34e and 2.34f) with η for the case when $\alpha = 0.2$.

Two points worth noting from this figure are:

1. There are two values of η at which μ_1 or μ_2 is ∞. The values of ω_m corresponding to these infinite ordinates are the natural frequencies ω_{n1} and ω_{n2}.
2. When $\eta = 1$, i.e. $\omega_m = \bar{\omega}_{n2}$, $a_1 = 0$

In other words, when the values m_2 and k_2 are such that $\sqrt{\frac{K_2}{m_2}}$ is equal to the frequency (ω_m) of exciting force acting on mass m_1, then the amplitude of mass m_1 will be zero. When $\bar{\omega}_{n2} = \omega_m$, while $a_1 = 0$, the amplitude of mass m_2 may be obtained (from Eq. 2.34b) as

$$a_2 = \left|\frac{P_0}{K_2}\right| \tag{2.35a}$$

The amplitude of mass m_2 is thus equal to its static displacement (displacement of m_2 under the static influence of P_0).

Application: The above theoretical treatment will be useful in the application of an undamped vibration neutralizer for a rigid block foundation as explained in Section 7.3c.

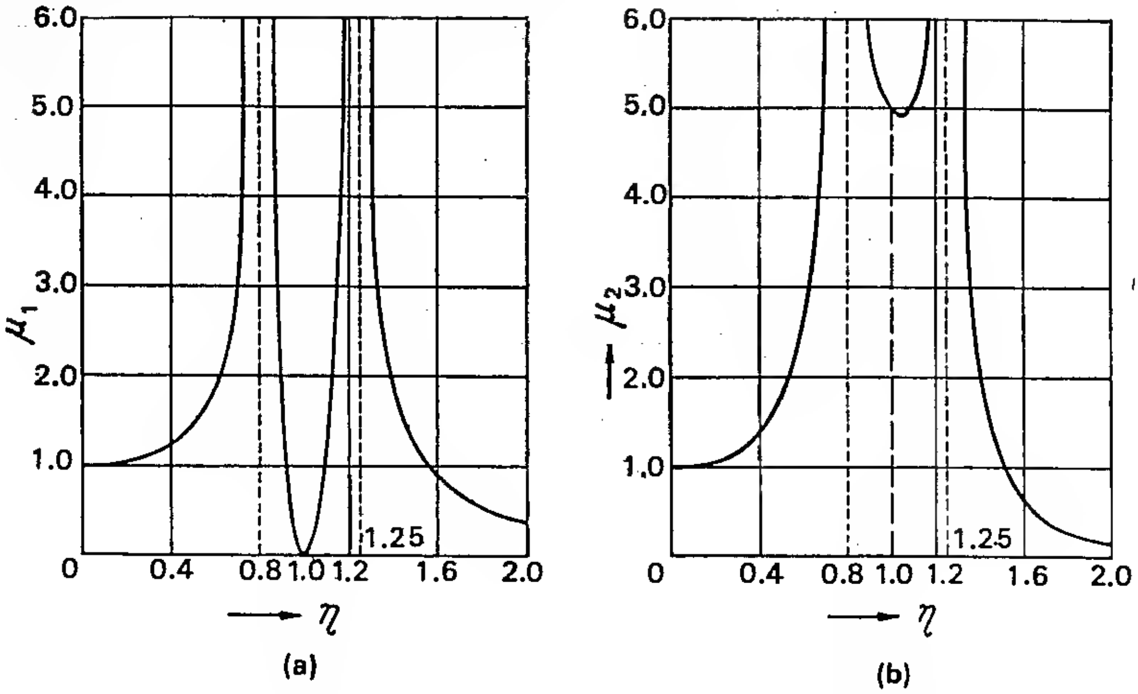


Fig. 2.5: Response Curves for an Undamped Two-Degree Freedom System for the Case when $\alpha=0.2$ and $\frac{K_2}{m_2}=\frac{K_1}{m_1}$.

2.3.2 *Damped Case*

a. Free Vibrations

Consider the system shown in Fig. 2.3b. Viscous dampers with damping coefficients C_1 and C_2 are additionally introduced here. It is difficult to precisely assess the values of C_1 and C_2 in practice and consequently they are not generally considered in practical designs based on multiple degree freedom systems. However, the following theoretical treatment will be helpful in cases where the influence of damping cannot be neglected, and this data can be obtained from field measurements or otherwise. The equations of motion for the system shown in Fig. 2.3b may be written as under:

$$m_1 \ddot{z}_1 + C_1 \dot{z}_1 + K_1 z_1 + K_2 (z_1 - z_2) + C_2 (\dot{z}_1 - \dot{z}_2) = 0 \tag{2.36a}$$

$$m_2 \ddot{z}_2 + C_2 (\dot{z}_2 - \dot{z}_1) + K_2 (z_2 - z_1) = 0 \tag{2.36b}$$

Both z_1 and z_2 are harmonic functions and can be represented by vectors. Writing the vectors as complex numbers and substituting

$$z_1 = a_1 \, e^{i\omega_n t} \tag{2.37a}$$

$$z_2 = a_2 \, e^{i\omega_n t} \tag{2.37b}$$

in Eqs. 2.37a and 2.37b, and solving, the following governing equation is obtained for the natural frequencies of the system.

$$\{F(\omega_m^2)\}^2 + 4\omega_m^2 \{\zeta_1 \bar{\omega}_{n1} (\bar{\omega}_{n2}^2 - \omega_m^2) \sqrt{1+\alpha} + \zeta_2 \bar{\omega}_{n2} (\bar{\omega}_{n1}^2 - \omega_m^2)(1+\alpha)\}^2 = 0 \tag{2.38}$$

where,

$$F(\omega_m^2)=\omega_m^4-\omega_m^2(1+\alpha)(\bar{\omega}_{n1}^2+\bar{\omega}_{n2}^2+4\,\zeta_1\,\zeta_2\bar{\omega}_{n1}\,\bar{\omega}_{n2}\sqrt{1+\alpha}+\bar{\omega}_{n1}^2\,\bar{\omega}_{n2}^2(1+\alpha) \tag{2.39}$$

where $\bar{\omega}_{n2}$, $\bar{\omega}_{n1}$ and α are already defined in Eqs. 2.23a, 2.23b and 2.24 respectively; ζ_1 and ζ_2 are damping ratios defined by

$$\frac{C_1}{m_1}=2\,\zeta_1\sqrt{\frac{K_1}{m_1}} \tag{2.40a}$$

$$\frac{C_2}{m_2}=2\,\zeta_2\sqrt{\frac{K_2}{m_2}} \tag{2.40b}$$

Corollary: When $\zeta_1=0$, and $\zeta_2=0$, Eq. 2.39 reduces to the form given by Eq. 2.22 for the undamped case.

b. Forced Vibrations

CASE 1: When the harmonic force $P_0 \sin \omega_m t$ acts on mass m_1.

The equations of motion for the system may be written as

$$m_1\,\ddot{z}_1+C_1\,\dot{z}_1+K_1\,z_1+K_2\,(z_1-z_2)+C_2\,(\dot{z}_1-\dot{z}_2)=P_0\sin\omega_m t \tag{2.41a}$$

$$m_2\,\ddot{z}_2+C_2\,(\dot{z}_2-\dot{z}_1)+K_2\,(z_2-z_1)=0 \tag{2.41b}$$

Since the system moves at the frequency of the exciting force under steady-state conditions, the solution may be assumed in the form:

$$z_1=a_1\,e^{i\omega_m t} \tag{2.42a}$$

and

$$z_2=a_2\,e^{i\omega_m t} \tag{2.42b}$$

Substituting these relations in Eqs. 2.41a and 2.41b and solving, the following relations are obtained for a_1 and a_2

$$a_1=\frac{P_0}{m_1}\left[\frac{(\bar{\omega}_{n2}^2-\omega_m^2)+2i\zeta_2\,\bar{\omega}_{n2}\,\omega_m}{F(\omega_m^2)+2i\omega_n\{\zeta_1\,\bar{\omega}_{n1}\,(\bar{\omega}_{n2}^2-\omega_m^2)\,\sqrt{1+\alpha}+\zeta_2\,\bar{\omega}_{n2}\,(\bar{\omega}_{n1}^2-\omega_m^2)\,(1+\alpha)\}}\right] \tag{2.43a}$$

and

$$a_2=\frac{a_1\,(K_2+C_2\,i\,\omega)}{K_2-m_2\,\omega^2+C_2\,i\,\omega} \tag{2.43b}$$

where $F(\omega_m^2)$ is given by Eq. 2.39.

Using the principles of complex algebra, the modulus of a_1 and a_2 may be written as

$$a_1=\frac{P_0}{m_1}\sqrt{\frac{(\bar{\omega}_{n2}^2-\omega_m^2)^2+4\zeta_2^2\,\bar{\omega}_{n2}^2\,\omega_m^2}{\{F(\omega_m^2)\}^2+4\omega_m^2\{\zeta_1\bar{\omega}_{n1}(\bar{\omega}_{n2}^2-\omega_m^2)\sqrt{1+\alpha}+\zeta_2\bar{\omega}_{n2}(\bar{\omega}_{n1}^2-\omega_m^2)(1+\alpha)\}^2}} \tag{2.44a}$$

$$a_2=\frac{P_0}{m_1}\sqrt{\frac{\omega_{n2}^4+4\,\zeta_2^2\,\bar{\omega}_{n2}^2\,\omega_m^2}{\{F(\omega_m^2)\}^2+4\omega_m^2\{\zeta_1\bar{\omega}_{n1}(\bar{\omega}_{n2}^2-\omega_m^2)\sqrt{1+\alpha}+\zeta_2\bar{\omega}_{n2}(\bar{\omega}_{n1}^2-\omega_m^2)(1+\alpha)\}^2}} \tag{2.44b}$$

Particular Case: When $\zeta_1=0$ (i.e., the damping in the lower system is neglected) and $\zeta_2=\zeta$, the amplitude of mass m_1 subjected to a harmonic force $P_0 \sin \omega_m t$ is given by

$$a_1=\frac{P_0}{m_1}\left[\frac{(\bar{\omega}_{n2}^2-\omega_m^2)^2+4\zeta^2\bar{\omega}_{n2}^2\,\omega_m^2}{\{f(\omega_m^2)\}^2+4\,\omega_m^2\,\zeta^2\,\omega_{n2}^2\,\{(1+\alpha)^2\,(\bar{\omega}_{n1}^2-\omega_m^2)^2\}}\right]^{\frac{1}{2}} \tag{2.45a}$$

and

$$a_2=\frac{P_0}{m_1}\left[\frac{\bar{\omega}_{n2}^4+4\,\zeta^2\,\bar{\omega}_{n2}^2\,\omega_m^2}{\{f(\omega_m^2)\}^2+4\,\omega_m^2\,\zeta^2\,\bar{\omega}_{n2}^2\,\{(1+\alpha)^2\,(\bar{\omega}_{n1}^2-\omega_m^2)^2\}}\right]^{\frac{1}{2}} \tag{2.45b}$$

where $f(\omega_m^2)$ is given by Eq. 2.32
or in terms of basic parameters, substituting $C_1=0$ and $C_2=C$

$$a_1=P_0\left[\frac{(K_2-m_2\,\omega_m^2)^2+C^2\,\omega_m^2}{\{(K_1-m_1\,\omega_m^2)\,(K_2-m_2\,\omega_m^2)-K_2m_2\omega_m^2\}^2+C^2\omega_m^2\{K_1-m_1\omega_m^2-m_2\omega_m^2\}^2}\right]^{\frac{1}{2}} \tag{2.46a}$$

and

$$a_2=P_0\left[\frac{K_2^2+C^2\,\omega_m^2}{\{(K_1-m_1\,\omega_m^2)\,(K_2-m_2\,\omega_m^2)-K_2m_2\omega_m^2\}^2+C^2\omega_m^2\{K_1-m_1\omega_m^2-m_2\omega_m^2\}^2}\right]^{\frac{1}{2}} \tag{2.46b}$$

Expressed in non-dimensional form, Eqs. 2.46a and 2.46b may be further written as

$$\mu_1=\frac{|a_1|}{a_{st}}=\left[\frac{(1-\eta_2^2)^2+4\,\zeta^2\,\eta_2^2}{[\alpha\,\eta_1^2-(\eta_1^2-1)\,(\eta_2^2-1)]^2+4\zeta^2\,\eta_2^2\,\,(\eta_1^2-1+\alpha\,\eta_1^2)^2}\right]^{\frac{1}{2}} \tag{2.47a}$$

and

$$\mu_2=\frac{|a_2|}{a_{st}}=\left[\frac{1+4\,\zeta^2\,\eta_2^2}{[\alpha\,\eta_1^2-(\eta_1^2-1)\,(\eta_2^2-1)]^2+4\,\zeta^2\,\eta_2^2\,(\eta_1^2-1+\alpha\,\eta_1^2)^2}\right]^{\frac{1}{2}} \tag{2.47b}$$

where

$$a_{st}=\frac{P_0}{K_1} \tag{2.48a}$$

$$\eta_1=\frac{\omega_m}{\sqrt{K_1/m_1}} \tag{2.48b}$$

$$\eta_2=\frac{\omega_m}{\sqrt{K_2/m_2}} \tag{2.48c}$$

$$\alpha=m_2/m_1 \tag{2.48d}$$

$$\zeta=C/C_c \tag{2.48e}$$

For the case $\dfrac{K_2}{m_2}=\dfrac{K_1}{m_1}$ (considered in preceding case), $\eta_1=\eta_2$ $(=\eta$, say), Fig. 2.6 shows the variation of μ_1 with η for various damping values (ζ).

It is interesting to note from Fig. 2.6 that irrespective of the degree of damping, all the response curves pass through two fixed points S_1 and S_2, the abscissa of which may be obtained as roots of the following equation

$$\eta^4-2\,\eta^2\left(\frac{1+\beta^2+\alpha\,\beta^2}{2+\alpha}\right)+\frac{2\,\beta^2}{2+\alpha}=0 \tag{2.49}$$

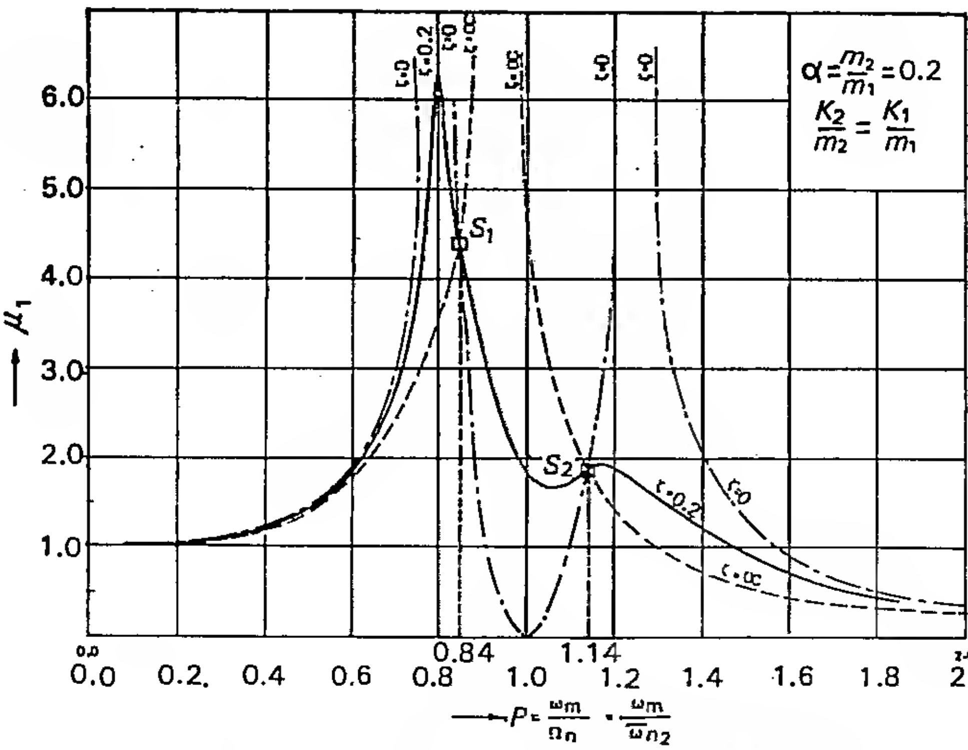


Fig. 2.6: Response of Mass m_1 for Various Damping Ratios (ζ)

where $\beta = \eta_1/\eta_2$

For the particular case considered above since $\eta_1 = \eta_2$, $\beta = 1.0$.
Substituting, the abscissae of the fixed points S_1 and S_2 in Fig. 2.6 are given by

$$\eta^4 - 2\,\eta^2 + \left(\frac{2}{2+\alpha}\right) = 0 \tag{2.50}$$

with $\alpha = 0.2$ in this case.
Tables 2.2 and 2.3 give the values of μ_1 and μ_2 for various values of frequency ratio η, mass ratio α and damping ratio ζ for the particular case when $\eta_2 = \eta_1$ or the relation $\frac{K_2}{m_2} = \frac{K_1}{m_1}$ is satisfied.

Application: The theory explained in the above particular case is used in the design of auxiliary mass-vibration dampers, which will be explained in Section 7.3c. The data contained in Tables 2.2 and 2.3 will be useful in the choice of appropriate parameters for the design of auxiliary mass-vibration dampers for a rigid block foundation.

2.4 Multiple-Degree Freedom System

Although the vibration analysis of a multiple-degree freedom system is relatively more complicated and often necessitates the use of a digital computer, the theoretical approach for the analysis of such a system for the undamped case is given in this section for the benefit of interested readers. Matrix notation* is used here for a concise presentation.

*Readers not familiar with this notation may refer to standard books on matrix algebra or Section 28, Vol 2 of Ref. C 1.6.

Table 2.2

VARIATION OF DYNAMIC FACTOR (μ_1) FOR MASS m_1

$$\left(\frac{K_2}{m_2} = \frac{K_1}{m_1}\right)$$

η \ α	0.1				0.2				0.3				0.4				0.5			
ζ	0	0.05	0.1	0.2	0	0.05	0.1	0.2	0	0.05	0.1	0.2	0	0.05	0.1	0.2	0	0.05	0.1	0.2
0.1	1.011	1.011	1.011	1.011	1.012	1.012	1.012	1.012	1.013	1.013	1.013	1.013	1.014	1.014	1.014	1.014	1.015	1.015	1.015	1.015
0.2	1.046	1.046	1.046	1.046	1.051	1.051	1.051	1.050	1.055	1.055	1.055	1.055	1.060	1.060	1.060	1.060	1.065	1.065	1.065	1.065
0.3	1.111	1.111	1.111	1.111	1.123	1.123	1.123	1.123	1.136	1.136	1.136	1.136	1.149	1.149	1.148	1.149	1.162	1.162	1.162	1.162
0.4	1.218	1.218	1.218	1.218	1.247	1.247	1.247	1.247	1.277	1.277	1.277	1.277	1.309	1.309	1.309	1.308	1.343	1.343	1.342	1.342
0.5	1.395	1.395	1.395	1.394	1.463	1.463	1.463	1.461	1.538	1.538	1.537	1.534	1.622	1.621	1.620	1.616	1.714	1.714	1.712	1.706
0.6	1.713	1.712	1.711	1.706	1.896	1.894	1.891	1.877	2.122	2.119	2.112	2.087	2.410	2.405	2.392	2.348	2.787	2.780	2.757	2.683
0.7	2.416	2.411	2.396	2.351	3.146	3.126	3.074	2.924	4.509	4.441	4.265	3.826	7.956	7.597	6.804	5.347	33.775	21.522	13.203	7.684
0.8	5.488	5.288	4.869	4.180	∞	19.796	10.603	6.486	5.769	5.778	5.798	5.848	2.848	2.911	3.090	3.685	1.890	1.937	2.068	2.524
0.9	4.231	4.574	5.341	6.826	1.510	1.670	2.077	3.223	0.918	1.016	1.263	1.959	0.660	0.730	0.906	1.394	0.515	0.569	0.706	1.080
1.0	0.000	0.995	1.961	3.714	0.000	0.498	0.981	1.857	0.000	0.332	0.654	1.238	0.000	0.249	0.490	0.928	0.000	0.199	0.392	0.743
1.1	2.731	2.786	2.872	2.960	1.061	1.162	1.373	1.738	0.659	0.729	0.887	1.199	0.479	0.531	0.653	0.910	0.374	0.417	0.516	0.732
1.2	8.871	5.312	3.371	2.287	4.661	3.546	2.524	1.799	1.846	1.752	1.580	1.352	1.151	1.136	1.105	1.051	0.836	0.838	0.842	0.850
1.3	2.247	2.149	1.942	1.594	4.996	3.654	2.451	1.565	22.33	4.426	2.358	1.386	3.452	2.626	1.809	1.171	1.870	1.674	1.357	0.983
1.4	1.323	1.305	1.259	1.143	1.813	1.725	1.536	1.203	2.878	2.439	1.828	1.197	6.977	3.462	1.971	1.127	16.438	3.492	1.830	1.022
1.5	0.935	0.929	0.912	0.865	1.124	1.103	1.050	0.919	1.408	1.349	1.214	0.952	1.887	1.709	1.390	0.956	2.857	2.231	1.537	0.930
1.6	0.716	0.714	0.707	0.685	0.812	0.804	0.783	0.722	0.937	0.919	0.873	0.755	1.107	1.068	0.974	0.776	1.352	1.265	1.084	0.783
1.7	0.576	0.574	0.571	0.558	0.631	0.628	0.618	0.585	0.699	0.691	0.671	0.610	0.782	0.768	0.731	0.631	0.888	0.862	0.797	0.646
1.8	0.477	0.476	0.474	0.468	0.512	0.511	0.505	0.486	0.554	0.549	0.539	0.505	0.602	0.595	0.577	0.522	0.659	0.648	0.618	0.536
1.9	0.405	0.404	0.403	0.697	0.429	0.427	0.424	0.412	0.456	0.453	0.447	0.426	0.486	0.483	0.472	0.439	0.521	0.515	0.499	0.451
2.0	0.349	0.349	0.348	0.344	0.366	0.365	0.363	0.355	0.385	0.383	0.379	0.366	0.405	0.403	0.397	0.376	0.429	0.425	0.416	0.385

Table 2.3

VARIATION OF DYNAMIC FACTOR (μ_2) FOR MASS m_2

$$\left(\frac{K_2}{m_2} = \frac{K_1}{m_1}\right)$$

η	α →	0.1				0.2				0.3				0.4				0.5			
	ζ →	0	0.05	0.1	0.2	0	0.05	0.1	0.2	0	0.05	0.1	0.2	0	0.05	0.1	0.2	0	0.05	0.1	0.2
0.1		1.021	1.021	1.021	1.021	1.022	1.022	1.022	1.022	1.022	1.022	1.022	1.022	1.024	1.024	1.024	1.024	1.026	1.026	1.026	1.026
0.2		1.090	1.090	1.090	1.090	1.095	1.095	1.094	1.094	1.098	1.098	1.098	1.098	1.104	1.104	1.104	1.104	1.109	1.109	1.109	1.109
0.3		1.221	1.221	1.220	1.219	1.234	1.234	1.234	1.233	1.247	1.247	1.247	1.245	1.262	1.262	1.262	1.261	1.277	1.277	1.277	1.275
0.4		1.450	1.450	1.448	1.442	1.485	1.484	1.482	1.476	1.519	1.519	1.517	1.511	1.559	1.558	1.556	1.550	1.598	1.598	1.596	1.589
0.5		1.860	1.860	1.852	1.832	1.951	1.949	1.943	1.920	2.049	2.047	2.040	2.014	2.162	2.160	2.152	2.123	2.286	2.283	2.274	2.241
0.6		2.677	2.669	2.647	2.566	2.962	2.952	2.924	2.825	3.312	3.299	3.264	3.137	3.765	3.748	3.700	3.533	4.355	4.332	4.265	4.037
0.7		4.737	4.694	4.574	4.197	6.169	6.089	5.869	5.219	8.833	8.639	8.135	6.822	15.600	14.793	12.991	9.544	66.225	41.910	25.209	13.714
0.8		15.244	14.384	12.517	9.113	∞	53.85	27.257	14.139	16.009	15.701	14.890	12.735	7.911	7.920	7.943	8.033	5.252	5.268	5.316	55.02
0.9		22.271	21.846	20.736	17.824	7.943	7.973	8.064	8.415	4.828	4.847	4.901	5.110	3.473	3.484	3.517	3.641	2.711	2.718	2.739	2.820
1.0		10.000	10.000	10.000	10.000	5.000	5.000	5.000	5.000	3.333	3.333	3.333	3.333	2.500	2.500	2.500	2.500	2.000	2.000	2.000	2.000
1.1		13.000	11.824	9.668	6.633	5.053	4.930	4.623	3.894	3.133	3.092	2.983	2.685	2.273	2.253	2.200	2.040	1.783	1.771	1.739	1.641
1.2		20.161	11.731	6.917	3.897	10.593	7.830	5.178	3.064	4.190	3.865	3.239	2.300	2.615	2.509	2.268	1.790	1.900	1.850	1.727	1.448
1.3		3.256	3.086	2.721	2.079	7.240	5.245	3.434	2.041	32.330	6.350	3.300	1.807	5.003	3.771	2.535	1.527	2.711	2.404	1.901	1.282
1.4		1.378	1.358	1.307	1.179	1.888	1.795	1.595	1.240	2.995	2.536	1.897	1.233	7.267	3.603	2.047	1.162	17.123	3.635	1.901	1.053
1.5		0.748	0.746	0.741	0.727	0.899	0.886	0.853	0.773	1.126	1.082	0.985	0.800	1.509	1.372	1.129	0.804	2.286	1.792	1.249	0.782
1.6		0.459	0.461	0.466	0.481	0.520	0.519	0.517	0.509	0.600	0.593	0.575	0.531	0.709	0.689	0.642	0.546	0.867	0.817	0.715	0.552
1.7		0.305	0.307	0.314	0.336	0.334	0.336	0.339	0.352	0.369	0.369	0.369	0.414	0.411	0.411	0.402	0.380	0.470	0.461	0.438	0.389
1.8		0.213	0.215	0.222	0.244	0.229	0.231	0.237	0.255	0.247	0.248	0.252	0.264	0.269	0.269	0.270	0.276	0.294	0.293	0.290	0.281
1.9		0.155	0.157	0.163	0.184	0.164	0.166	0.172	0.190	0.174	0.176	0.181	0.197	0.186	0.188	0.192	0.203	0.200	0.200	0.203	0.208
2.0		0.116	0.118	0.124	0.142	0.122	0.124	0.129	0.146	0.128	0.130	0.135	0.151	0.135	0.137	0.141	0.155	0.143	0.144	0.148	0.159

2.4.1 Free Vibrations

Consider the system shown in Fig. 2.7. Under free vibration, there is no exciting force on any of the masses. The system is said to possess n degrees of freedom leading to n equations of motion which may be written in the form

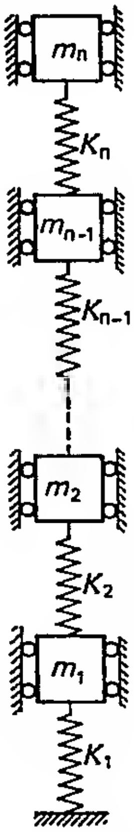


Fig. 2.7: Multiple Degree Freedom System.

$$\left.\begin{array}{l} m_1 \ddot{z}_1 + K_1 z_1 + K_2 (z_1 - z_2) = 0 \\ m_2 \ddot{z}_2 + K_2 (z_2 - z_1) + K_3 (z_2 - z_3) = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ m_n \ddot{z}_n + K_n (z_n - z_{n-1}) = 0 \end{array}\right\} \tag{2.51}$$

The equation system (2.51) may be written in matrix form as

$$[M]\{\ddot{Z}\} + [K]\{Z\} = 0 \tag{2.52}$$

where (M) is the diagonal mass matrix given by

$$[M] = \begin{bmatrix} m_1 & 0 \ldots\ldots\ldots\ldots & 0 \\ 0 & m_2 \ldots\ldots\ldots\ldots & \\ 0 & 0\ m_3 \ldots\ldots\ldots & \\ \ldots & \ldots\ldots\ldots\ldots & \\ \ldots & \ldots\ldots\ldots\ldots & \\ 0 & 0\ \ 0 \ldots\ldots\ldots & m_n \end{bmatrix} \tag{2.53}$$

$[K]$ is the stiffness matrix which, in its gencral form, is represented by

$$[K] = \begin{bmatrix} K_{11} & K_{12} \ldots\ldots\ldots & K_{1n} \\ K_{21} & K_{22} \ldots\ldots\ldots & K_{2n} \\ \ldots & \ldots\ldots\ldots\ldots & \\ \ldots & \ldots\ldots\ldots\ldots & \\ K_{n1} & K_{n2} \ldots\ldots\ldots & K_{nn} \end{bmatrix} \tag{2.54}$$

K_{ij} are the stiffness coefficients which can be evaluated for a given structural system. For the chain-like system shown in Fig. 2.7,

$$[K] = \begin{bmatrix} K_1 + K_2 & -K_2 \ldots\ldots & 0 \\ -K_2 & K_2 + K_3 & -K_3 . 0 \\ \ldots\ldots & \ldots\ldots\ldots & \\ \ldots\ldots & \ldots\ldots\ldots & \\ \ldots\ldots & \ldots\ldots\ldots & K_n \end{bmatrix} \tag{2.55}$$

Substituting $z_1 = a_1 \sin \omega t$, $z_2 = a_2 \sin \omega t$ and so on, in Eq. 2.52 and simplifying.

$$\{[K] - [M]\omega^2\}\ \{A\} = 0 \tag{2.56}$$

where

$$\{A\} = \begin{bmatrix} a_1 \\ a_2 \\ \ldots \\ \ldots \\ a_n \end{bmatrix} \tag{2.57}$$

The algebraic problem represented by Eq. 2.56 is called the "matrix eigen value problem." It is also called the "real eigen value problem" to distinguish it from the complex eigen value problem obtained when the damping matrix is also considered in the equations of motion (Eq. 2.51). Eq. 2.56 represents a set of homogeneous equations (right-hand side equal to zero), the condition for obtaining a non-trivial solution being that the determinant formed by the coefficients of the left-hand side of the equation system should vanish. This gives the relation in its general form as

$$\begin{vmatrix} K_{11}-m_1\omega^2 & K_{12}\ldots\ldots\ldots\ldots K_{1n} \\ K_{21} & K_{22}-m_2\omega^2\ldots\ldots K_{2n} \\ \ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots \\ K_{n1} & K_{n2}\ldots\ldots K_{nn}-m_n\omega^2 \end{vmatrix} = 0 \tag{2.58}$$

Eq. 2.58 on expansion gives n roots for ω^2, say $\omega_1^2, \omega_2^2\ldots\omega_n^2$ such that $\omega_1^2<\omega_2^2<\ldots\omega_n^2$. The fundamental natural frequency is ω_1 and $\omega_2, \omega_3\ldots\omega_n$ are the higher-order frequencies of the multiple degree freedom system. The terms $\omega_1, \omega_2\ldots\omega_n$ are also called the "eigen values" of the system.

Substituting each value of ω^2 at a time in the equation system, one can evaluate the relative values of $a_1, a_2\ldots a_n$. It may be noted that the absolute values of $a_1, a_2\ldots a_n$ cannot be obtained since the equations are homogeneous. There are numerous methods available for the solution of eigen value problems. Standard computer programmes are also available for solving the eigen value problem involving large matrices, as in the case when the number of degrees of freedom is too large to be handled by manual calculation.

If $\{\mathbf{V_r}\}$ denotes the column vector with relative components $a_1^r, a_2^r,\ldots a_n^r$ corresponding to a value ω_r (r^{th} eigen value) then $\{\mathbf{V_r}\}$ is called the eigen vector (also called modal vector or mode shape) corresponding to the eigen value ω_r.

The following important relations, known as "orthogonality conditions of eigen vectors," will be useful:

$$\{\mathbf{V_r}\}^T [K] \{\mathbf{V_s}\}=0 \tag{2.59a}$$

and

$$\{\mathbf{V_r}\}^T [M] \{\mathbf{V_s}\}=0 \tag{2.59b}$$

where r and s are two distinct modes.

The superscript T denotes the transpose of the matrix contained in flower brackets.

To obtain the displacement matrix $\{Z_t\}$ at any instant t after the free motion is set in, the appropriate initial conditions are to be applied.

Let $\{Z_0\}$ and $\{\dot{Z}_0\}$ denote the initial displacement and velocity vectors at time $t=0$. The following expression for $\{Z_t\}$ may be derived in terms of the eigen values and eigen vectors of the system

$$\{Z_t\} = \sum_{r=1}^{n} \frac{\{\mathbf{V_r}\}\{\mathbf{V_r}\}^T [M]}{\{\mathbf{V_r}\}^T [M] \{\mathbf{V_r}\}} \left[\{Z_0\} \cos \omega_r t + \frac{1}{\omega_r} \{\dot{Z}_0\} \sin \omega_r t \right] \tag{2.60}$$

Eq. 2.60 gives the displacements $z_1, z_2\ldots z_n$ at any time t. It may be noted that the matrix product $\{\mathbf{V_r}\}^T[M]\{\mathbf{V_r}\}$ in the denominator is a scalar quantity.

A useful check on the calculation is provided by the following identity,

$$\sum_{r=1}^{n} \frac{\{V_r\}\{V_r\}^T [M]}{\{V_r\}^T [M] \{V_r\}} = [I] \tag{2.61}$$

The right-hand side is the identity matrix, also called the "unit matrix".

2.4.2 Forced Vibrations

Consider the system shown in Fig. 2.7 with harmonic exciting forces $P_1 \sin \omega_m t$, $P_2 \sin \omega_m t \ldots P_n \sin \omega_m t$ acting on masses $m_1, m_2 \ldots m_n$ respectively. The amplitudes of exciting force are represented by the force vector $\{F\}$ where

$$\{F\} = \begin{bmatrix} P_1 \\ P_2 \\ \cdot\cdot \\ \cdot\cdot \\ P_n \end{bmatrix} \tag{2.62}$$

The equation of motion of the system may be written in matrix form thus:

$$[M]\{\ddot{Z}\} + [K]\{Z\} = \{F\} \tag{2.63}$$

The steady-state solution of Eq. 2.63 may be expressed in the form

$$\{Z\} = \{a\} \sin \omega_m t \tag{2.64}$$

where $\{a\}$ is the unknown column vector of amplitudes.
Substituting Eq. 2.64 in Eq. 2.63 and simplifying, the following set of equations is obtained:

$$\{[K] - \omega^2 [M]\}\{a\} = \{F\} \tag{2.65}$$

or

$$\{a\} = \{[K] - \omega^2 [M]\}^{-1} \{F\} \tag{2.66}$$

where, the superscript $^{-1}$ denotes the inversion of the square matrix contained in the flower brackets of Eq. 2.66.

NOTE: Since damping has not been considered in equation system 2.63, if ω_m is equal to one of the natural frequencies of the system, the matrix $\{[K] - \dot{\omega}^2[M]\}$ becomes a singular matrix (value of its determinant becomes zero) and therefore cannot be inverted.

Alternative solution: The natural frequencies ω_r ($r=1, 2, \ldots n$) and natural modes $\{V_r\}$ are first determined as explained in the preceding section. The amplitudes can be obtained from the following relation.

$$\{a\} = \sum_{r=1}^{n} \frac{1}{(\omega_r^2 - \omega_m^2)} \left[\frac{\{V_r\}\{V_r\}^T}{\{V_r\}^T [M] \{V_r\}} \right] \{F\} \tag{2.67}$$

2.5. Transient Response

2.5.1 *Response of Single-Degree Freedom System*

Consider the motion of a spring mass system (Fig. 2.1) under the influence of a general transient force $F(\tau)$ shown in Fig. 2.8. The variation of force with the time as shown in the diagram may be considered to be made up of pulses of short duration $\Delta\tau$.

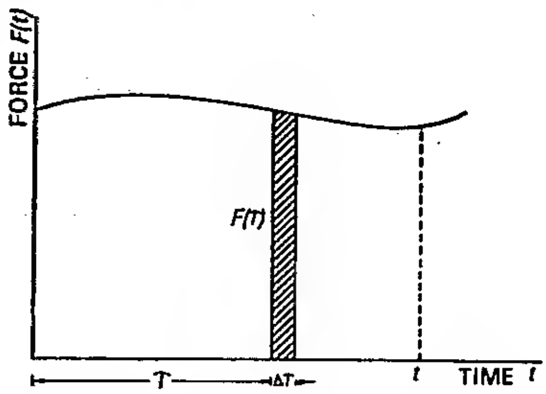


Fig. 2.8: A General Force–Time Relationship.

The response Δz of the system subjected to a pulse having a momentum Δ_s may be written as

$$\Delta z = \frac{\Delta_s}{m\,\omega_n} \sin \omega_n (t-\tau) \tag{2.68}$$

where ω_n is the natural frequency of the system and τ is the period upto which the system has been at rest before the action of the pulse.

Since $$\Delta_s = F(\tau)\,\Delta\tau$$

$$\Delta z = \frac{F(\tau)\,\Delta\tau}{m\,\omega_n} \sin \omega_n (t-\tau) \tag{2.69}$$

The response of the system subjected to the cumulative action of such a series of pulses is given by

$$z = \int_0^t \frac{F(\tau)\,d\tau}{m\,\omega_n} \sin \omega_n (t-\tau) \tag{2.70}$$

Eq. 2.70 is called the "Duhamel's integral" or "convolution integral".

NOTE: If the system was not at rest at $t=0$, the free vibration term $(A \sin \omega_n t + B \cos \omega_n t)$ should also be added to the right-hand side of Eq. 2.70 to obtain the total displacement at any time t. Thus in general.

$$z = A \sin \omega_n t + B \cos \omega_n t + \int_0^t \frac{F(\tau)\,d\tau}{m\,\omega_n} \sin \omega_n (t-\tau) \tag{2.71}$$

Particular case: Consider the response of a single-degree undamped system subjected to a rectangular pulse shown in Fig. 2.9. The load P_0 is suddenly applied and kept on the system for a duration T.

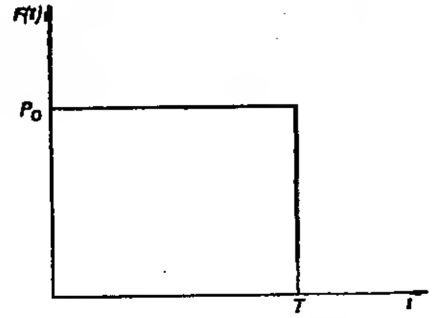


Fig. 2.9: A Rectangular Pulse.

Fig. 2.10 shows the variation of dynamic factor $\mu(=z/z_{st}$, where z_{st} is the static displacement, $P_0/K)$ with the period ratio T/T_n where T_n is the natural period of the system.

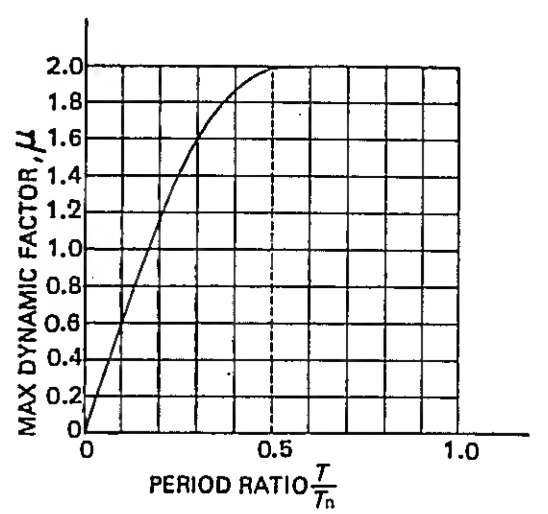


Fig. 2.10: Transient Response for a Single-Degree System Due to Rectangular Pulse.

Application: The foregoing theoretical treatment will be useful for the dynamic analysis of block foundations supporting impact causing machinery such as hammers, presses, etc. (See Example 3 in Section 4.5.7).

2.5.2 Response of Multiple-Degree Freedom System

Response of a multiple-degree freedom system subjected to a transient force vector $\{\mathbf{F}(t)\}$ may be obtained as follows. Let the matrix of initial displacements and velocities at the time $t=0$ be denoted by the $\{Z\}$, and $\{\dot{Z}\}$. If Vr is the eigen vector corresponding to the eigen value (or circular natural frequency ω_r,) then the column matrix $[Z]_t$ containing displacements of the system at any time t is given by the following general relation

$$\{Z\}_t = \sum_{r=1}^{n} \frac{\{\mathbf{V}_r\}\{\mathbf{V}_r\}^T[M]}{\{\mathbf{V}_r\}^T[M]\{\mathbf{V}_r\}}\left[\{z\}_0 \cos \omega_r t + \frac{1}{\omega_r}\{\dot{z}\}_0 \sin \omega_r t\right]$$

$$+ \sum_{r=1}^{n} \frac{\{\mathbf{V}_r\}\{\mathbf{V}_r\}^T}{\omega_r\{\mathbf{V}_r\}^T[M]\{\mathbf{V}_r\}} \int_0^t F(\tau) \sin \omega_r (t-\tau)\, d\tau \qquad (2.72)$$

It may be noted that the first part of the right-hand side of Eq. 2.72 denotes the displacement under free vibration (Eq. 2.60) and the second part is the response due to the transient force (Eq. 2.70.)

Eq. 2.72 will be useful only if the integrals involving the forcing function can be evaluated. Numerical integration using a digital computer will be necessary if the force–time relation is of a random nature. For methods of numerical integration, the reader may refer to standard books on numerical analysis.*

*S. H. Crandall, *Engineering Analysis*, McGraw-Hill, New York, 1956.

CHAPTER THREE

Evaluation of Design Parameters

3.1 Importance of Design Parameters

THE VARIOUS parameters influencing the design of a machine foundation are : (a) centre of gravity, (b) moment of inertia of the base, (c) mass moment of inertia, (d) effective stiffness of the base support, and (e) damping. While the parameters mentioned in (a), (b), (c) above may be called "geometrical properties of the machine foundation system", the parameters (d) and (e) may be termed physical properties of the elastic base of the foundation.

The terms like centre of gravity, moment of inertia and mass moment of inertia hardly need any introduction. As stated in Chapter 1, the eccentricity of the centre of gravity of a machine foundation with reference to the vertical axis passing through the centre of elasticity of the base support induces coupling of vibratory modes and this complicates the design procedure. It is, therefore, desirable in design practice to ensure that the eccentricities in the two horizontal directions (x and y) are within permissible limits. This will be further illustrated in the worked examples given in Chapter 4.

The moment of inertia of the base of the foundation and mass moment of inertia influence the dynamic calculations for the rocking (or twisting) mode of vibration. The moment of inertia and the mass moment of inertia are direction-dependent in the sense that their expressions differ with the chosen reference axis.

The effective stiffness and damping offered by the base support depend on the type of the flexible base provided under the foundation—whether soil, springs, elastic-pads, etc.

a. Soil

As will be explained in Chapter 4, there are principally two schools of thought based on which the effective stiffness of soil under a machine foundation can be evaluated.

The elastic half space theory requires the determination of shear modulus (G) and Poissons ratio (ν) of soil preferably by an *in situ* dynamic test. The *in situ* dynamic test for the determination of shear modulus (G) as suggested by the Current Indian Standard Code IS 5249-1969 is given in Sec. 3.3.

The expressions relating G and ν with the spring stiffness of soil in the various modes of vibration, viz. vertical translation, horizontal sliding, rocking motion in a vertical plane (XZ and YZ planes), and twisting in the horizontal plane, are given in Sec. 4.2 (b).

The theory based on the undamped linear spring analogy for soil, as proposed by Barkan[cl.1] requires the evaluation of certain soil parameters which are listed below:

i. Coefficient of elastic uniform compression (C_z)
ii. Coefficient of elastic uniform shear (C_τ)
iii. Coefficient of elastic non-uniform compression (C_θ)
iv. Coefficient of elastic non-uniform shear (C_ψ)

The soil parameters mentioned above are used for the evaluation of the spring stiffness of soil in various modes of vibration. The relevant expressions are given in Sec. 3.3.4.

The coefficient of elastic uniform compression (C_z) is defined as the ratio of compressive stress applied to a rigid foundation block to the "elastic" part of the settlement induced consequently. It has been found[cl.1] that within a certain range of loading, there is a proportional relationship between the elastic settlement and the external uniform pressure on soil, the constant of proportionality being designated as the coefficient of elastic uniform compression.

The coefficient of elastic uniform shear (C_τ) may likewise be defined as the ratio of average shear stress at the foundation contact area to the "elastic" part of the sliding movement of the foundation.

The coefficients described above are functions of soil-type and of size and shape of the foundation. However, for practical purposes they are often assumed to be functions of soil-type only.

Damping is a measure of energy dissipation in a given system. Being a physical property of a system damping can be evaluated only by tests. Two methods for the determination of damping are explained in Sec. 3.3.5.

b. Other Elastic Supports

For other types of elastic supports normally used under machine foundations, such as rubber pads, cork sheets, spring coils, etc., the stiffness, damping, permissible bearing pressure and such other design parameters shall be supplied by the manufacturers of these products. The stiffness in various modes may, however, be evaluated by using certain formulae which will be given later. It is desirable that a test certificate is demanded from the suppliers of these products so that designers may use their data with confidence in design calculations.

3.2 Geometrical Properties of Machine Foundations

3.2.1. Centre of Gravity

The machine and body of the foundation may be divided into a number of segmental masses m_i having regular geometrical shapes. Let the coordinates of the centre of gravity of each mass element m_i referred to some arbitrary axes be (x_i, y_i, z_i). Then the coordinates $(\bar{x}, \bar{y}, \bar{z})$ of the common centre of gravity of machine and foundation are given by

$$\bar{x} = \frac{\sum_i m_i x_i}{\sum_i m_i} \tag{3.1a}$$

$$\bar{y} = \frac{\sum_i m_i y_i}{\sum_i m_i} \tag{3.1b}$$

$$\bar{z} = \frac{\sum_i m_i z_i}{\sum_i m_i} \tag{3.1c}$$

3.2.2. *Moment of Intertia of Base Area*

a. If the base of the foundation is of rectangular shape having dimensions L and B (Fig. 3.1a) the moment of inertia I_x, I_y and I_z are given by

$$I_x = LB^3/12 \tag{3.2a}$$
$$I_y = BL^3/12 \tag{3.2b}$$
$$I_z = I_x + I_y \tag{3.2c}$$

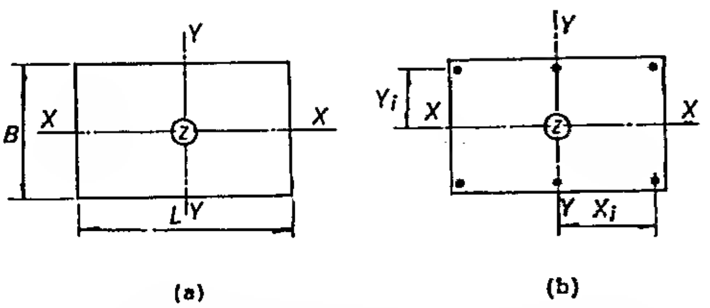


Fig. 3.1: Foundation on Elastic Supports—(a) Uniformly Distributed, (b) Point Supported.

b. If the foundation is supported at N number of isolated points as in Fig. 3.1b, the moment of inertia of the group I' is given by
Then

$$I'_x = \sum_i y_i^2 \tag{3.3a}$$

$$I'_y = \sum_i x_i^2 \tag{3.3b}$$

$$I'_z = I'_x + I'_y = \sum_i (y_i^2 + x_i^2) \tag{3.3c}$$

$\sum_i$ denotes summation over N supports ($i = 1, N$)

3.2.3. *Mass Moment of Inertia*

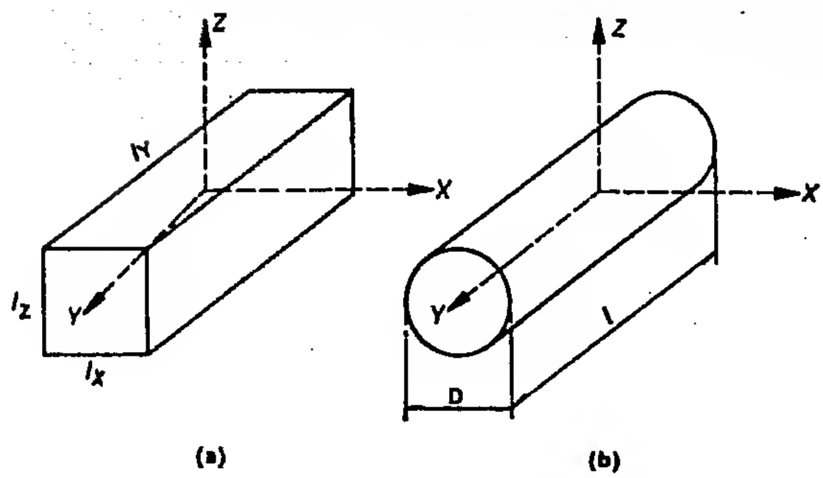
Table 3.1 gives the expressions for mass moments of inertia for rectangular and cylindrical elements (Fig. 3·2) about their centroidal axes.

Table 3.1

MASS MOMENT OF INERTIA (φ) REFERRED TO CENTROIDAL AXES

Shape of element having mass m	φ_x	φ_y	φ_z
Rectangular prism (Fig. 3.2a)	$\frac{m}{12}(l_y^2 + l_z^2)$	$\frac{m}{12}(l_x^2 + l_z^2)$	$\frac{m}{12}(l_x^2 + l_y^2)$
Solid circular cylinder (Fig. 3.2b)	$\frac{m}{12}(\frac{3}{4}D^2 + l^2)$	$\frac{m}{8}D^2$	$\frac{m}{12}(\frac{3}{4}D^2 + l^2)$

Fig. 3.2: Typical Geometrical Shapes—(a) Rectangular Prism, (b) Solid Circular Cylinder.



The mass moment of inertia φ_0 about a parallel axis at a distance S from the centre of gravity is given by

$$\varphi_0 = \varphi + mS^2 \tag{3.4}$$

3.3 Physical Properties of the Elastic Base and their Experimental Evaluation

3.3.1 Equipment Required for Dynamic Tests

Before describing the actual procedure for the experimental determination of the physical properties, a brief account is given here of the major items of equipment that are involved in any dynamic test. The equipment can be broadly classified into two categories—one required for inducing a known pattern of vibration (e.g., sinusoidal waveform) and the other required for measuring the vibration response.

a. Equipment for Inducing Vibration

The principal unit of this group of equipment is the vibrator—also called the oscillator. Oscillators are of different types, depending on the principle on which each type works, viz. mechanical, electromagnetic, hydraulic, etc. For the particular application to machine foundations, a mechanical type oscillator is commonly used. The principle of this oscillator has been briefly described in Chapter 1 (*see* Fig. 1.3).

The associated equipment required for inducing vibration with a mechanical oscillator includes an electrical motor and a speed control unit. The mechanical oscillator consists of two shafts so arranged that they rotate in opposite directions at the same speed when one of them is driven by a motor through a belt or a flexible shaft. Such an arrangement induces a unidirectional vibratory force at the base of the oscillator. Depending on the orientation of the two counter-rotating shafts, either a vertical or horizontal dynamic force (passing through the centre of gravity of the oscillator) can be realised. By varying the voltage supplied to the motor with the help of a speed control unit the speed of the motor and hence that of the oscillator can be varied. This in turn causes a change in frequency of vibration induced by the oscillator.

Plate I shows the test set up used for inducing vertical or horizontal dynamic force one at a time. The set up includes a mechanical oscillator, a motor, and a speed control unit. The oscillator shown here has a dynamic force range of 0–2400 kg, the upper limit

corresponding to a rotating frequency of 50 cps. The force induced by this oscillator is frequency dependant for a given setting of eccentric masses on the two rotating shafts. By varying the eccentricity of these masses by means of an external control, it is possible

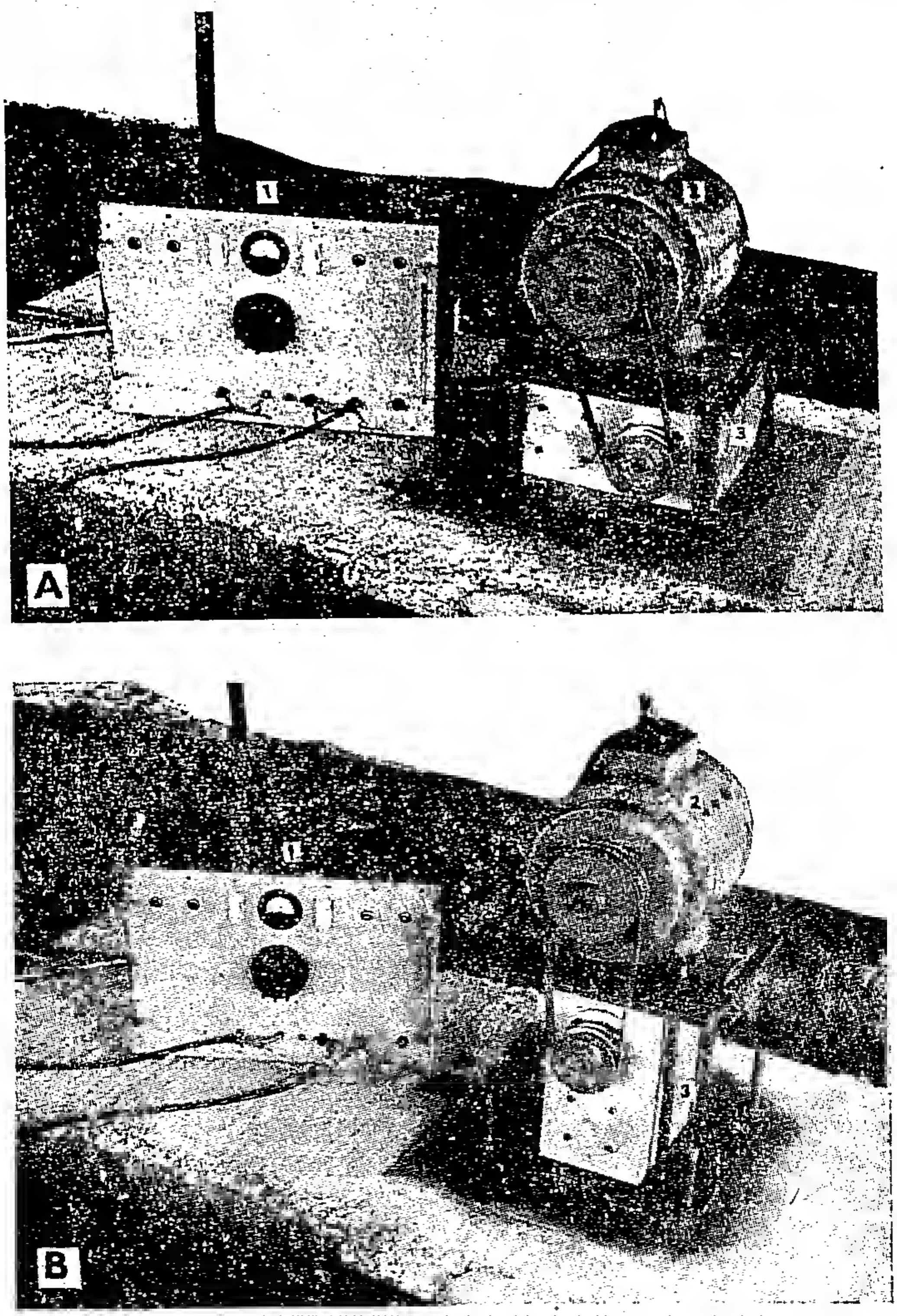


Plate I: Set up for Inducing Vertical (A) and Horizontal (B) Vibrations.
1. Speed Control Unit, 2. Motor, 3. Mechanical Oscillator.

to vary the amplitude of dynamic force even at the same frequency.

The horse power and the rated speed of the motor should be sufficient to realise the peak dynamic force and the maximum frequency of vibration to be induced by the oscillator. A 5 HP motor having a rated speed of 3000 rpm and an appropriate speed control unit should be adequate to meet the requirements of the oscillator described above.

b. Equipment for Measuring Vibration Response

The equipment under this group includes essentially a transducer, an amplifier and a recorder. The transducer (also known as vibration pick-up) converts the physical quantity to be measured into an electrical signal which is related to the physical quantity through a calibration factor. The voltage signal sensed by the transducer is amplified by an electronic unit known as "preamplifier." The amplified signal is then fed to the recorder for recording the waveform or to an oscilloscope for a visual display of the same. If the transducer is sufficiently sensitive and is of self-generating type (i.e., voltage is induced in the transducer cable with the movement of transducer) the use of a preamplifier may be dispensed with.

Vibration transducers may be either displacement, velocity or acceleration type depending on whether the electrical voltage signal induced in it is proportional to one or other of these physical quantitites. They may be further classified as resistive type (e.g., strain gauge based transducer—*see* Plate II) inductive type or piezo-electric type depending on the principle of design and construction of the transducer.

The choice of the physical quantity to be measured as well as the appropriate transducer and the associated measuring equipment necessitate experience and skill and above all engineering judgement.

Plate II shows two numbers of acceleration type strain gauge based transducers, a dual channel carrier amplifier, and a portable dual channel pen recorder. This portable set up was used by the authors for a number of vibration measurements both inside and outside the laboratory. The "geophone" which is suggested to be used in the dynamic test for determination of shear modulus (G) in Sec. 3.3.2 is a velocity type inductive transducer.

The transducers shown in Plate II, enable measurement of absolute accelerations upto a limit of $\pm$ 25 g (g is unit of acceleration due to gravity), the sensitivity being 205 μV (open circuit) per volt of excitation (of bridge) per g. The particular advantage of this type of transducer is that it can be easily calibrated even in the field by rotating the sensing axis through 180° thus causing a variation of acceleration from $-g$ to $+g$.

The amplifier and recorder system shown in Plate II have a combined sensitivity of 5μV/div, the recorder alone having a sensitivity of 1 mV/div. One division of the chart paper to which the above sensitivity values refer is equal to 0.8 mm (the chart width of 40 mm is divided into 50 divisions). The recorder has 4 chart speeds, viz. 1, 5, 25 and 125 mm/sec. The frequency response of the recorder is flat from DC to 40 cps if full width of channel (40 mm) is used for tracing.

Plate III shows the view of a double beam oscilloscope with camera attached to its screen. This oscilloscope has two identical channels each having a sensitivity factor of 1 mV/div to 10 V/div and a time base of 0.2 μ s/div to 5 sec/div in a number of steps.

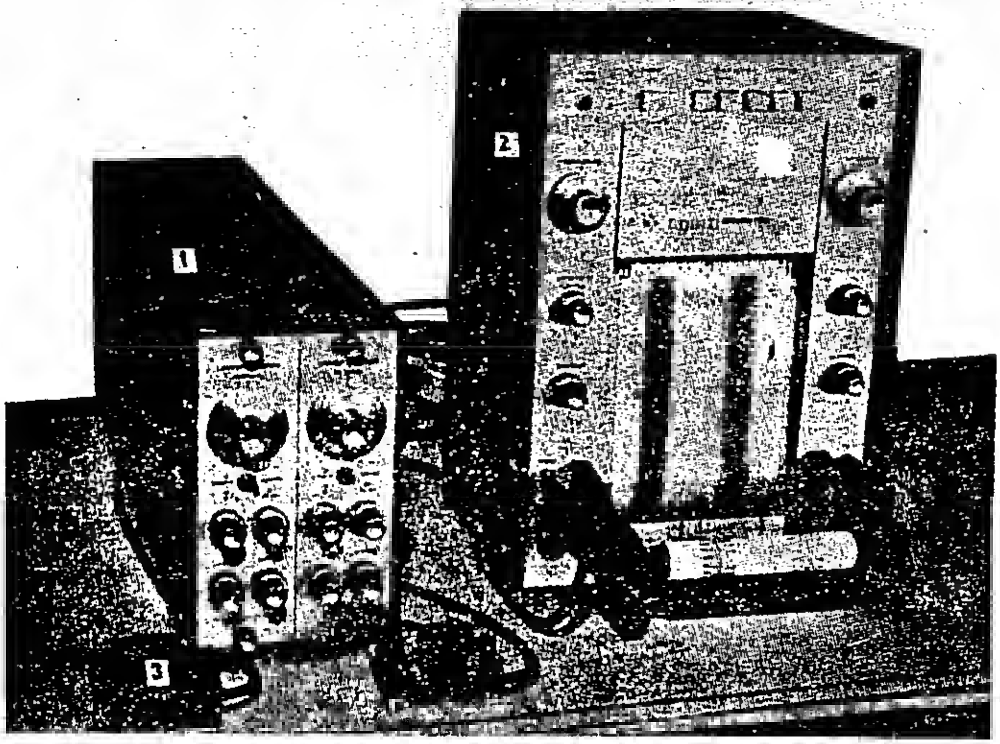


PLATE II: Set up for Recording Vibration Response. 1. Carrier Amplifier, 2. Pen Recorder, and 3. Strain Gauge Type Accelerometers.

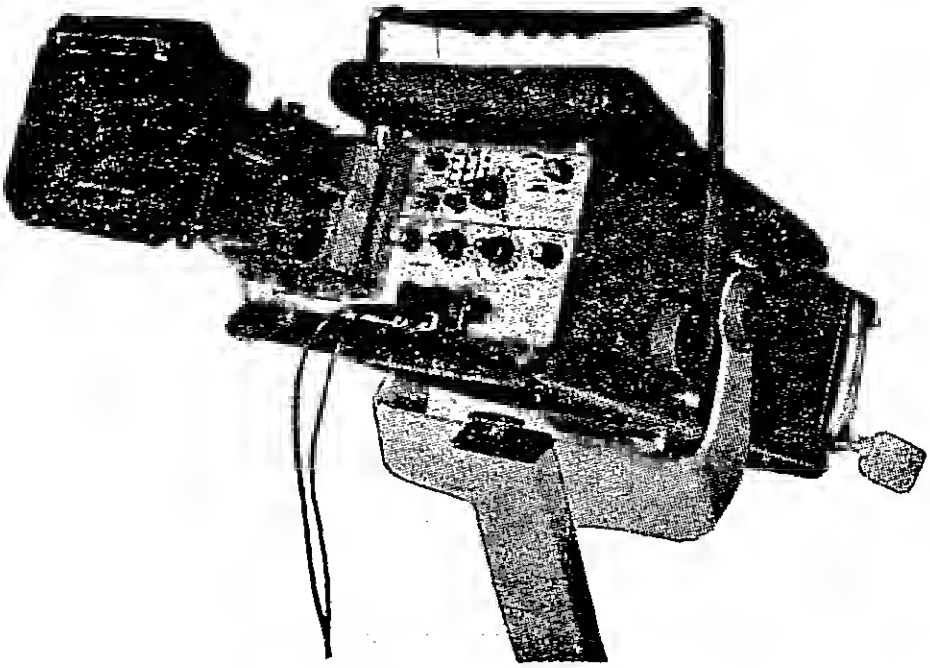


PLATE III: Double-Beam Oscilloscope with Camera.

3.3.2 Experimental Determination of Soil Constants

a. Shear Modulus (*G*) and Poisson Ratio (ν)

IS: 5249-1969[C4.6] suggests the following method for the *in situ* determination of shear modulus of soil:

A plain cement concrete block (M150) of size 1.5 m × 0.75 m × 0.7 m shall be cast at site at the particular depth where the machine foundation is to be laid (Fig. 3.3). A mechanical

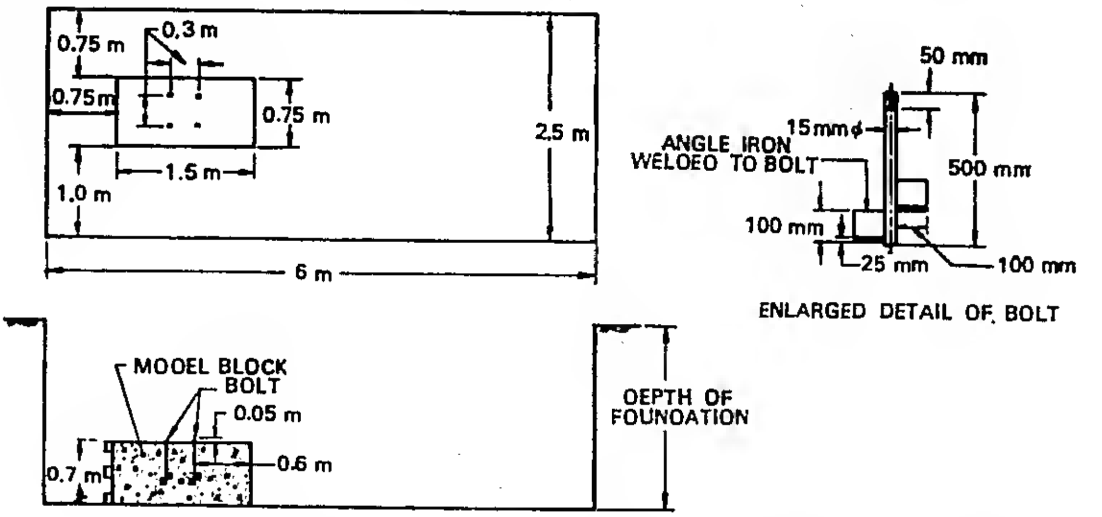


Fig. 3.3: Test Pit with Concrete Block (From IS: 5249-1969; with permission of the Indian Standards Institution, New Delhi).

oscillator should be mounted on the block so that the block is subjected to purely sinusoidal vertical vibration (Plate IA). The oscillator is set to work at a certain low frequency

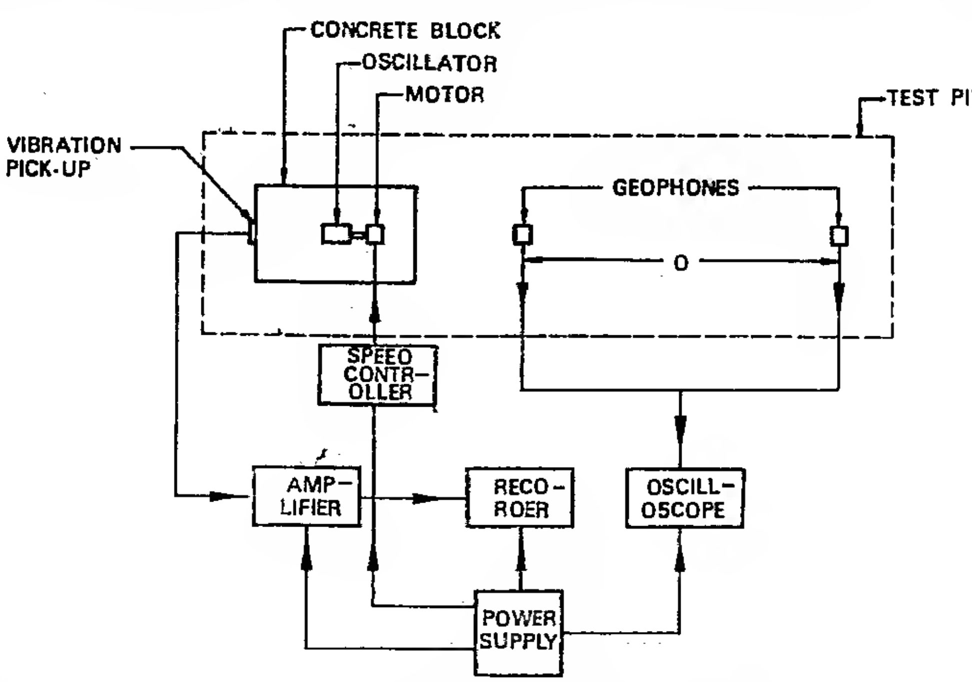


Fig. 3.4: Typical Experimental Set up for *in situ* Dynamic Soil Testing (After IS: 5249-1969; with permission of the Indian Standards Institution, New Delhi).

value. Two geophones of identical characteristics, one connected to the vertical plates and the other to the horizontal plates of an oscilloscope are so positioned along a ray drawn from the block in the longitudinal direction that the Lissajous figure on the oscilloscope screen becomes a circle (Plate IV). The nearest geophone may be at a distance of 30 cm from the block and the farther one varied in position till this condition is achieved. Fig. 3.4 shows the block diagram for the testing arrangement. The distance (D) between the two positions of geophones is then measured. It can be proved from the theory of wave propagation that the wave length λ in this particular case is four times the measured distance D. The velocity V_s of the propagating shear wave can be obtained from the expression $V_s=\lambda\ f$ where f is the frequency of vibration (in cps) which is the same as the frequency of rotation of oscillator. This can be obtained from vibration record or by means of a tachometer.

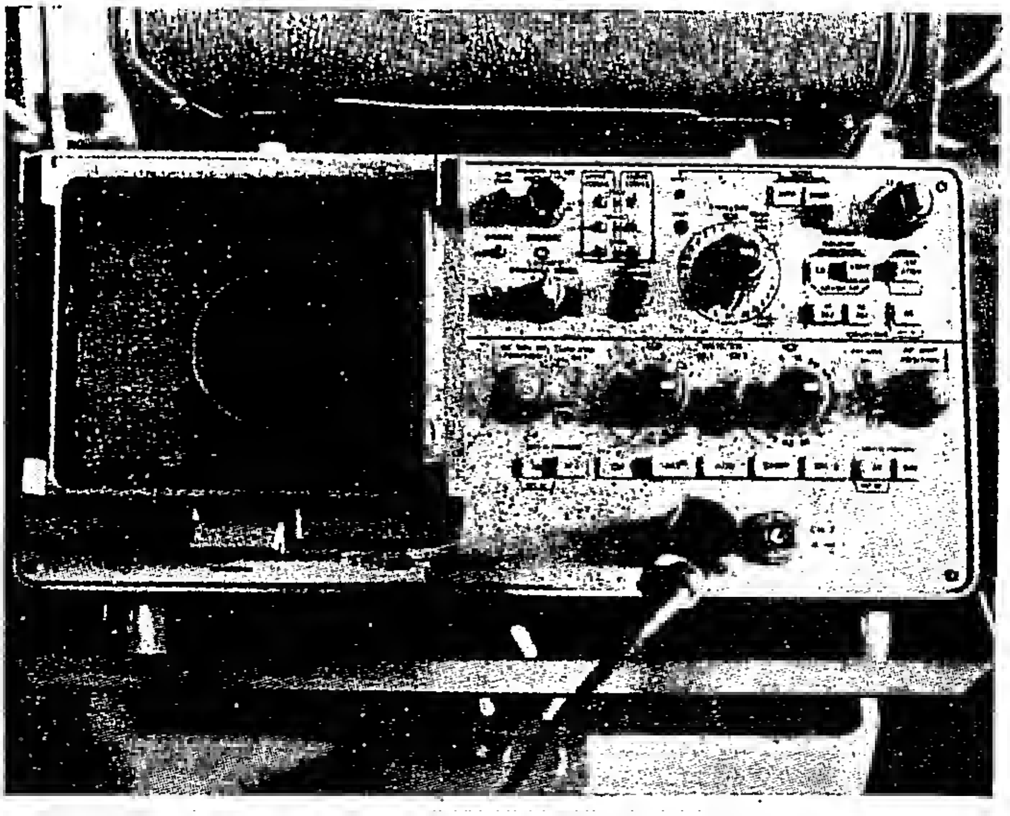


PLATE IV: Lissajous Figure Showing a Phase Difference of $\pi/2$.

Alternatively, the output from the two geophones may be connected to the two vertical amplifiers of a double beam oscilloscope shown in Plate III. The distance between the geophones is so adjusted that the two traces on the oscilloscope screen appear 180° out of phase as shown in Plate V. The distance between the geophones will then be equal to half the wavelength (λ) of vibration. The shear wave velocity may be calculated as before.

The elastic modulus (E) and shear modulus (G) of the soil medium may be calculated from the following relations:

$$E = 2\rho V_s^2\ (1 + \nu) \tag{3.5a}$$

$$G = \rho V_s^2 \tag{3.5b}$$

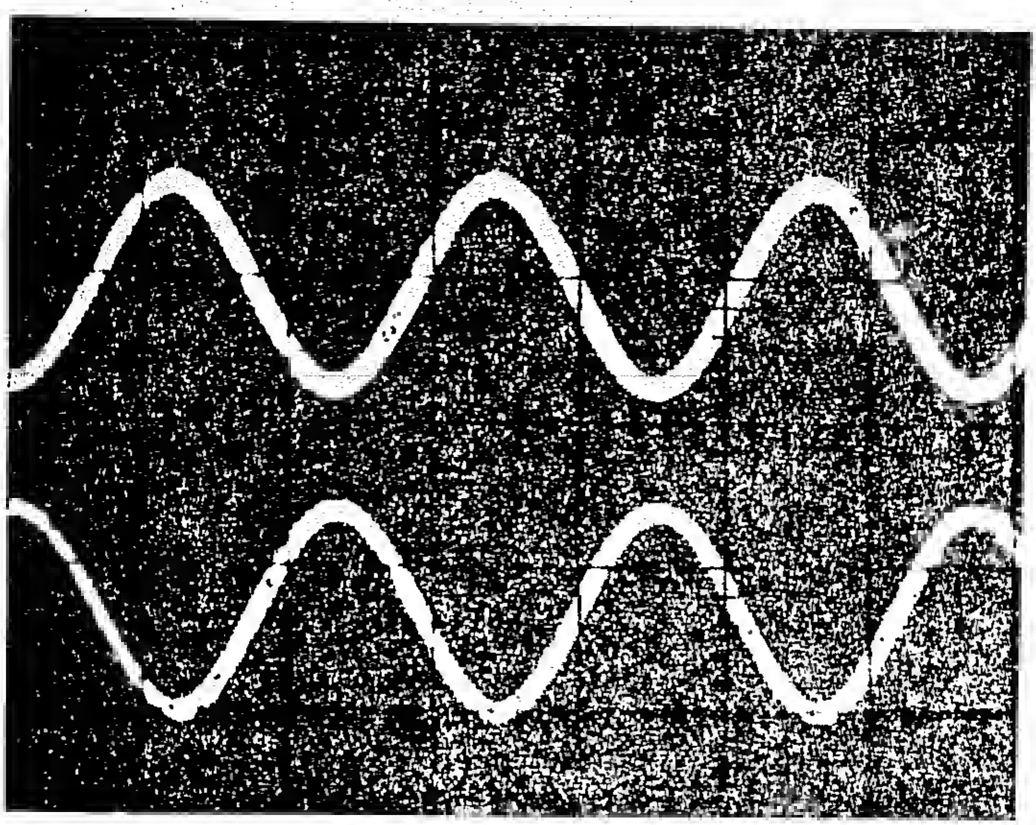


PLATE V: Two Waveforms with a Phase Difference of π.

where

ρ is density of soil

and ν is Poisson ratio of soil.

The following values of Poisson ratio (ν) may be used[C4.6] in Eq. 3.5a

Clay : 0.5

Sand: 0.30 to 0.35

Rock: 0.15 to 0.25

As a general rule, Poisson ratio may be assumed as 0.3 for cohesionless soils and 0.4 for cohesive soils.

The test described above is carried out with the frequency of oscillator set to the operating frequency of the actual machine. The ratio of dynamic force to static weight of concrete test block and the oscillator taken together should be kept the same as that in the actual machine foundation.

Aliter: The shear modulus (G) can also be obtained from the experimentally determined value of coefficient of elastic uniform compression using the following relations:

$$E = 2G\,(1+\nu) \tag{3.6}$$

and

$$C_z = \frac{\alpha E}{1-\nu^2}\,\frac{1}{\sqrt{BL}} \tag{3.7}$$

where α is given in Table 3.2 for various values of aspect ratio (L/B), L and B are the length and breadth of the rectangular model block used in the test.

For the chosen size of the model block (1.5 m$\times$0.75 m$\times$0.7 m) the value of α is 1.09.

Table 3.2

FACTOR α FOR RECTANGULAR FOUNDATIONS

(After Barkan, 1962)

L/B	α
1	1.06
1.5	1.07
2	1.09
3	1.13
5	1.22
10	1.41

b. Coefficient of Elastic Uniform Compression (C_z)

The following method is also based on the provisions given in IS: 5249-1969.[C4.6]

A concrete block (1.5 m$\times$0.75 m$\times$0.7 m) is constructed at the proposed depth of the machine foundation. A mechanical oscillator with a variable speed drive is centrally fixed on the block to induce purely vertical sinusoidal vibrations in the block (Plate IA). A vibration pick-up is fixed on top of the block and the amplitude records are obtained by means of an oscillograph for different frequencies of excitation. The frequency is gradually increased in steps till the resonant stage is passed through. The frequency corresponding to the peak amplitude is the resonant frequency f_n. The coefficient of elastic uniform compression (C_z) is obtained from the formula

$$C_z = \frac{4\pi^2 f_n^2 m}{A_b} \tag{3.8}$$

where

m is mass of the test block plus mounted mechanical equipment

f_n is resonant frequency is cps

A_b is contact area of the test block with soil.

The ratio of dynamic force to static weight (of concrete block and oscillator) should be kept the same as that in the actual machine foundation.

c. Coefficient of Elastic Uniform Shear (C_τ)

IS 5249-1969[C4.6] suggests the following method. The oscillator is so mounted on the concrete test block that it generates horizontal sinusoidal vibrations in the direction of the longitudinal axis of block (see Plate IB). Three vibration pick-ups are mounted on the block—one each at top, middle and bottom along the vertical central line of the transverse face of the block (Fig. 3.5) such that the sensing axes of the vibration :-ups are parallel to the longitudinal axis of the block. The oscillator is worked at :rent frequencies and for each frequency, the amplitude response is obtained on an lograph.

orizontal amplitude versus frequency curves plotted for each of the three pick-ups

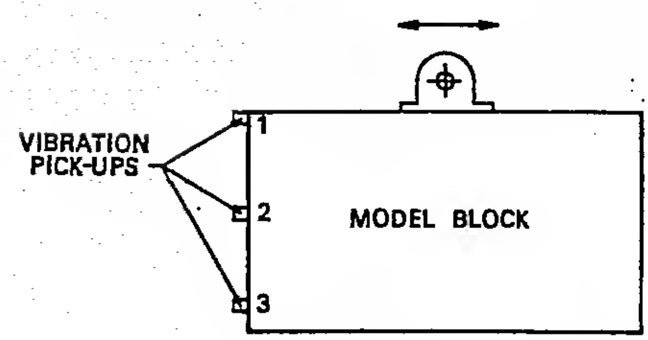


Fig. 3.5: Set up For Horizontal Vibration Test (From IS: 5249-1969; with permission of the Indian Standards Institution, New Delhi).

show two peaks, the frequencies at which correspond to the two resonant frequencies (f_x) of the coupled system (sliding and rocking).

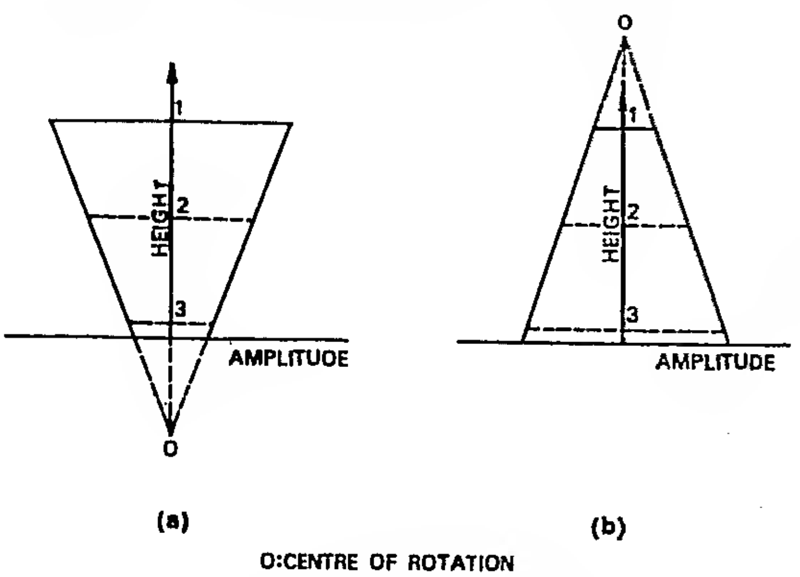


Fig. 3.6: Plot of Amplitude versus Height—(a) First Mode, (b) Second Mode.

To determine the mode of vibration, the resonant amplitude at either resonant frequency is plotted against height of the location of the pick-up above the base of the block (Fig. 3.6). If the plot obtained corresponds to Fig. 3.6a, then the particular resonant frequency (f_{x1}) corresponds to the first (or fundamental) mode of the coupled motion. However, if it corresponds to that shown in Fig. 3.6b, then the resonant frequency corresponds to the second mode (f_{x2}).

The coefficient of elastic uniform shear of soil (C_τ) is then obtained from

$$C_\tau = \frac{8\pi^2 \alpha f_x^2}{(P+Q) \pm \sqrt{(P+Q)^2 - 4PQ\alpha}} \tag{3.9}$$

The positive sign shall be taken when f_x is the second natural frequency (f_{x2}) of the coupled motion and negative sign when f_x is the first natural frequency (f_{x1}). The following definitions apply:

$P \quad = A_t/m$

$\alpha \quad = \varphi/\varphi_0$

$Q \quad = 3.46\ I/\varphi_0$

$\varphi \quad$ = mass moment of inertia of block about the horizontal axis passing through the centre of gravity of block and perpendicular to the direction of vibration;

φ_0 =mass moment of inertia of block about the horizontal axis passing through the centre of gravity of contact area of block and soil perpendicular to the direction of vibration;

I =moment of inertia of foundation contact area about the horizontal axis passing through the centre of gravity of contact area and perpendicular to the direction of vibration;

A_f =base area of the foundation.

3.3.3 *Use of Soil Constants and their Relationship*

The values of soil constants C'_Z, C'_τ, etc. to be used in the design of actual machine foundation may be obtained from the following relations (IS: 5249-1969):

$$C'_Z = C_Z\sqrt{A_b/A_f} \tag{3.10a}$$

or

$$C'_\tau = C_\tau\sqrt{A_b/A_f} \tag{3.10b}$$

where C_z is the value obtained from the test on concrete block explained earlier, and A_b is the base area of the concrete block used in the text. Actual tests have shown, however, that the inverse area relationship given by Eqs. (3.10a) and (3.10b) is not applicable beyond a certain area limit which may be taken as 10m², after which the value of C_z may be assumed constant for design purposes. This is in view of the fact that the variation of C_z for large base areas, often encountered in machine-foundation problems is not well established.

Where exact data concerning soil are not available, the values of C_z given in the Table 3.3 may be used for preliminary designs.

The values given in Table 3.3 may be used for foundations with base areas of 10 m², or more. If the base area is less than 10 m², the values tabulated shall be multiplied by $\sqrt{10/A_f}$, where A_f is the actual base area of the machine foundation.

Table 3.3

RECOMMENDED DESIGN VALUES FOR C_Z
(IS: 2974-Pt. I-1969)

Category of soil	Permissible bearing capacity σ_p of soil in kg/cm²	Coefficient of elastic uniform compression (C_Z) in kg/cm³
Weak soils	1	2
	2	4
Medium soils	3	5
	4	6
Strong soils	5	7
Rocks	>5	>7

Having determined one of the soil constants say C_z from the *in situ* testing of soil, the other dynamic soil constants may be evaluated approximately using the following relations suggested by Barkan.

(i) Coefficient of elastic non-uniform compression (C_θ) $\quad 2\,C_z \quad$ (3.11a)

(ii) Coefficient of elastic uniform shear (C_τ)	$0.5\ C_z$	(3.11b)
(iii) Coefficient of elastic non-uniform shear (C_ψ)	$0.75\ C_z$	(3.11c)

3.3.4 Expression for Spring Stiffness of Elastic Supports

a. Soil

The spring coefficients (K) for the various modes of vibration are calculated as follows:

i. For vertical motion	$K_z = C_z A_f$	(3.12a)
ii. For horizontal (or sliding) motion	$K_\tau = C_\tau A_f$	(3.12b)
iii. For rocking motion	$K_\theta = C_\theta I_{x\ (\text{or } y)}$	(3.12c)
iv. For torsional motion (rotation about vertical axis)	$K_\psi = C_\psi I_z$	(3.12d)

where A_f is area of horizontal contact surface between foundation and soil, I is second moment of contact area about the horizontal axis (x or y) passing through the centroid of the base and normal to the plane of rocking, and I_z is second moment of contact area about the vertical axis passing through the centroid of the base.

b. Elastic Pads

Let A and t denote the contact area and thickness of one elastic pad. If E and G represent the modulus of elasticity and shear modulus of the material of the pad respectively, then the stiffness factors, with the usual notation, are given by the following expressions:

i. For vertical translation	$K_z = EA/t$	(3.13a)
ii. For horizontal translation	$K_x = GA/t$	(3.13b)
iii. For rotational motion in a vertical plane xz (or yz)	$K_\theta = \dfrac{E I_y\ (\text{or } x)}{t}$	(3.13c)
iv. For twisting motion in the horizontal plane xy	$K_\psi = \dfrac{G I_z}{t}$	(3.13d)

If the foundation is supported on N elastic pads symmetrically arranged under the foundation (as in Fig. 3.1b), the resulting stiffness would be as under

i. Vertical translation	$K_z = N\dfrac{EA}{t}$	(3.13e)
ii. Horizontal translation	$K_x = N\dfrac{GA}{t}$	(3.13f)
iii. Rotational motion in xz (or yz) plane	$K_\theta = \dfrac{I'_x}{N} K_z$	(3.13g)
iv. Twisting motion	$K_\psi = \dfrac{I'_z}{N} K_x$	(3.13h)

It may be noted that A in Eq. 3.13 (e) and (f) refers to the area of each pad and I' in Eqs. 3.13 (g) and (h) to the moment of inertia of the group of supports about respective axes (Eq. 3.3).

e. Steel Springs

Let d be the diameter of spring wire, D the diameter of spring coil, n the number of windings in each coil, h the height of each spring coil, and G the shear modulus of the material of the wire (for steel, $G \simeq 8 \times 10^6$ T/m^2). The stiffness factors of the spring coil are expressed as follows:

i. *Vertical stiffness (K_s) of one spring:*

$$K_s = \frac{1}{n} \frac{Gd^4}{8D^3} \tag{3.14a}$$

If there are N springs in the coil, the resultant vertical stiffness (K_z) is given by $N.k_s$. The permissible load (P) on the spring coil is given by

$$P = \frac{\tau \pi d^3}{8 \alpha D}$$

where α is a factor given by

$$\alpha = 1 + 1.25\ (d/D) + 0.875\ (d/D)^2 + (d/D)^3$$

and τ is permissible shear stress in the spring material.

Table 3.4 contains the value of K_z (for $n=1$) and P for a spring coil for different values of D and d.

Table

DATA OF

Permissible load on spring (P),

Diameter of coil (D) mm	6 mm		10 mm		12 mm		14 mm		16 mm		Diameter of 20 mm	
	P kg	K_s kg/mm	P kg	K_s kg/mm	P kg	K_s kg/mm	P kg	K_s kg/mm	P kg	K_s kg/mm	P kg	K_s kg/mm
20	172.09	168.08										
30	131.39	49.8	507.12	384.26								
40	105.25	21.0	426.58	162.11	688.37	336.15						
50	87.54	10.76	364.96	83.0	597.73	172.11	898.89	318.85	1269.67	543.95		
60	74.86	6.23	317.84	48.03	525.55	99.60	798.13	184.52	1138.73	314.78	2028.46	768.52
70	—		281.05	30.25	467.82	62.72	715.33	116.2	1027.75	198.23	1857.28	483.96
80	—		251.68	20.26	421.00	42.02	646.98	77.84	934.31	132.80	1706.31	324.22
90			227.76	14.23	382.43	295.11	589.96	54.67	855.31	93.27	1574.62	227.71
100			207.93	10.38	350.18	21.51	541.84	39.86	789.98	67.99	1459.86	166.00
120			—		299.42	12.45	465.39	23.07	679.88	39.35	1270.00	96.40
140			—		—		407.55	14.53	597.28	24.78	1124.19	60.50
160							—		532.30	16.60	1006.72	40.52
180									—		911.04	28.46
200											831.725	20.75
220												
240												
260												
280												
300												
320												

ii. *Horizontal stiffness (K_x) of one spring:*

$$K_x = K_s \left[\frac{1}{0.385\, \alpha \left\{ 1 + \frac{0.77}{D^2} h^2 \right\}} \right] \tag{3.14b}$$

Where α is the coefficient to be obtained from Fig. 3.7 for known values of h/D and δ_s/h, h is height of spring coil, and δ_s is the static compression of the spring coil under vertical load.

If there are N springs in the coil, the resultant horizontal stiffness is $N.k_x$.

iii. *Stiffness (K_θ) against rotation in vertical plane for a group of springs:*

$$K_\theta = \frac{I'}{n} \frac{Gd^4}{8\, D^3} = I'_{x\,(\text{or } y)}\, K_s \tag{3.14c}$$

where I' is moment of inertia of the group of isolated spring supports about the axis of rotation x or y (Eq. 3.3a and 3.3b).

3.4

CYLINDRICAL SPRINGS*
Axial Stiffness (K_s for one winding)†

spring wire (d)											
22 mm		24 mm		26 mm		28 mm		30 mm		32 mm	
P kg	K_s kg/mm	P kg	K_s kg/mm	P kg	K_s kg/mm	P kg	K_s kg/mm	P kg	K_s kg/mm	P kg	K_s kg/mm
2376.39	708.57										
2195.05	474.69	2750.00	672.00								
2034.00	333.39	2562,14	472.18	3159.85	650.36	3827,30	874.77	4564.05	1152.77		
1891.87	243.04	2390.94	344.22	2958.54	474.11	3595.55	637.71	4302.31	840.38	5060.00	1088.00
1655.41	140.64	2102.20	199.20	2613.97	274.37	319.25	369.04	3839,19	486.33	4554.91	629.57
1468.65	88.57	1871.30	125.44	2334.77	172.78	2861.33	232.40	3452.88	306.26	4110.99	396.46
1318.41	59.33	1684.02	84.04	2106.35	115.75	2887.91	155.69	3130.90	205.17	3737.72	265.60
1195.35	41.67	1529.72	59.02	1917.02	81.30	2359.84	109.35	2860.54	144.10	3421.23	186.54
1092.90	30.38	1400.71	43.03	1758.00	59.26	2167.38	79.71	2631.28	105.05	3151.91	135.99
1006.39	22.83	1291.40	32.33	1622.79	44.53	2003.15	59.89	2434.92	78.92	2920.35	102.19
—		1197.68	24.90	1506.54	34.30	1861.55	46.13	2265.12	60.79	2719.50	78.70
		—		1405.62	26.98	1738.33	36.28	2117.00	47.81	2543.87	61.90
				—		1630.18	29.05	1986.75	38.28	2389.12	49.56
						—		1871.38	31.13	2251.81	40.29
								—		2129.22	33.2

* The values given in this table are calculated for G=830,000 kg/cm² and T=6000 kg/cm².
† For a spring coil with n windings, divide the tabulated values by n.

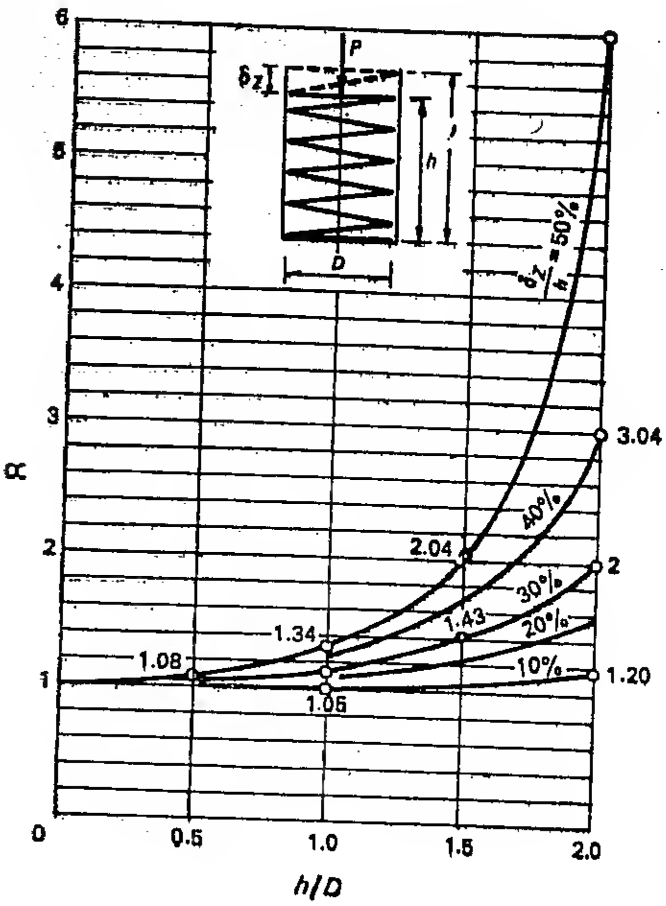


Fig. 3.7: Factor α as Function of h/d and δ_z/h for Steel Springs (After Major, A., *Vibration Analysis and Design of Foundations for Machines and Turbines*, Akademiai Kiado, Budapest, 1962; with permission).

iv. *Torsional stiffness* (K_ψ) *of a group of springs;*

$$K_\psi = \frac{I_z'}{n} \frac{Gd^4}{8D^3}\left[\frac{1}{0.385\,\alpha\left\{1+\frac{0.77\,h^2}{D^2}\right\}}\right] \tag{3.14d}$$

I_z' is obtained for the group of springs from Eq. (3.3c),

d. Piles

i. *Vertical Stiffness*

Bearing pile: The natural frequency of vertical vibrations (ω_n) of an end bearing pile carrying a load W may be obtained from the relation

$$\beta \tan \beta = \alpha \tag{3.15}$$

where α is the ratio of self weight of the pile to the external load carried by it, and

$$\beta = \omega_n H \sqrt{\frac{\gamma}{Eg}} \tag{3.16}$$

E and γ are the modulus of elasticity and density of the material of the pile; and H is the height of the pile.

Table 3.5 shows the values of β (corresponding to the first mode of vibration) for various values of α for which equation 3.15 is valid. For a pile of known characteristics, the natural frequency may be obtained from the above tabulated data.

Alternatively, the natural frequency of an end bearing pile may be read from Fig. 3.8 suggested by Richart[08.41]. This figure shows the variation of natural frequency (f_n in cpm) with pile length (H in feet) for different materials under different ranges of direct stress.

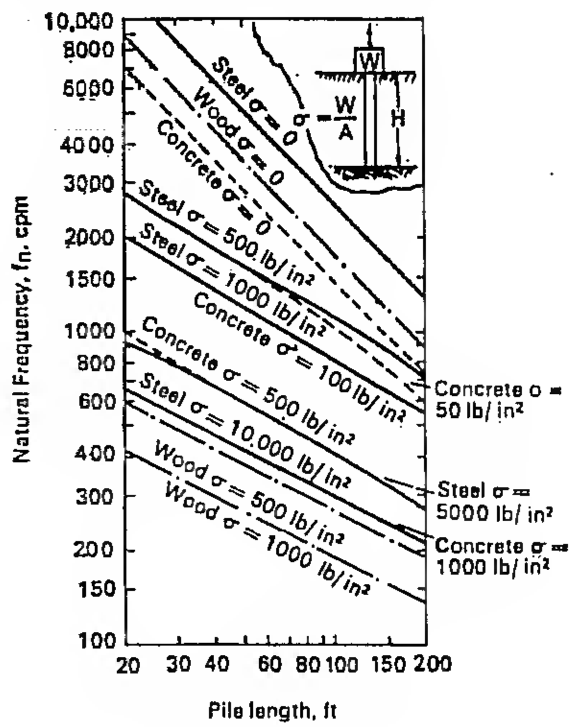


Fig. 3.8: Characteristics of Vertical Vibration of End Bearing Piles under Axial Loads (From Richart, F. E., Jr., "Foundation Vibrations", *Trans. ASCE*, 127, Pt. I, pp. 863-98, 1962; with permission).

The vertical stiffness (K_z) of a single pile may be obtained from the computed natural frequency, thus

$$K_z = \frac{\omega_n^2}{g}\left(1+\frac{\alpha}{3}\right)W \tag{3.17}$$

Table 3.5

COEFFICIENTS FOR NATURAL FREQUENCY OF PILES

α	0.01	0.10	0.50	0.70	0.90	1.00	1.50	2.00	3.00	4.00	5.00	10.00	20.00	100	∞
β	0.10	0.32	0.65	0.75	0.82	0.86	0.98	1.08	1.20	1.27	1.32	1.42	1.52	1.57	$\pi/2$

Equation 3.17 assumes the validity of a single-degree freedom system with one-third of the weight of the pile added to W for calculations. It can be verified that the error induced in this assumption is less than 1 per cent for a value of $\alpha=1$ and this error decreases further as α assumes still less values.

Friction pile: The expression for K_z of a single pile is given by

$$K_z = C_p A_s \tag{3.18}$$

Where C_p is the coefficient of elastic resistance and A_s is the surface area of the pile.

The value of C_p depends on the spacing of the piles and the character of the surrounding soil. Its approximate value when the spacing of the piles exceeds about six times the diameter (or the cross-sectional dimension of the pile) may be taken from the following data.[C4.9]

Nature of soil	C_p (t/m^3)
(a) Soft and sandy clays	500
(b) Sandy soils	2500
(c) Loess and sandy clay	3000

If the pile spacing is small compared to the diameter then a multiplying correction factor (α) given in Table 3.6 should be applied to Eq. 3.18.

Table 3.6

CORRECTION FACTOR α FOR A PILE GROUP

S/D	α
3	0.35
4.5	0.58
6	0.63
∞	1.00

ii. *Horizontal stiffness:* Assuming that the pile is free from the surrounding soil and fixed at top and bottom over a length h as in Fig. 3.9, the lateral stiffness of a single pile is given by

$$K_{x\,(\text{or }y)} = \frac{12\,E}{h^3}\,I_{x\,(\text{or }y)} \tag{3.19}$$

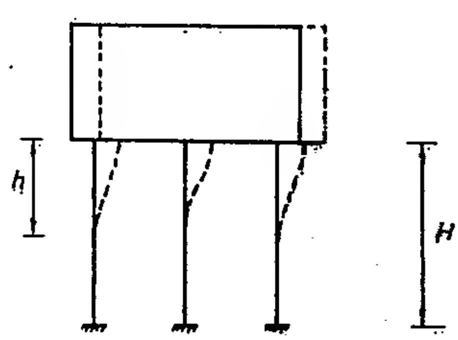


Fig. 3.9: Block Foundation on Piles.

where I_x (or y) is moment of inertia of the pile cross-section about the x (or y) axis; h is generally taken $\frac{1}{4}$ to $\frac{1}{3}$ of the total depth of pile (H).

iii. *Rotary stiffness:* The stiffness factor (K_θ) against rotation of a pile group in yz (or xz) plane is given by

$$K_\theta = I'_{x\,(\text{or }y)}\,K_z \tag{3.20}$$

where K_z is vertical stiffness of a single pile and $I'_{x\,(\text{or }y)}$ is the moment of inertia of pile group about x (or y) axis (Eq. 3.3).

iv. *Torsional stiffness:* The stiffness factor (K_ψ) for torsional vibration is given by

$$K_\psi = K_{x\,(\text{or}\,y)}\, I'_z \tag{3.21}$$

3.3.5 Damping Coefficient (ζ)

a. *From forced vibration test:* A mechanical oscillator is mounted on a concrete block (Fig. 3.3) in such a manner that it induces pure vertical vibrations. The vertical amplitude response is obtained from a pick-up mounted on its top for various frequencies of excitation till the "resonance" is passed through. A graph (Fig. 3.10A) is drawn between amplitude and frequency of excitation. The frequency (f_n) corresponding to the peak amplitude represents the "resonant frequency".

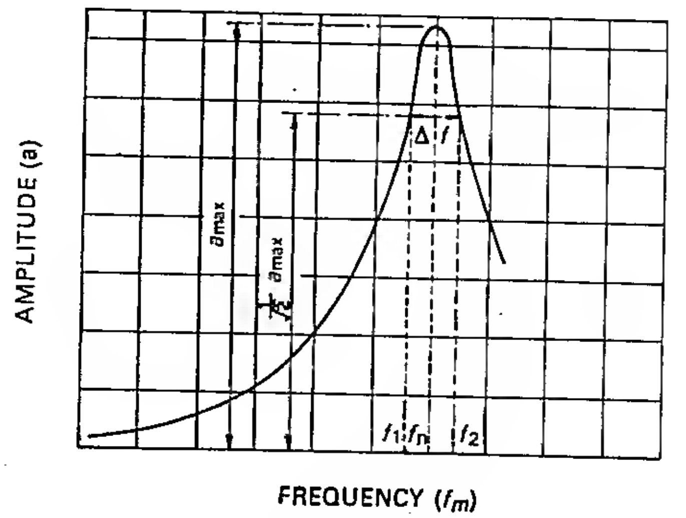


Fig. 3.10A: Response Curve under Forced Vibration.

The damping factor (ζ) can now be obtained using the relation

$$\zeta = \frac{\Delta_f}{2 f_n} \tag{3.22}$$

where Δ_f is the intercept between the two points on the response curve at which the amplitude is 0.707 (or $1/\sqrt{2}$) times the peak amplitude and f_n is the resonant frequency (at which the amplitude is the maximum).

b. *From free vibration test:* Free vibrations are induced in the block in some suitable way, such as by hitting the block on top with a hammer. The decay curve (Fig. 3.10B) is

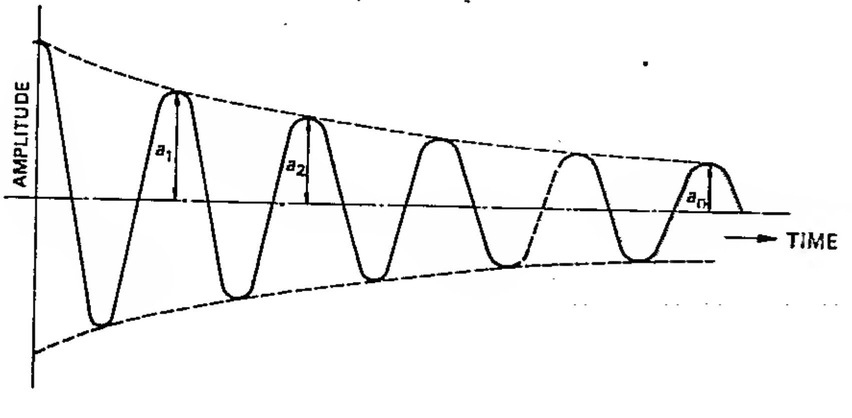


Fig. 3.10B: Decay Curve under Free Vibration.

obtained on a vibration recorder connected to a vibration pick-up which is fixed to the concrete block. The damping factor is obtained from the formula

$$\zeta = \frac{1}{2\pi} \log \frac{a_1}{a_2} \tag{3.23}$$

where a_1 and a_2 are the vibration amplitudes at two successive peaks on the decay curve.

CHAPTER FOUR

Analysis and Design of Block-Type Machine Foundations

4.1 Modes of Vibration of a Block Foundation

A BLOCK FOUNDATION has, in general, six degrees of freedom and, therefore, six natural frequencies (one corresponding to each mode of vibration). Three of them are translations along the three principal axes and the other three are rotations about the three axes (Fig. 4.1). The vibratory modes may be 'decoupled' or 'intercoupled' depending on the relative positions of the centre of gravity of the machine foundation and the centroid of its base area. The natural frequency is determined in a particular mode (decoupled or intercoupled) and compared with the operating frequency.

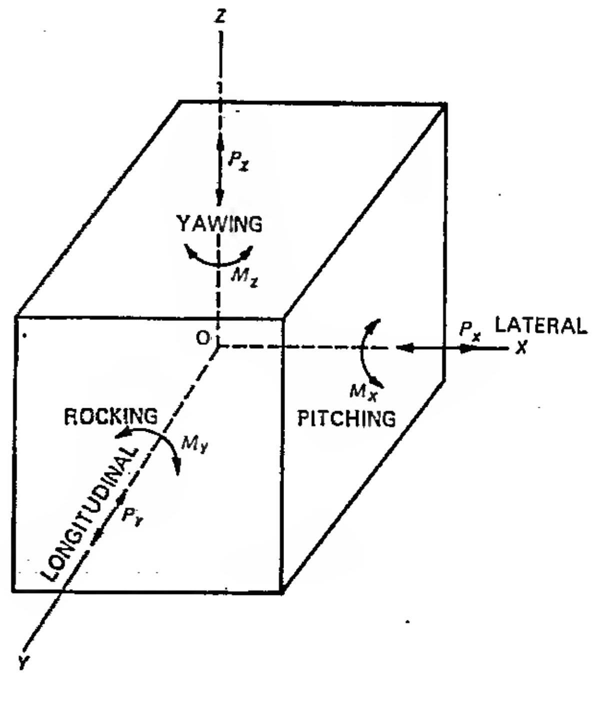


Fig. 4.1: Modes of Vibration of a Block Foundation.

4.2 Review of Methods for Dynamic Analysis

a. Empirical Methods

These methods are based on experimental data collected from practice.

i. Tschebotarioff[C3.49] gave an approximate relation between contact area of a machine foundation and a variable which he termed as "reduced natural frequency" (f_{nr}). The latter is defined as the product of the natural frequency and square root of contact pressure.

$$f_{nr}=f_n\sqrt{\sigma} \quad \text{where } \sigma=\frac{W}{A_f}, \text{ t/ft}^2 \tag{4.1}$$

W is weight of foundation and A_f is the base area. This correlation (shown in Fig. 4.2) is used to determine the natural frequcny in terms of weight of the machine plus foundation and the contact area. The graph is given only for four particular types of soil and its use for any other type of soil may not be justified.

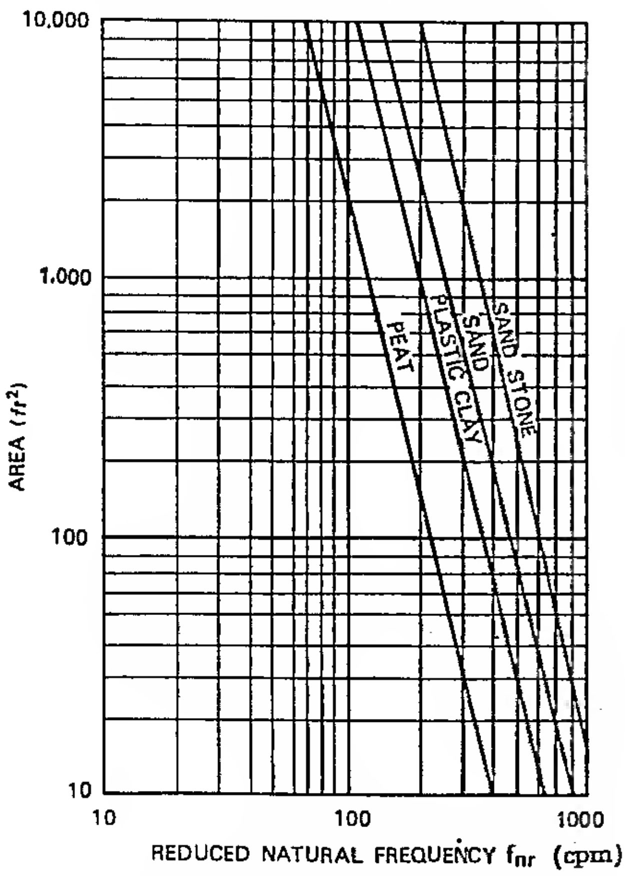


Fig. 4.2: Graphical Data on Engine Foundations (After Tschebotarioff, G.F., "Performance Records of Engine Foundations", *ASTM Special Technical Publication No.* 156, 1953; with permission).

ii. Alpan[C3.1] made use of Tschebotarioff's data and developed an expression for natural frequcncy, in the form:

$$f_n=\frac{\alpha}{\sqrt{W}}\left(A_f\right)^{1/4} \tag{4.2}$$

where f_n is natural frequency, W is weight of machine and foundation (in kg), A_f is contact area (m²), α is a constant equal to 3900 for peats, 69,000 for plastic clays, 82,000 for sands

and 111,000 for sand stones. In view of the empirical nature of these methods, they may be used only for preliminary design purposes when the soil constants are not readily available.

b. Methods Based on Considering Soil as a Semi-infinite Elastic Solid

The soil is considered to be a semi-infinite isotropic elastic solid (also called the "elastic half space") which is subjected to vibrations from an oscillator having a circular base. The term "oscillator" here corresponds to the machine foundation. The pertinent properties of soil as elastic body are: (1) the shear modulus G, (2) Poisson ratio ν, and (3) the mass density ρ. The theory predicts the amplitudes of motion resulting from a periodic force. Under this hypothesis, three types of pressure distribution are assumed at the basc of the foundation: (1) uniform, (2) parabolic, and (3) that produced by a rigid base (Fig. 4.3).

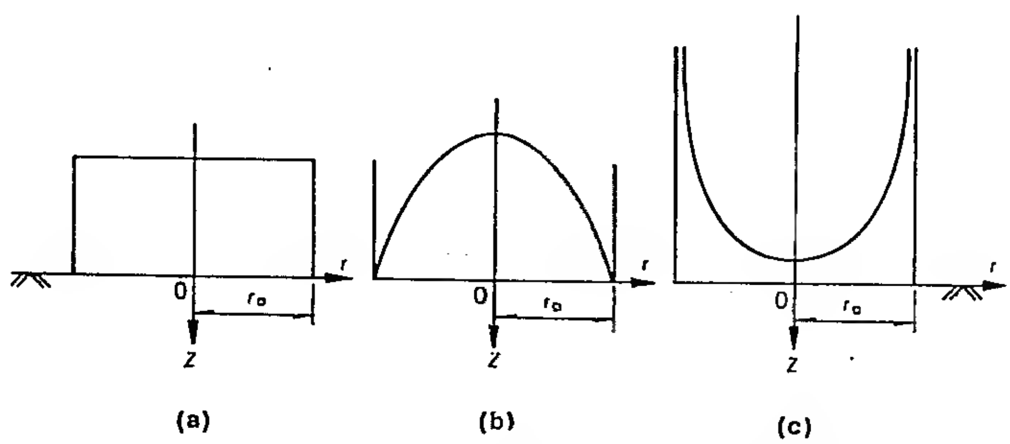


Fig. 4.3: Types of Pressure Distribution—**(a)** Uniform, **(b)** Parabolic, **(c)** Rigid base.

i. Among the available solutions to this problem for vertical vibrations the one that was offered originally by Sung[C3.47] and later developed by Richart[C3.41] is popular. This approach is based on the assumption of a rigid base-type pressure distribution under the foundation. Two types of exciting force are considered—one type in which the amplitude of exciting force is constant and the other in which it is dependent upon exciting frequency. As stated in Chapter 2, only the latter case is of interest in the machine foundations. However, for academic interest the case of constant force excitation is also discussed below. Figs. 4.4 shows the non-dimensional plots drawn with the frequency factor (ω_0) and amplitude factor (a_1 and a_2) taken respectively on the abscissa and the mass ratio (b_z) as ordinate for various values of Poisson ratio (ν).

The notation used is as follows:

Frequency factor $(\omega_0)=\omega_z r_0\sqrt{\rho/G}$ (4.3)

Mass ratio $(b_z)=m/\rho r_0^3$ (4.4)

where ρ is the mass density of soil, m is the mass of machine foundation, r_0 is the radius of equivalent circular base $\left(\sqrt{\frac{A_f}{\pi}}\right)$ and A_f is the base area of foundation.

Amplitude factor (a_1, a_2)

$$a_1=a_z G\frac{r_0}{P_0} \text{ for constant force } (P_0) \text{ excitation} \tag{4.5a}$$

$$a_2 = a_z \frac{\rho r_0^3}{m_e e} \text{ for rotating mass } (m_e e \omega_m^2) \text{ excitation} \tag{4.5b}$$

where a_z is the resonant (vertical) amplitude.

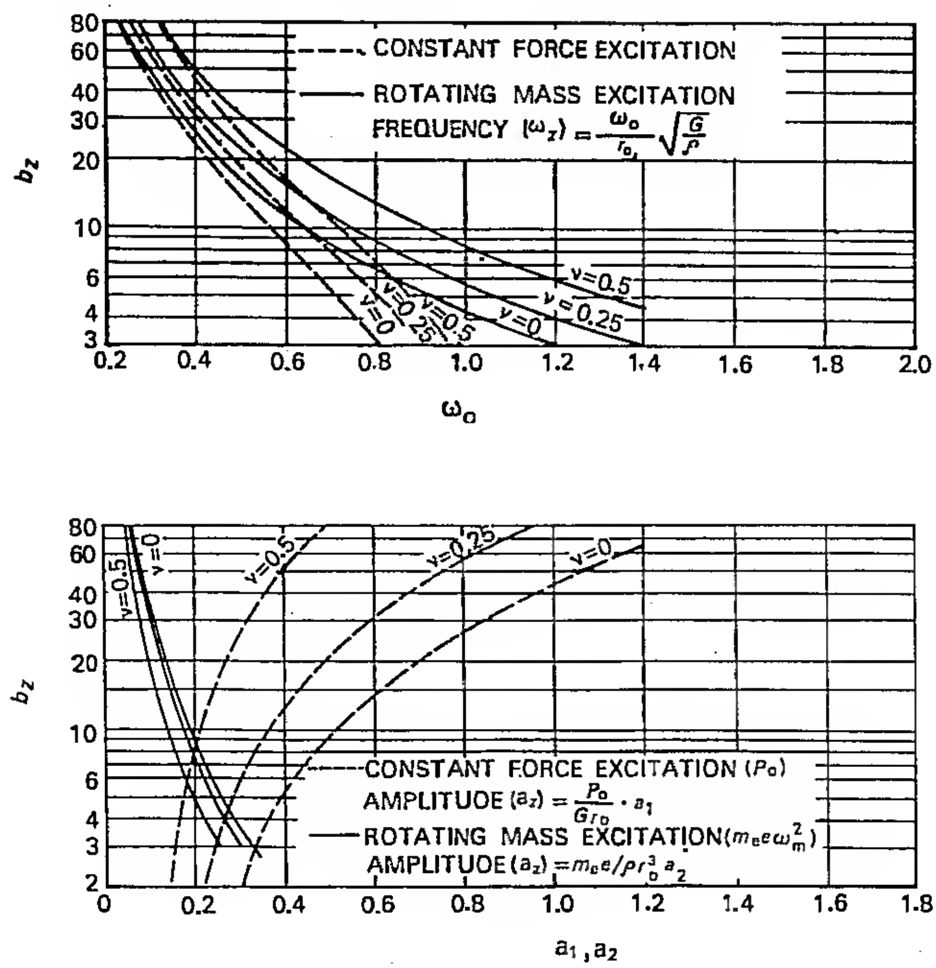


Fig. 4.4

ii. Arnold, Bycroft and Warburton[C3.2] have examined the sliding and rocking modes of vibration of a cylindrical foundation resting on the surface of a semi-infinite elastic medium. The pressure distribution under the foundation was assumed to be of the rigid base pattern. Only one case of Poisson ratio (equal to zero) was considered for showing the amplitude–frequency relations. Figs. 4.5 and 4.6 show the characteristics of sliding and rocking oscillations of the foundation. The figures are self-explanatory. For foundations with non-circular bases, the radius of equivalent circular base is obtained as below:

For translation $$r_0 = \sqrt{\frac{A_f}{\pi}} \tag{4.6a}$$

For rotation about one of the horizontal axes $$r_\theta = \sqrt[4]{\frac{4I_0}{\pi}} \tag{4.6b}$$

where I_0 is the moment of inertia of the base about the axis of rotation and A_f is the base area of the foundation.

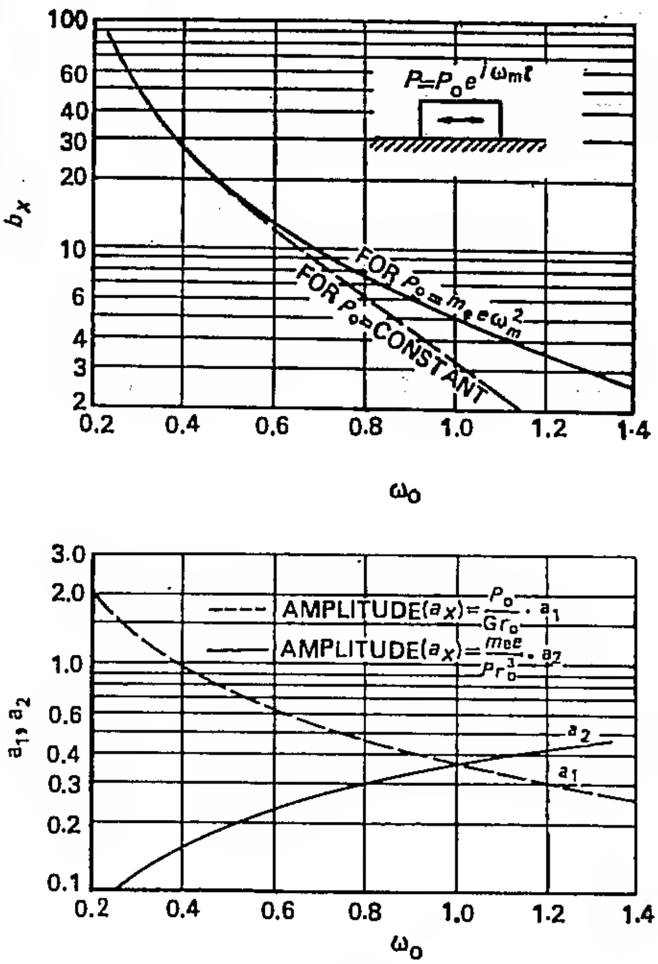


Fig. 4.5

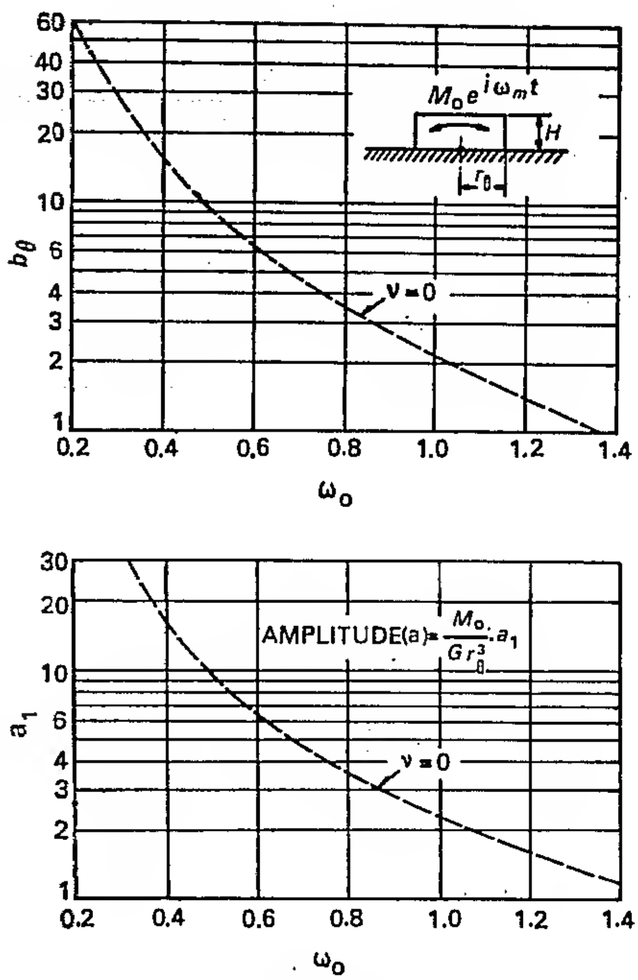


Fig. 4.6

Figs. 4.4–4.6: Characteristics of Vertical **(4.4)**, Pure Sliding **(4.5)**, and Rocking Oscillations **(4.6)**, as Functions of Inertia Ratio b. (From Richart, F. E., Jr., "Foundation Vibrations", *Trans. ASCE*, 127 (I), 863-98, 1962; with permission).

The mass ratio b_x in Fig. 4.5 is given by

$$b_x = m/\rho r_0^3 \tag{4.6c}$$

The frequency factor ω_0 and the inertia ratio b_θ for rotary mode in Fig. 4.6 are given by

$$\omega_0 = \omega_\theta r_\theta \sqrt{\rho/G} \tag{4.6d}$$

and

$$b_\theta = \frac{\varphi_0}{\rho r_\theta^5} \tag{4.6e}$$

where r_θ is given by equation 4.6b.

The amplitude (a_x) under pure sliding oscillations is obtained from relations similar to Eqs. 4.5a and 4.5b with a_z replaced by a_x.

For rocking oscillations (Fig. 4.6) the amplitude of rocking moment (M_0) was assumed

to be constant. In machine foundations, however, the exciting moment is a function of the operating frequency.

iii. Hsieh[CS.22] gave an analytical treatment to the problem of a circular foundation under the supposition of rigid base-type pressure distribution underneath it. He expressed the equations of motion for translation of such a foundation as follows :

Translation

$$m\ddot{x}+\sqrt{\rho G}\,r_0^2F_2\dot{x}+F_1Gr_0x=P_0\sin\omega_m t \tag{4.7a}$$

Rocking

$$\varphi\ddot{\theta}+\sqrt{\rho G}\,r_0^4F_2\dot{\theta}+Gr_0^3F_1\theta=M_0\sin\omega_m t \tag{4.7b}$$

where

F_1, F_2 are functions contained in Table 4.1

α_1, α_2 are contained in the expression for F_1 such that $F_1=\alpha_1-\alpha_2\omega_0^2$

ω_0 is the frequency factor$=\omega_n r_0\sqrt{\rho/G}$

and P_0 is the amplitude of exciting force.

Table 4.1

FUNCTIONS F_1 and F_2 (Hsieh, 1962)

Mode	Poisson's ratio ν	F_1	F_2
Vertical	0	$4.0-0.5\,\omega_0^2$	$3.3+0.4\,\omega_0$
$(0<\omega_0<1.5)$	0.25	$5.3-1.0\,\omega_0^2$	$4.4+0.8\,\omega_0$
	0.5	$8.0-2.0\,\omega_0^2$	6.9
Horizontal	0	$4.5-0.2\,\omega_0^2$	$2.4+0.3\,\omega_0$
$(0<\omega_0<2.0)$	0.25	$4.8-0.2\,\omega_0^2$	$2.5+0.3\,\omega_0$
	0.5	$5.3-0.1\,\omega_0^2$	$2.8+0.4\,\omega_0$
Rocking $(0<\omega_0<1.5)$	0	$2.5-0.4\,\omega_0^2$	$0.4\,\omega_0$
Torsion $(0<\omega_0<2.0)$	all	$5.1-0.3\,\omega_0^2$	$0.5\,\omega_0$

Substituting $F_1=\alpha_1-\alpha_2\omega_0^2$, Eqs. 4.7a and 4.7b reduce to the following forms

$$(m+\alpha_2\rho r_0^3)\ddot{x}+\sqrt{\rho G}\,r_0^2F_2\dot{x}+\alpha_1Gr_0x=P_0\sin\omega_m t \tag{4.7c}$$

$$(\varphi+\alpha_2\rho r_0^5)\ddot{\theta}+\sqrt{\rho G}\,r_0^4F_2\dot{\theta}+\alpha_1Gr_0^3\theta=M_0\sin\omega_m t \tag{4.7d}$$

Comparing Eqs. 4.7c and 4.7d with the standard equation of motion for a single-degree system Eq. (2.9) the following conclusions can be reached:

a. The term $\alpha_2\rho r_0^3$ represents the "effective mass of soil" participating in vibration for translatory modes. The corresponding expression for "mass moment of inertia of soil" for rotational modes is $\alpha_2\rho r_0^5$.

b. The term $\sqrt{\rho G}r_0^2F_2$ plays the role of the equivalent damping coefficient (C) for translatory modes. The corresponding term for rotatory modes is $\sqrt{\rho G}r_0^4F_2$.

The term $\alpha_1 G r_0$ is equivalent spring constant of soil for translatory modes. The corresponding expression for rotatory modes is $\alpha_1 G r_0^3$.

Knowing the effective mass, damping and stiffness terms of the equivalent single degree freedom system, the natural frequencies and amplitudes can be obtained using the expressions given in Table 2.1 . For uniform and parabolic pressure distributions under the footing, Hsieh suggests that a radius of αr_0 may be used in the above expressions where α is 0.78 and 0.59 respectively.

iv. Ford and Haddow[C3.13] gave an analytical method for determining natural frequencies of a machine foundation. This method is based on Rayleigh's principle of conservation of energy on the assumption that the system is conservative. The expression for natural frequency of vertical vibration is given by

$$f_z = \frac{1}{2\pi}\sqrt{\frac{2G(1+\nu)\beta_0 g}{(\rho g/\beta_0)+\sigma}} \tag{4.8}$$

where

β_0 is decay factor which represents the rate of decay of amplitude with depth below the surface

ρ is the mass density

g is the acceleration due to gravity

and σ is bearing pressure on soil.

The value of β_0 is obtained from

$$\beta_0 = \frac{C}{\alpha\sqrt{A_f}\,(1-\nu^2)} \tag{4.9}$$

where

α is a constant (Table 4.2) depending on shape of foundation.

C is a soil constant (C=2 for sands and 1.5 for clays)

and A_f is the base area of foundation in ft².

Table 4.2

FACTOR α BASED ON SHAPE OF FOUNDATION

(After Ford and Haddow, 1960)

Shape	Circle	Square	Rectangle					
Ratio L/B	—	1.0	1.5	2	3	5	10	100
α	0.96	0.95	0.94	0.92	0.88	0.82	0.71	0.37

The expression for horizontal natural frequency is as follows:

$$f_x = f_y = \frac{1}{2\pi}\sqrt{\frac{G\beta_0 g}{\rho g/\beta_0+\sigma}} \tag{4.10}$$

The parameters contained in this expression are same as those defined earlier.

v. Use of modified mass ratio: By using the so-called "modified mass ratio" for vertical vibration introduced by Lysmer and for other modes by Hall[C3.19] the influence of Poisson ratio is considered directly [compare with paragraphs (i) and (ii) above]. Using

the relations contained in Table 4.3, a unique set of curves could be plotted for magnification factors μ and μ' (which are related to the amplitudes) and frequency factor (ω_0) as functions of modified mass ratio (B) for all values of Poisson ratio (ν). In the charts presented by Richart earlier for vertical vibrations (Fig. 4.4) different curves were drawn for different Poisson ratios. Figs. 4.7 to 4.10 show plots of frequency factor (ω_0) and magnification factor (μ, μ') separately for constant force excitation (shown in broken lines) and for rotating mass excitations (shown in solid lines) as functions of modified mass ratio.

The notations used in Table 4.3 is as under:

Frequency Factor (ω_0)

a. For vertical translation $\quad \omega_0=\omega_z r_0\sqrt{\rho/G}$ (4.11a)

b. For horizontal translation $\quad \omega_0=\omega_x r_0\sqrt{\rho/G}$ (4.11b)

c. For rocking $\quad \omega_0=\omega_\theta r_\theta\sqrt{\rho/G}$ (4.11c)

d. For twisting $\quad \omega_0=\omega_\psi r_\psi\sqrt{\rho/G}$ (4.11d)

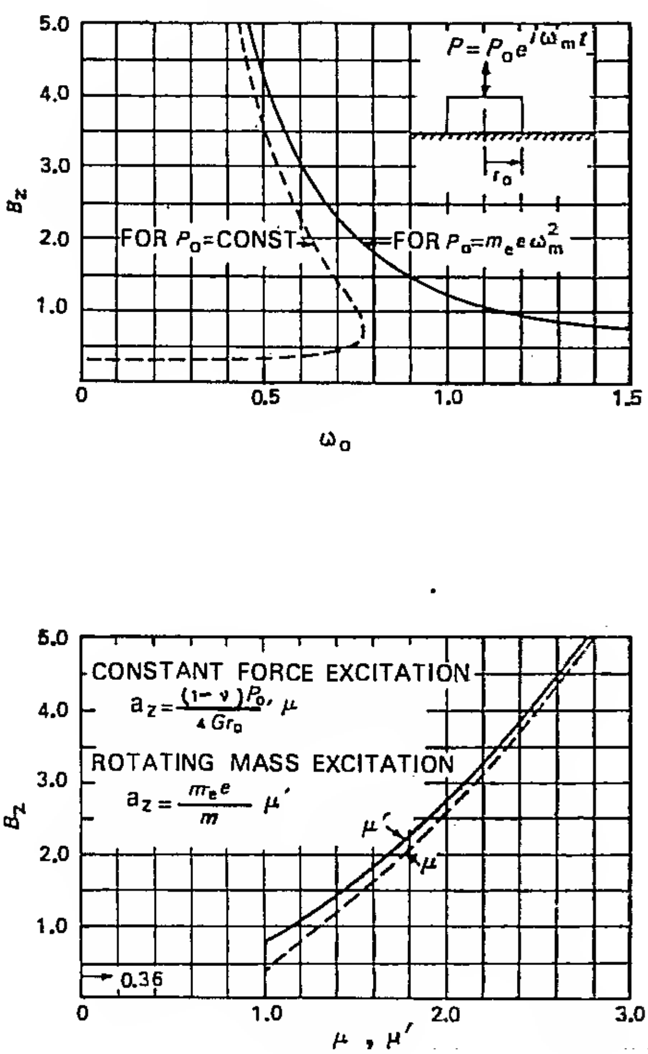


Fig. 4.7

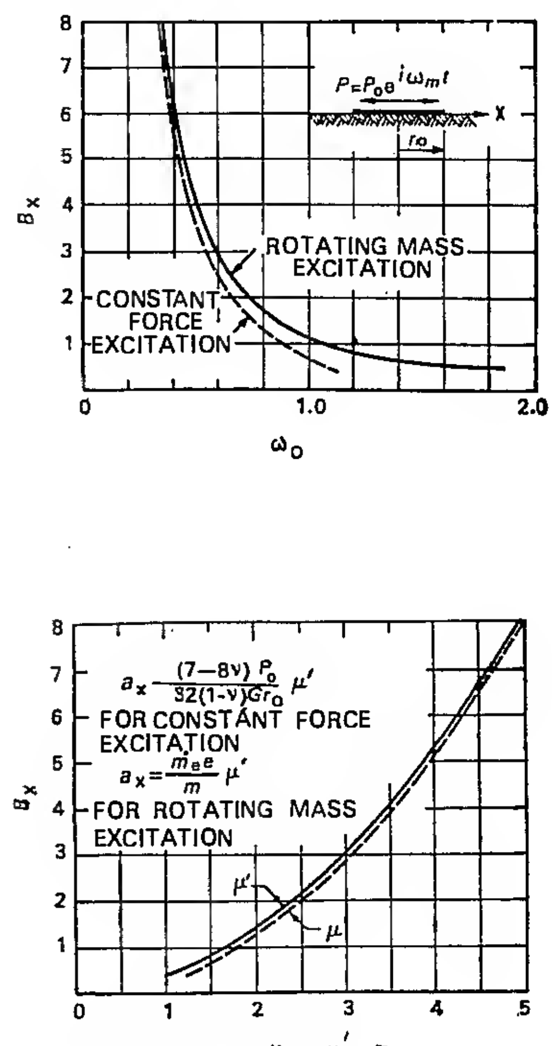


Fig. 4.8

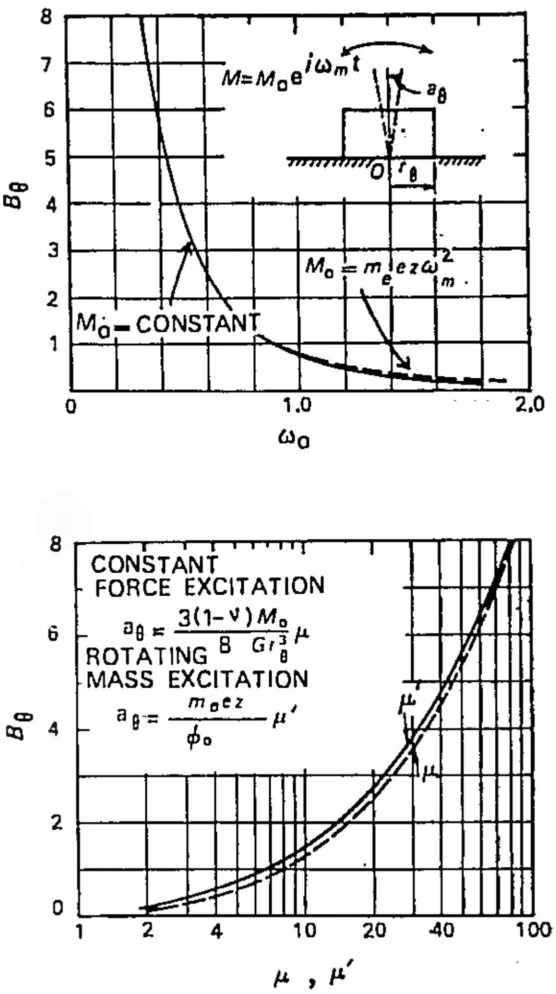


Fig. 4.9

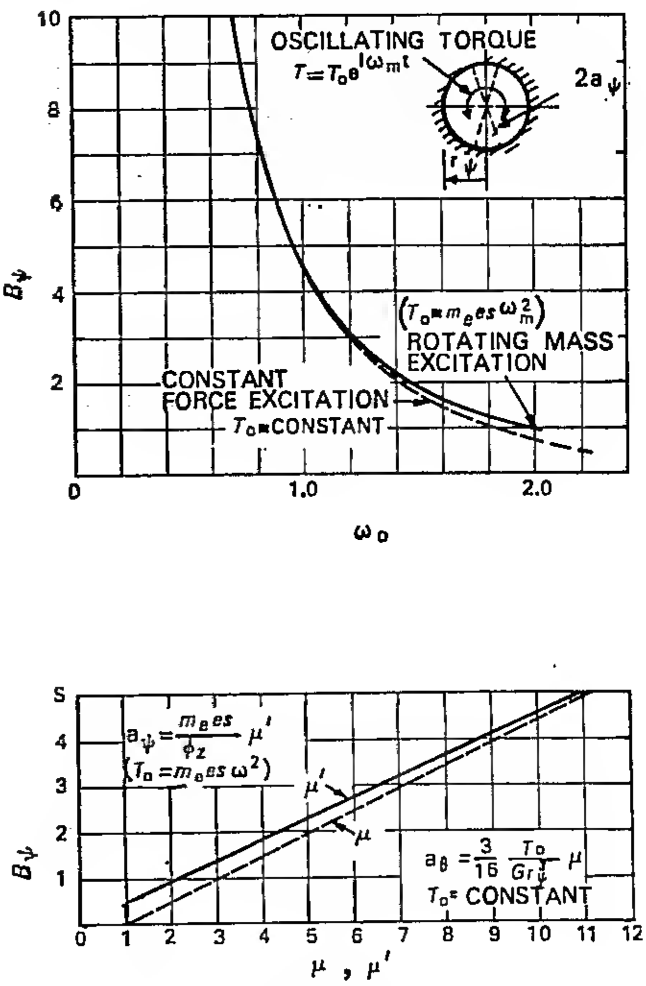


Fig. 4.10

Figs. 4.7–4.10: Characteristics of Vertical (**4.7**), Pure Sliding (**4.8**), Rocking (**4.9**), and Torsional Oscillations (**4.10**) as Functions of Modified Inertia Ratio *B*. (From Richart, F. E., Jr., *et al.*, *Vibration of Soils and Foundations*, Prentice-Hall, New Jersey, USA, 1970; with permission).

For a rectangular base, the equivalent circular radius (r_0) is obtained from the following relations

For translation
$$r_0=\sqrt{A_f/\pi} \tag{4.12a}$$

For rocking
$$r_\theta=\sqrt[4]{\frac{4I_0}{\pi}} \tag{4.12b}$$

For torsion
$$r_\psi=\sqrt[4]{\frac{2I_z}{\pi}} \quad \text{where} \quad I_z=I_x+I_y \tag{4.12c}$$

vi. Lumped-parameter system equivalent to elastic half-space model: A lumped-parameter system conventionally represented by a mass spring and a dash-pot is used as the basic model for analyzing the motion of a rigid block foundation. The equation of motion for such a system is given by

$$m\ddot{x}+C\dot{x}+Kx=P(t) \tag{4.13}$$

where m is equivalent mass, C is equivalent damping constant, K the equivalent spring constant and $P(t)$ the time-dependent force.

Table 4.3

EXPRESSIONS FOR MASS RATIO AND AMPLITUDES
(After Richart *et al.*, 1970)

Mode	Mass ratio B	Resonant amplitude for: Constant force excitation	Resonant amplitude for: Rotating mass excitation	Remarks
Vertical	$B_z=\frac{(1-\nu)}{4}\frac{m}{\rho r_0^3}$	$\frac{(1-\nu)P_0}{4Gr_0}\mu$	$\frac{m_e e}{m}\mu'$	
Rocking	$B_\theta=\frac{3(1-\nu)}{8}\frac{\varphi_0}{\rho r_0^5}$	$\frac{3(1-\nu)}{8}\frac{M_0}{Gr_0^3}\mu$	$\frac{m_e e z}{\varphi_0}\mu'$	$z=\frac{M_0}{m_e e\omega_m^2}$
Sliding	$B_x=\frac{(7-8\nu)}{32(1-\nu)}\frac{m}{\rho r_0^3}$	$\frac{(7-8\nu)}{32(1-\nu)}\frac{P_0}{Gr_0}\mu$	$\frac{m_e e}{m}\mu'$	
Torsional	$B_\psi=\frac{\varphi_z}{\rho r_\psi^5}$	$\frac{3}{16}\frac{T_0}{Gr_\psi^3}\mu$	$\frac{m_e e s}{\varphi_z}\mu'$	$s=\frac{T_0}{m_e e\omega_m^2}$

The mass of the machine and the foundation is taken as the lumped mass. Typical equivalent lumped-parameters are given in Fig. 4.11.

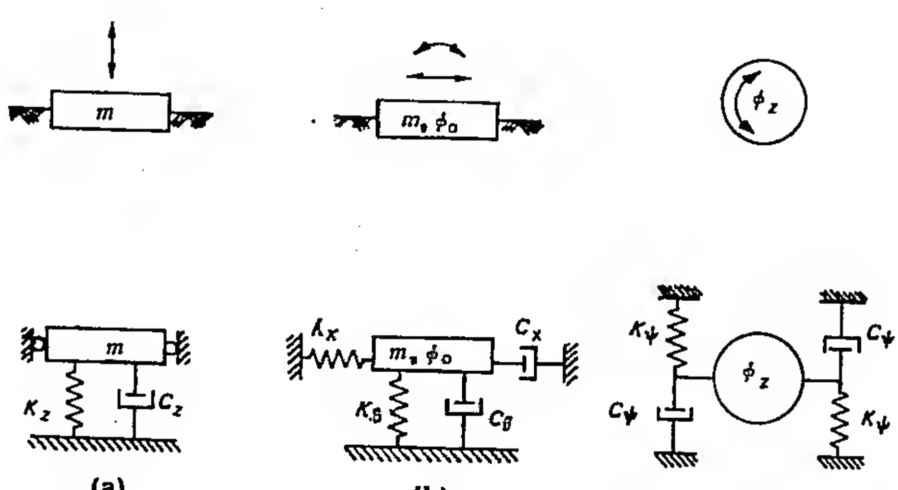


Fig. 4.11: Equivalent Lumped Parameter Systems for **(a)** Vertical, **(b)** Sliding-cum-Rocking, **(c)** Torsional Modes.

The spring constants for a rigid circular footing resting on elastic half-space for different modes of vibration are as follows:[C1.17]

Vertical
$$K_z=\frac{4Gr_0}{1-\nu} \tag{4.14a}$$

Horizontal
$$K_x=\frac{32(1-\nu)Gr_0}{7-8\nu} \tag{4.14b}$$

Rocking
$$K_\theta=\frac{8Gr_0^3}{3(1-\nu)} \tag{4.14c}$$

Torsion $$K_{\psi}=\frac{16}{3}Gr_{\psi}^{3} \tag{4.14d}$$

For a rectangular footing having base dimensions ($L\times B$) and resting on elastic half-space, the spring constants may be determined from the following expressions:

Vertical $$K_{z}=\frac{G}{1-\nu}\alpha_{z}\sqrt{LB} \tag{4.15a}$$

Horizontal $$K_{x}=2(1+\nu)G\alpha_{x}\sqrt{LB} \tag{4.15b}$$

Rocking $$K_{\theta}=\frac{G}{1-\nu}\alpha_{\theta}BL^{2} \tag{4.15c}$$

The parameters α_z, α_x and α_θ may be evaluated from Fig. 4.12.

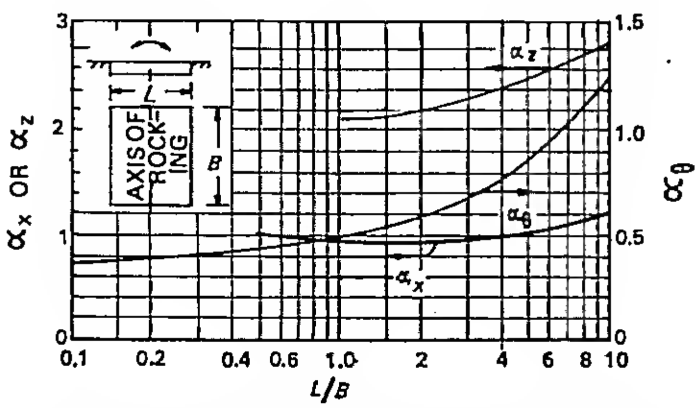


Fig. 4.12: Useful Parameters for Rectangular Foundations (From Richart, F. E., Jr., *et al.*, *Vibration of Soils and Foundations*, Prentice-Hall, New Jersey, USA, 1970; with permission).

The resonant frequency and amplitudes may be obtained from the expressions given in Table 2.1. The damping ratio (ζ) may be obtained from the expressions in Table 4.4.

Table 4.4

EQUIVALENT DAMPING RATIOS

(After Richart *et al.*, 1970)

Mode	Mass ratio (B_i)	Damping ratio (ζ_i)
Vertical	$B_z=\frac{(1-\nu)}{4}\frac{m}{\rho r_0^3}$	$\zeta_z=\frac{0.425}{\sqrt{B_z}}$
Sliding	$B_x=\frac{7-8\nu}{32(1-\nu)}\frac{m}{\rho r_0^3}$	$\zeta_x=\frac{0.288}{\sqrt{B_x}}$
Rocking	$B_\theta=\frac{3(1-\nu)}{8}\frac{\varphi_0}{\rho r_\theta^5}$	$\zeta_\theta=\frac{0.15}{(1+B_\theta)\sqrt{B_\theta}}$
Torsional	$B_\psi=\frac{\varphi_z}{\rho r_\psi^5}$	$\zeta_\psi=\frac{0.5}{1+2B_\psi}$

c. Considering Soil as a Spring

These methods are based on the assumption that a certain mass of soil vibrates along with the foundation. Interpreting this behaviour as an undamped mass-spring system, the circular natural frequency (ω_n) in vertical mode is given by

$$\omega_n=\sqrt{\frac{K_s}{m+m_s}} \tag{4.16}$$

where K_s is soil spring constant, m is mass of machine foundation and m_s the apparent mass of soil vibrating with the foundation.

The investigations carried out by various research workers have confirmed the fact that a certain mass of soil vibrates with the foundation, and this should be accounted for in the analysis. There is, however, no agreement on the magnitude of the effective mass of the soil participating in vibration.

i. Pauw[C3.37] has suggested that the effective zone of soil beneath the foundation may be assumed as a truncated pyramid (Fig. 4.13) extending to infinite depth.

The other assumptions in Pauw's method are as follows:

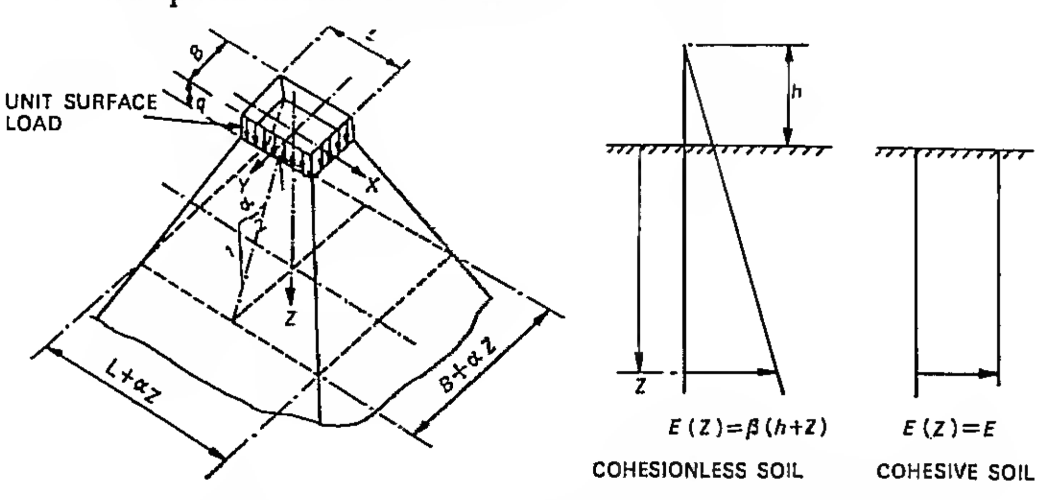


Fig. 4.13: Assumptions in Pauw's Analysis (From Pauw, A., "A Dynamic Analogy for Foundation Soil Systems", *ASTM Special Technical Publication*, No. 156, 1953; with permission).

For cohesionless soils, the modulus of elasticity (E) is proportional to effective depth. For cohesive soils the value of E is constant and the stress distribution over any section parallel to the contact surface is uniform.

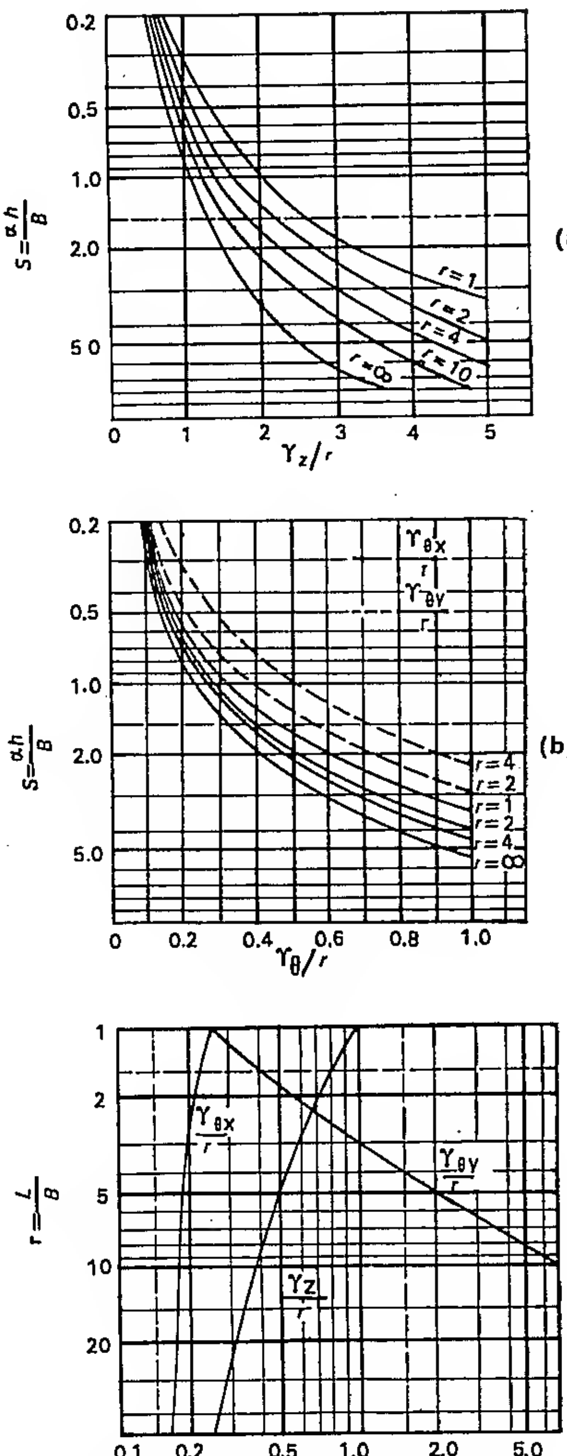
Stiffness factor for translation: According to this method, the spring factors K_z, K_x and K_y for translatory modes of vibration in the respective directions (Fig. 4.14) are given by the expressions contained in Table 4.5.

The values of γ_z can be obtained from Fig. 4.14a for cohesionless soils and from Fig. 4.14c for cohesive soils. The notation used is as follows:

L and B are the sides of the rectangular foundation

$$s=\alpha\frac{h}{B} \tag{4.17}$$

$$r=L/B \tag{4.18}$$



(a)

$K_z = \beta B^2 \gamma_z$
$K_x = \beta' B^2 \gamma_z$
$K_y = \beta' B^2 \gamma_z$.
$\beta' = \beta/[2(1+\nu)]$

(b)

$K_{zy} = \beta B^4 \gamma_{\theta x}$ [rotation about x axis]
$K_{xz} = \beta B^4 \gamma_{\theta y}$ [rotation about y axis]
$K_{xy} = \beta' B^4 (\gamma_{\theta x} + \gamma_{\theta y})$ [rotation about z axis]

— $\frac{\gamma_{\theta x}}{r}$

-- $\frac{\gamma_{\theta y}}{r}$

(c)

$K_z = E\alpha B\gamma_z$
$K_x = G\alpha B\gamma_z$
$K_y = G\alpha B\gamma_z$
$K_{yz} = E\alpha B^3 \gamma_{\theta x}$
$K_{xz} = E\alpha B^3 \gamma_{\theta y}$
$K_{xy} = G\alpha B^3 (\gamma_{\theta x} + \gamma_{\theta y})$

$$G = \frac{E}{2(1+\nu)}$$

Fig. 4.14: Equivalent Soil Spring Constants (From Pauw, A., "A Dynamic Analogy for Foundation Soil Systems", *ASTM Special Technical Publication*, No. 176. 1953; with permission)—**(a)**, **(b)** For Cohesionless Soils, **(c)** For Cohesive Soils.

h is equivalent surcharge defined by the ratio of foundation pressure to unit weight of soil
α is a factor which defines the slope of the truncated pyramid and is generally taken as unity
β denotes the rate of increase of Young's modulus with depth (zero in cohesive soils)

and $\beta' = \beta/2(1+\nu)$. (4.19)

Table 4.5

EXPRESSIONS FOR SPRING CONSTANTS

(After Pauw, 1953)

Mode	Spring constant	For cohesionless soils	For cohesive soils
Translation	K_z	$\beta B^2 \gamma_z$	$E \alpha B \gamma_z$
	$K_x = K_y$	$\beta' B^2 \gamma_z$	$G \alpha B \gamma_z$
Rocking	$K_{\theta x}$	$\beta B^4 \gamma_{\theta x}$	$E \alpha B^3 \gamma_{\theta x}$
	$K_{\theta y}$	$\beta B^4 \gamma_{\theta y}$	$E \alpha B^3 \gamma_{\theta y}$
	K_ψ	$\beta' B^4 (\gamma_{\theta x} + \gamma_{\theta y})$	$G \alpha B^3 (\gamma_{\theta x} + \gamma_{\theta y})$

Stiffness factors for rotation: For rotational modes, the expressions for spring factors are given in Table 4.5.

The values of $\gamma_{\theta x}$ and $\gamma_{\theta y}$ can be obtained from Fig. 4.14b & c for cohesionless and cohesive soils separately.

Apparent soil mass: The expression for apparent soil mass m_s for translatory modes of vibration is given by

$$m_s = \frac{\rho B^3 C_m}{g\alpha} \tag{4.20}$$

where C_m is a function of s and r.

i. For non-cohesive soils, C_m is obtained from Fig. 4.15a. No graphical data is suggested by Pauw for cohesive soils.

The expression for mass moment of inertia of soil for rotational vibrations is given by

$$\varphi_s = \frac{\rho B^5 C_i}{12 g\alpha} \tag{4.21}$$

where C_i can be obtained from Fig. 4.15 a & b. In these figures, C_i^x, C_i^y and C_i^z denote the factors of mass moments of inertia about x, y and z axes respectively. These factors can be obtained from the Figs. 4.15a and 4.15b for cohesionless and cohesive soils separately.

For a rigid block foundation having six degrees of freedom the generalized coordinates $x, y, z, \theta_x, \theta_y$, and ψ are as shown in Fig. 4.16. The inertia parameters associated with these co-ordinates used as suffixes are

$$m_x = m_y = m_z = m^f + m^s \tag{4.22}$$

The superscripts 'f' and 's' denote foundation and soil respectively.

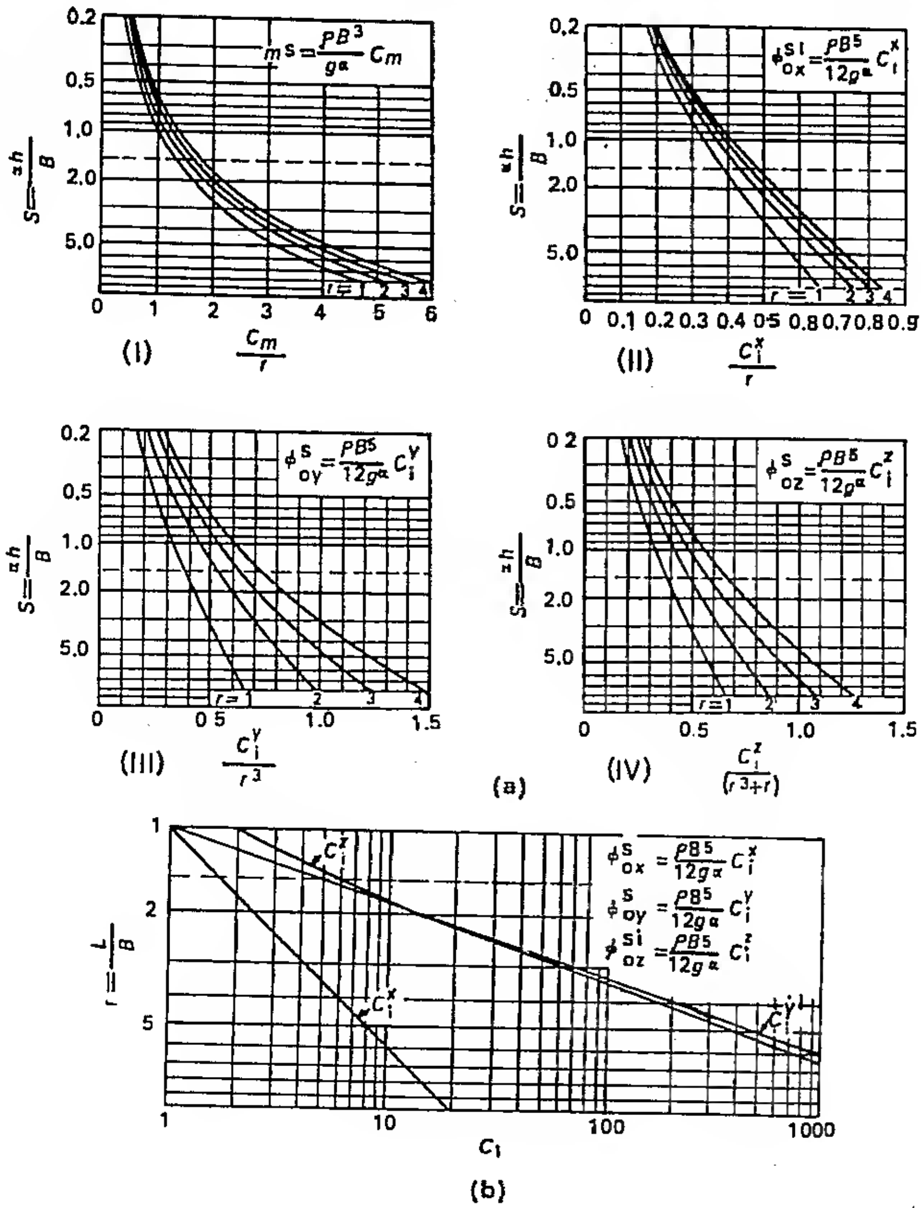


Fig. 4.15: Apparent Mass Factors (From Pauw, A., "A Dynamic Analogy for Foundation Soil Systems", *ASTM Special Technical Publication*, No. 156, 1953; with permission). (**a**) Cohesionless Soil: (**i**) Translation Modes, (**ii**) to (**iv**) Rotary Modes about x, y and z Axes, (**b**) Cohesive Soils: Rotary Modes).

Similarly, the mass moments of inertia φ_{ox}, φ_{oy}, φ_{oz} for rotation about x, y and z axes respectively are given by the following expressions:

$$\begin{aligned} \varphi_{ox} &= (\varphi_{ox})^f + (\varphi_{ox})^s \\ \varphi_{oy} &= (\varphi_{oy})^f + (\varphi_{oy})^s \\ \varphi_{oz} &= (\varphi_{oz})^f + (\varphi_{oz})^s \end{aligned} \tag{4.23}$$

For the foundation shown in Fig. 4.16, the six natural frequencies of the block foundation can be written as follows:

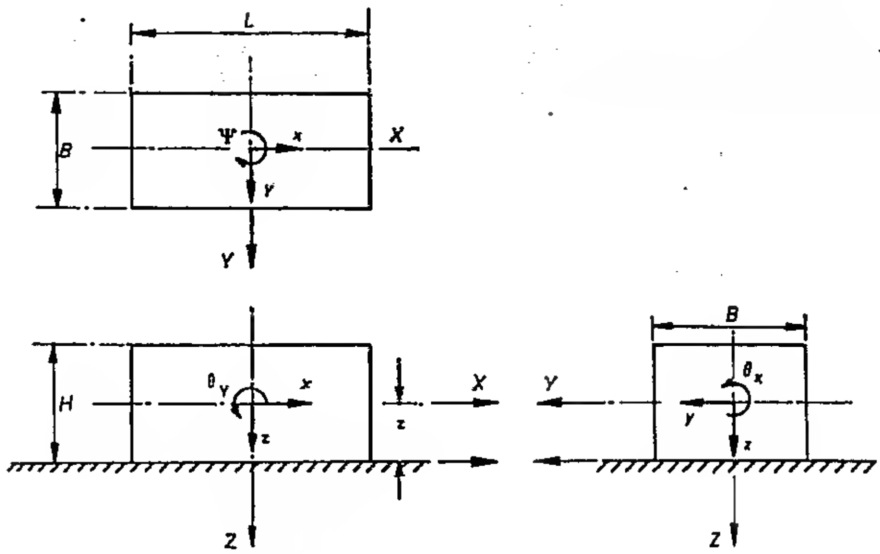


Fig. 4.16: Foundation Coordinates.

Vertical

$$\omega_z^2 = K_z/m_z \tag{4.24a}$$

Sliding and rocking in xz plane

$$\omega^2_{x,\theta y} = \frac{1}{2}\left[\frac{K_x}{m_x} + \frac{K_x S^2 + K_{\theta y}}{\varphi_{oy}} \pm \sqrt{\left\{\frac{K_x}{m_x} + \frac{K_x S^2 + K_{\theta y}}{\varphi_{oy}}\right\}^2 - \frac{4\,K_x K_{\theta y}}{m_x\,\varphi_{oy}}}\,\right] \tag{4.24b}$$

Sliding and rocking yz plane

$$\omega^2_{y,\theta x} = \frac{1}{2}\left[\frac{K_y}{m_y} + \frac{K_y S^2 + K_{\theta x}}{\varphi_{ox}} \pm \sqrt{\left\{\frac{K_y}{m_y} + \frac{K_y S^2 + K_{\theta x}}{\varphi_{ox}}\right\}^2 - \frac{4\,K_y K_{\theta x}}{m_y\,\varphi_{ox}}}\,\right] \tag{4.24c}$$

'S' denotes the height of centre of gravity above the base of the foundation.
Torsional (yawing)

$$\omega_\psi^2 = \frac{K_\psi}{\varphi_{oz}} \tag{4.24d}$$

The spring constants (K) and inertia parameters (m^s, φ_o^s) can be obtained from Pauw's charts (Figs. 4.14 and 4.15).

ii. Balakrishna Rao and Nagaraj[C3.5] suggested that the soil participating in the vibration may be assumed to be that enclosed within the γ kg/cm² pressure bulb where γ kg/cm³ is the unit weight of soil ($\gamma=\rho/g$). The pressure bulb is obtained by considering the sum of static and maximum positive dynamic load of the machine and the foundation block to act as a concentrated load at the mass-centre of the machine foundation. The boundary of this pressure bulb for the given load condition would vary with the nature of soil. If W is the total concentrated load (static+dynamic), then the depth (d_s) of the deepest point of the density pressure bulb is given by

$$d_s^2 = 0.4775 \times \frac{W}{\gamma} \tag{4.25}$$

and the mass of soil oscillating with the foundation is

$$m = \tfrac{4}{3}\rho\pi\left(\frac{d_B}{2}\right)^3 \tag{4.26}$$

the pressure bulb being a sphere for concentrated load.

iii. The practice prevalent in the U.S.S.R. for the design of machinery foundation is mainly based on the method proposed by Barkan. Barkan ignores the effects of "damping" and considers the soil as a linear weightless spring. According to him, damping has only a slight effect upon the calculated natural frequency, and if the operating and natural frequencies are well apart, the effect of damping on amplitudes can be neglected. Regarding the effect of soil inertia, Barkan had shown that the apparent mass of soil participating in foundation vibrations does not normally exceed 23 per cent of total mass of machine and foundation. Thus, according to him, the calculated natural frequency should not be in error by more than 10 per cent.

For foundations resting directly on soil, Barkan has introduced the following soil parameters which yield the spring stiffnesses of soil in various modes. These parameters are already defined in Section 3.4.

Coefficient of elastic uniform compression for vertical translatory mode	C_z
Coefficient of elastic non-uniform compression for rocking mode	C_θ
Coefficient of elastic uniform shear for horizontal translatory mode	C_τ
Coefficient of elastic non-uniform shear for twisting (or yawing) mode	C_ψ

If the common centre of gravity of machine and foundation and the centroid of the base area lie on the same vertical line in one of the principal planes of the foundation, it can be shown that the translatory motion of the foundation along the vertical axis (or the z axis) and the rotary motion about the same axis (yawing) are independent (or uncoupled). Therefore, the motion in these two modes can be represented by two separate single-degree systems. On the other hand, the sliding motion in the horizontal x (or y) axis and the rocking (rotary) motion about the y (or x) axis are interdependent (or coupled). Horizontal sliding and rocking motions of the foundation in xz and yz planes are, therefore, represented by two "two-degree freedom systems" separately.

The foregoing discussion leads to the conclusion that the dynamic analysis of a block foundation should be carried out for the following cases illustrated in Fig. 4.17:

a. Uncoupled translatory motion along (z) axis.

b. Coupled sliding and rocking motion of the foundation in x–z and y–z (vertical) planes passing through the common centre of gravity of machine and foundation.

c. Uncoupled twisting motion about z axis.

4.2.1 Discussion on the Various Methods

The empirical methods suggested by Tschebotarioff and Alpan (Sec. 4.2) can be used for preliminary design purposes if the soil beneath the foundation falls under any one of the four categories shown in Fig. 4.2. However, these methods can be used only to check the occurrence of resonance which in itself is not adequate for a satisfactory design.

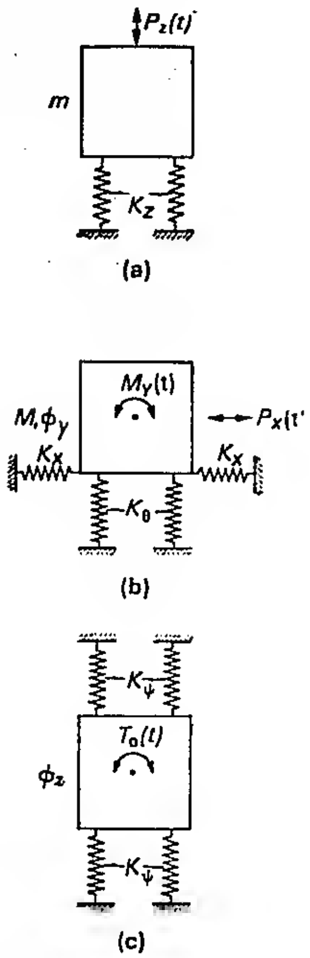


Fig. 4.17: Modes of Vibration—(a) Vertical, (b) Coupled Sliding and Rocking, and (c) Torsional.

Among the methods where soil is considered as a semi-infinite elastic solid (Sec. 4.2b) those of Richart using the concept of modified mass ratio and the approach of an equivalent lumped parameter system are the recent additions. The tables and charts given in Section 4.2b can be used for the calculation of natural frequencies and resonant amplitudes in the respective modes. The assumptions made in the theory that the soil is a homogenous elastic solid and that the contact pressure distribution under the foundation is of the rigid base pattern are not practically valid in all cases. Besides, using this approach the analysis can be carried out for each of the six modes independently, while, in practical cases, some of the vibratory modes (e.g., sliding and rocking modes) are coupled. These considerations limit the application of the above-mentioned methods in practical examples of machine foundations.

The methods of Pauw and Balakrishna Rao are based on the assumption that a certain mass of soil participates in vibration with the foundation. However, the concept of participating soil mass and its quantitative evaluation is not well established as yet.

Barkan's method, which is based on linear spring theory and which neglects the effects of damping and participating soil mass, is now popular in design offices. The method is relatively simple and predicts fairly closely the true behaviour of the foundation, as evidenced from the reported investigations on numerous existing foundations.

From the above considerations, it is recommended that in actual design practice the

method suggested by Barkan may be used for the dynamic analysis of block foundations. The theoretical basis of this method is given in Sec. 4.3.

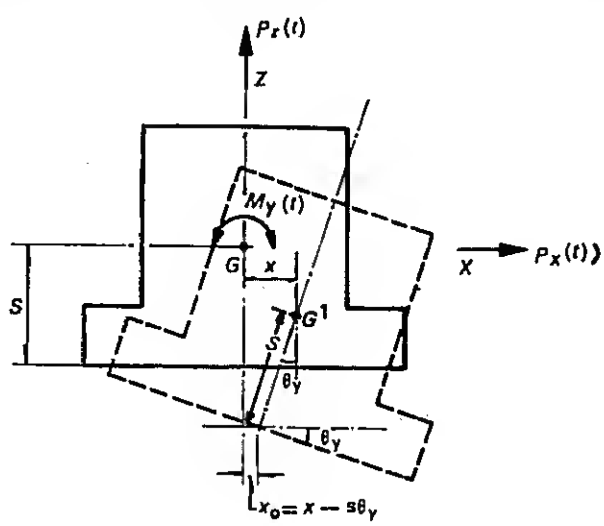
4.3 Recommended Method of Analysis for Block Foundations

As stated in Sec. 4.2.1, Barkan's method is recommended for the dynamic analysis of block-type machine foundations. The theoretical basis of this method is outlined below:

Let it be assumed that the combined centre of gravity of the machine and foundation lies in the same vertical line as the centroid of base plane. It was stated earlier that for this case, the vertical translation and twisting modes are uncoupled, while the sliding and rocking motions in each of the two vertical planes (xz and yz planes) passing through the common centre of gravity of machine and foundation are separately coupled. The motion of the foundation in the xz plane will be examined first.

Fig. 4.18 shows a block foundation having a mass m (W/g) and base area A_f and subjected to the action of oscillating loads $P_z(t)$, $P_x(t)$ and a moment $M_y(t)$ where t is the time para-

Fig. 4.18: Displacement of Foundation under Oscillating Forces in x-z Plane.



meter. Let the principal axes through the common centre of gravity G be chosen as the axes of co-ordinates and S is the height of G above the centre of elasticity of base support. Fig. 4.19 shows how S should be measured for different types of elastic support that may be used under the foundation. Let K_z, K_x and $K_{\theta y}$ denote respectively the stiffnesses of the elastic supports used in vertical compression, horizontal shear and against rotation (about the y axis). Let φ_y denote the mass moment of inertia of the foundation about the y axis. Then x, z and θ_y are respectively the displacements along x and z axes and rotation about the y axis.

The equations of motion of the foundation for the undamped case can be written as follows:

Vertical

$$m\ddot{z} + K_z z = P_z(t) \tag{4.27a}$$

Horizontal

$$m\ddot{x} + K_x(x - S\theta_y) = P_x(t) \tag{4.27b}$$

Rocking

$$\varphi_y\ddot{\theta} - K_x Sx + (K_{\theta y} - WS + K_x S^2)\theta_y = M_y(t) \tag{4.27c}$$

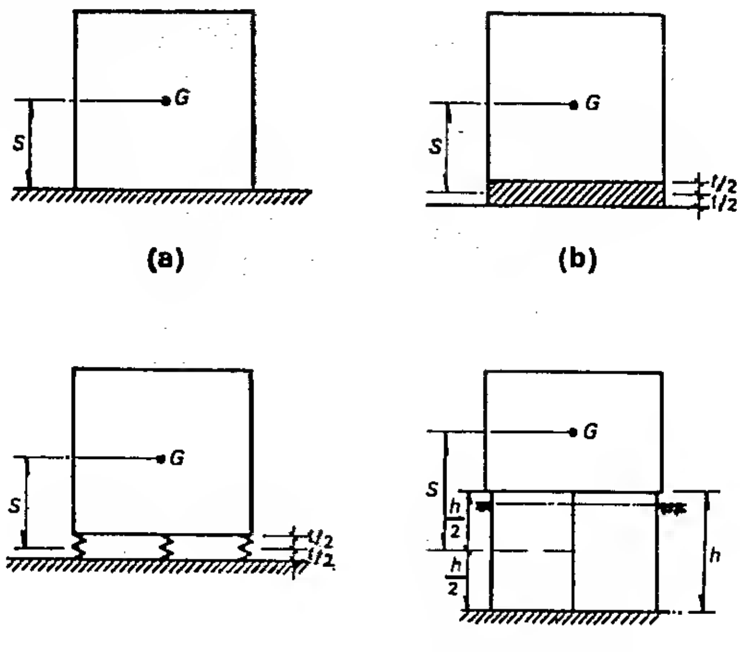


Fig. 4.19: Values of S for Foundation Resting on Different Supports—(**a**) on Soil, (**b**) on Elastic Bedding, (**c**) on Springs, (**d**) on Piles.

It may be seen that Eq. 4.27a representing the translatory motion along the z axis is independent of the other two coordinates x and θ while Eqs. 4.27b and 4.27c, which represent the horizontal sliding and rocking motions respectively, contain both x and θ. Eqs. 4.27b and 4.27c thus form a coupled set while Eq. 4.27a can be solved independently as a separate single-degree system.

To obtain the equations of motion in the yz plane, the suffixes x and y should be interchanged in Eqs. 4.27b and 4.27c. The equation of motion for torsion (rotation about the z axis) under the influence of an oscillating torsional moment $T_0 \sin \omega t$ is given by

$$\varphi_z \ddot{\psi} + K_\psi \psi = T_0 \sin \omega t \tag{4.27d}$$

where φ_z is the mass moment of inertia about the z axis, ψ is the angle of twist and K_ψ is the stiffness of the elastic support for rotation about vertical axis.

Eq. 4.27d, similar to Eq. 4.27a, is independent of the motion of the foundation in other modes and may be solved as a separate single-degree system. The solution of the equations of motion 4.27a to 4.27d leads to the following expressions for natural frequencies ana amplitudes for the various modes.

a. Vertical Translation (Fig. 4.17a)

i. The circular natural frequency (ω_z) for uncoupled vertical translation along the z axis is given by

$$\omega_z = \sqrt{K_z/m} \tag{4.28a}$$

For foundations resting directly on soils

$$\omega_z = \sqrt{\frac{C_z A_f}{m}} \tag{4.28b}$$

ii. The vertical amplitude (a_z) under the action of an exciting force $P_z \sin \omega_m t$, ω_m being the circular operating frequency is given by

$$a_z = \frac{P_z}{m\,(\omega_z^2 - \omega_m^2)} \tag{4.29}$$

b. Sliding and Rocking Motion in *xz* Plane

i. *Natural frequencies:* The two natural frequencies ω_{n_1}, ω_{n_2} which represent the coupled motion (sliding along *x* axis and rocking about *y* axis) in the *xz* plane are given by the roots of the following quadratic equation in ω_n^2

$$\omega_n^4 - \left(\frac{\omega_{\theta y}^2 + \omega_x^2}{\alpha_y}\right)\omega_n^2 + \frac{\omega_{\theta y}^2\,\omega_x^2}{\alpha_y} = 0 \tag{4.30}$$

where α_y is the ratio of the mass moment of inertia (φ_y) about the *y* axis passing through centre of gravity to the mass moment of inertia (φ_{0y}) about a parallel axis through the centre of elasticity of the base support.

$$\alpha_y = \varphi_y / \varphi_{0y} \tag{4.31}$$

$$\omega_{\theta y}^2 = (K_{\theta y} - WS)/\varphi_{0y} \tag{4.32a}$$

and

$$\omega_x^2 = K_x/m \tag{4.32b}$$

For foundations resting on soils

$$\omega_{\theta y}^2 = \frac{C_\varphi I_y - WS}{\varphi_{0y}} \tag{4.33a}$$

and

$$\omega_x^2 = \frac{C_\tau A_f}{m} \tag{4.33b}$$

The terms ω_x and $\omega_{\theta y}$ are called the "limiting frequencies" of the coupled motion; ω_x represents the natural circular frequency for "pure sliding" along the *x* axis when the foundation is assumed to possess infinite resistance to rocking (about the *y* axis) and $\omega_{\theta y}$ denotes the natural circular frequency for "pure rocking" (about the *y* axis) when the foundation is assumed to possess infinite resistance to sliding (along the *x* axis).

The two roots ω_{n_1} and ω_{n_2} of Eq. 4.30 are given by

$$\omega_{n1}^2 = \frac{1}{2\,\alpha_y}\left[\omega_{\theta y}^2 + \omega_x^2 + \sqrt{(\omega_{\theta y}^2 + \omega_x^2)^2 - 4\,\alpha_y\,\omega_{\theta y}^2\,\omega_x^2}\right] \tag{4.34a}$$

and

$$\omega_{n2}^2 = \frac{1}{2\,\alpha_y}\left[\omega_{\theta y}^2 + \omega_x^2 - \sqrt{(\omega_{\theta y}^2 + \omega_x^2)^2 - 4\,\alpha_y\,\omega_{\theta y}^2\,\omega_x^2}\right] \tag{4.34b}$$

The foundation vibrates with circular natural frequencies ω_{n_1} and ω_{n_2} (where $\omega_{n_1} > \omega_{n_2}$) about two centres of rotation — O_1 and O_2 (Fig. 4.20) — which are situated at distances

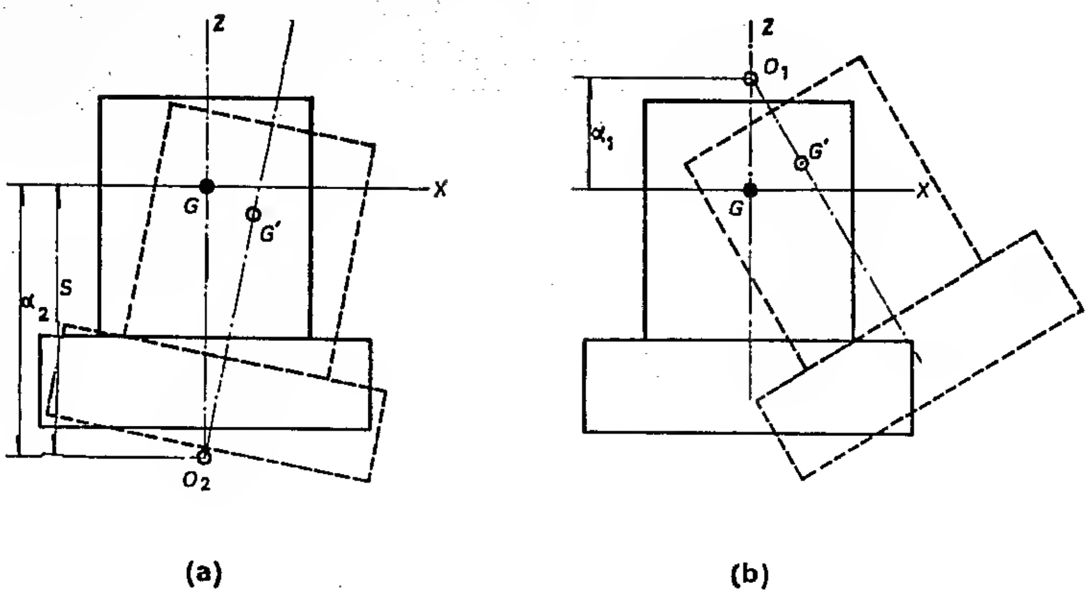


Fig. 4.20: Centres of Rotation for Coupled Sliding and Rocking Motion in x-z Plane—(a) First Mode, (b) Second Mode.

α_1 and α_2 respectively from the common centre of gravity where

$$\alpha_1 = \frac{\omega_x^2 S}{\omega_x^2 - \omega_{n1}^2} \tag{4.35a}$$

and

$$\alpha_2 = \frac{\omega_x^2 S}{\omega_x^2 - \omega_{n2}^2} \tag{4.35b}$$

It can be verified that

$$\alpha_1 \alpha_2 = \varphi_y / m \tag{4.35c}$$

ii. *Amplitudes:* The horizontal amplitude (a_x) and rotational amplitude ($a_{\theta y}$) of the foundation subjected to the simultaneous action of an exciting force $P_0 \sin \omega_m t$ and an exciting moment $M_y \sin \omega_m t$ are given by

$$a_x = \left[(K_{\theta y} - WS + K_x S^2 - \varphi_y \omega_m^2) P_x + (K_x S) M_y \right] \frac{1}{f(\omega_m^2)} \tag{4.36a}$$

and

$$a_{\theta y} = \left[(K_x S) P_x + (K_x - m\omega_m^2) M_y \right] \frac{1}{f(\omega_m^2)} \tag{4.36b}$$

where

$$f(\omega_m^2) = m \varphi_y (\omega_{n1}^2 - \omega_m^2)(\omega_{n2}^2 - \omega_m^2) \tag{4.37}$$

The net horizontal displacement (along the x axis) of the upper edge of the foundation is equal to

$$a_x + (H - S) a_{\theta y} \tag{4.38}$$

where H is height of foundation.

c. Sliding and Rocking Motion in yz Plane

The natural frequencies of the coupled sliding (along the y axis) and rocking (about the x axis) motion of the foundation are given by an equation similar to Eq. 4.30 obtained with the suffixes x and y interchanged in it.

The amplitudes a_y and $a_{\theta x}$ may likewise be obtained from Eqs. 4.36a and 4.36b with the suffixes x and y interchanged. The net horizontal amplitude (along the y axis) of the upper edge of foundation is then

$$a_y + (H-S)\, a_{\theta x} \tag{4.39}$$

d. Yawing [or Twisting Motion about z axis (Fig. 4.17c)]

As explained earlier, the yawing motion is uncoupled and the natural frequency (ω_ψ) for twisting mode and the amplitude under the action of a twisting moment $T_0 \sin \omega_m t$ are given by the following expressions

$$\omega_\psi = \sqrt{K_\psi/\varphi_z} \tag{4.40a}$$

For foundation resting on soils

$$\omega_\psi = \sqrt{\frac{C_\psi I_z}{\varphi_z}} \tag{4.40b}$$

$$a_\psi = \frac{1}{(\omega_\psi^2 - \omega_m^2)} \times \left\{\frac{T_0}{\varphi_z}\right\} T_0 \tag{4.41}$$

If the combined centre of gravity of the machine and foundation and the centroid of the foundation base do not lie in the same vertical line, the vertical vibration is not independent of horizontal vibration and rocking. In this case, vertical, horizontal and rocking vibrations in xz (or yz) planes are intercoupled and the three coupled natural frequencies ω_1, ω_2 and ω_3 (three in each plane xz and yz) are given by the roots of the following expression:

$$\omega_n^2 e_x^2 = \frac{\alpha\,(\omega_z^2-\omega_n^2)\,(\omega_{n1}^2-\omega_n^2)\,(\omega_{n2}^2-\omega_n^2)}{\omega_z^2\,(\omega_x^2-\omega_n^2)} \tag{4.42}$$

where ω_z, ω_{n1}, ω_{n2} are given by Eqs. 4.28a and 4.34, e_x is the eccentricity of the centroid of base area of foundation measured along the x axis from the centre of gravity of machine foundation and

$$\alpha = \varphi_y/m \tag{4.43}$$

To obtain the roots of Eq. 4.42, the expression on the right-hand side is evaluated for various values of ω_n^2 and plotted as shown by curves A and B in Fig. 4.21.

A straight line is drawn corresponding to the left-hand side of the above equation. The abscissas of the points of intersection of the two plots give the three unknown roots ($\bar{\omega}_1^2$, $\bar{\omega}_2^2$, $\bar{\omega}_3^2$) which are the three circular natural frequencies of the coupled motion of foundation.

NOTE: If the eccentricity is within five per cent of the length of the corresponding side of the foundation, the same may be neglected in calculations.

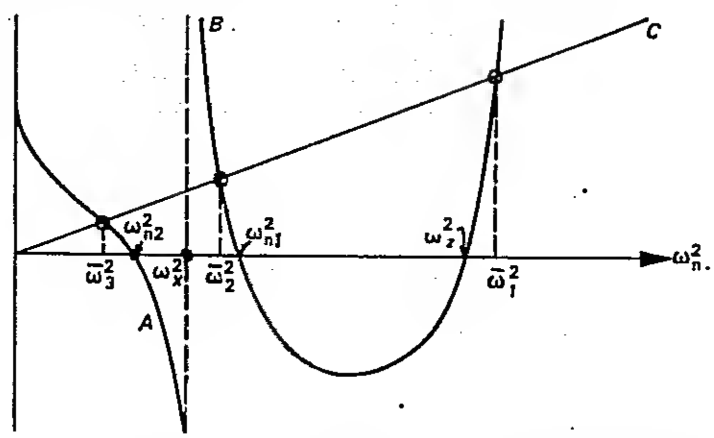


Fig. 4.21: Graph Illustrating Eq. 4.42 (After Major, A., *Vibration Analysis and Design of Foundations for Machines and Turbines*, Akademiai Kiado, Budapest, 1962; with permission).

4.4 Foundations for Machines Inducing Periodical Forces (Example: Reciprocating Machinery)

Reciprocating engines having crank-type mechanism include the following category of machines: (a) steam engines, (b) diesel engines, (c) displacement compressors, and (d) displacement pumps.

Foundations of reciprocating machinery are generally of block-type with openings provided where necessary for functional reasons. The classification of machines which induce periodical forces and the type of foundations to be provided for such machines have been discussed in Section 1.1.

4.4.1 Special Considerations in Planning

The dimensions of the foundation should be such that for low-speed machines (operating speed less than 500 rpm) the natural frequency is high, and vice-versa. To obtain a high natural frequency, the foundation must have a large base area and a small self-weight. The foundation recommended in such cases is either of box or caisson type. To obtain a low natural frequency, the foundation must be fairly massive or should be supported on springs or other suitable materials.

The outline dimensions of the foundation (in-plan) are generally furnished by the machine manufacturers. The height of the foundation may be fixed tentatively on the basis of soil strata *in situ* and the operating levels for the machinery. The dimensions so chosen may have to be altered, if required, in the design stage to satisfy the accepted design criteria which will be given in Section 4.4.2.

The following points will be considered while planning the foundations for reciprocating engines:

i. The eccentricity of the common centre of gravity of machine and foundation referred to the centroid of base area should not exceed 5% of the corresponding base dimension of the foundation.

ii. To decrease the transmission of vibrations to adjacent structures, it is necessary to provide an air gap around the foundation. Where the elastic under layers such as spring casings are provided below the foundation, the latter should be placed in a reinforced concrete trough and due provision made to give access to these under layers for periodical inspection or replacement.

iii. In order to reduce the horizontal amplitudes, the height of foundation should be selected as small as possible. A larger base dimension is selected in the direction of the rocking moment, if any, acting on the foundation.

iv. If several machines are located close-by in the same machine hall, a common foundation may be recommended for all of the machines, particularly when the underlying soil is soft. However, the analysis of vibrations of the foundation for such a group of machines is complicated. For practical purposes, the common foundation may be considered as broken up into sections corresponding to individual foundations and the computations carried out as if they were separate foundations. The permissible amplitudes may then be increased to 0.25 mm[C1.1].

4.4.2 Design Criteria

The principal design criteria for the foundations subjected to periodical forces are as follows:

i. The natural frequency should be at least 30 per cent away from the operating speed of the machine.

ii. The amplitude of the foundation should not normally exceed 0.2 mm.

iii. The stress on soil (or other elastic layers such as cork, springs, etc. where used) under the combined influence of static and dynamic loads should be within the respective permissible values. For preliminary designs, the bearing pressure on soil due to static loads alone may be taken as 0.4 times the corresponding safe bearing capacity.

The minimum possible dimensions of the foundation should be selected satisfying the above design criteria.

4.4.3 Design Data

The data to be supplied by the machine manufacturers include the following:

i. Normal speed and power of engine.

ii. Magnitude and position of static loads of the machine and the foundation.

iii. Magnitude and position of the dynamic loads which occur during the operation of the machine. Alternatively, the designer should be supplied with all of the data necessary for the computation of exciting forces (see Sec. 4.4.4).

iv. Position and sizes of openings provided in the foundation for anchor bolts, pipe lines, flywheel, etc.

v. Any other specific information which the machine suppliers may wish to add considering the speciality of the particular machine. These may include permissible differential settlements, permissible amplitudes of motion, etc. (if they differ from the normal design values).

4.4.4 Calculation of Induced Forces and Moments

Let a block foundation be acted upon by an exciting force P, the components of which in the respective directions are P_x, P_y and P_z (Fig. 4.22). Let (x_e, y_e, z_e) denote the coordinates of the point of application of the force referred to the principal axes passing through the common centre of gravity (G) as axes of coordinates. The unbalanced moments M_x, M_y and M_z about the respective axes can be expressed as

$$M_x = P_z y_e + P_y z_e \tag{4.44a}$$

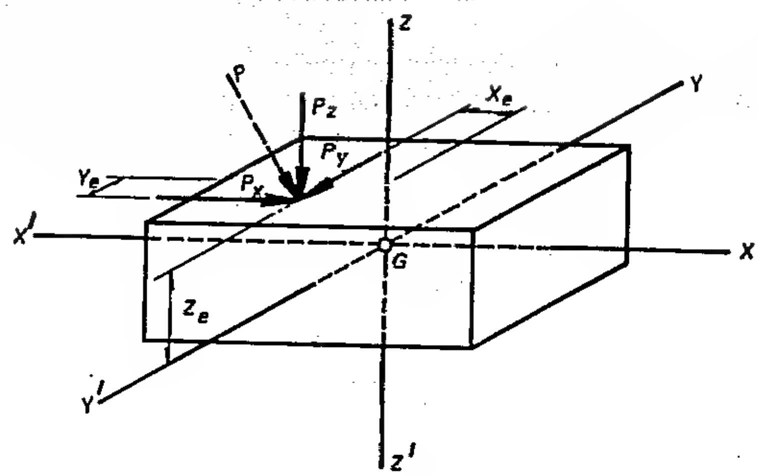


Fig. 4.22: General Representation of Exciting Forces on a Block Foundation.

$$M_y = P_z x_e + P_x z_e \tag{4.44b}$$

$$M_z = P_x y_e + P_y x_e \tag{4.44c}$$

The method of evaluating the induced forces for a simple crank mechanism has been explained in Sec. 1.5. It may be recalled that reciprocating masses produce both primary inertial forces as well as forces corresponding to higher harmonics. The latter are generally neglected since their contribution is small.

a. Multi-cylinder Engines

For a multi-cylinder engine having parallel cylinders, the induced forces (neglecting higher harmonics) are given by the following expressions:

i. Parallel to cylinder axis (P_1)

$$P_1 = \sum_{n=1}^{m} \left[r_n \omega_m^2 (m_{rot} + m_{rec}) \cos (\omega_m t + \beta_n) \right] \tag{4.45a}$$

ii. Perpendicular to the shaft axis (P_2)

$$P_2 = \sum_{n=1}^{m} \left[r_n \omega_m^2 m_{rot} \sin (\omega_m t + \beta_n) \right] \tag{4.45b}$$

where r_n is radius of crank for the nth cylinder, ω_m is the angular speed of rotation, m_{rec} and m_{rot} are the total reciprocating and rotating masses respectively, β_n is the wedging angle (angle between the crank of the nth cylinder and the first crank) and m is the number of cylinders in the engine. The wedging angles for various crank settings are given in Table 4.6.

For arbitrary position of cylinders, the induced forces shall be considered for each cylinder separately. Apart from the induced forces, there also occurs induced moments, the magnitudes of which are evaluated using Eq. (4.44). For multi-cylinder engines the algebraic sum of the induced moments in each cylinder should be considered.

The following example illustrates the calculation of exciting forces and moments for a two-cylinder vertical engine.

Table 4.6

WEDGING ANGLES FOR MULTI-CYLINDER ENGINE

Type of engine and crank setting	Wedging angles (β_n)
1. Two-cylinder engines	
(a) Cranks in same direction	$\beta_1=0;\ \beta_2=2\pi$
(b) Cranks forming 90°	$\beta_1=0;\ \beta_2=\pi/2$
(c) Cranks forming 180°	$\beta_1=0;\ \beta_2=\pi$
2. Three-cylinder engines	$\beta_1=0;\ \beta_2=2\pi/3,\ \beta_3=4\pi/3$
3. Four-cylinder engines	$\beta_1=0;\ \beta_2=\pi,\ \beta_3=\pi,\ \beta_4=2\pi$
4. Six-cylinder engines	$\begin{cases}\beta_1=0,\ \beta_2=2\pi/3,\ \beta_3=4\pi/3\\ \beta_4=4\pi/3,\ \beta_5=2\pi,\ \beta_6=8\pi/3\end{cases}$

Consider a two-cylinder vertical engine (Fig. 4.23) having identical cylinders with the cranks set 90° apart. The induced forces in the cylinders along the axis of the cylinders (vertical axis) are P_{z1} and P_{z2} and those in the perpendicular direction are P_{x1} and P_{x2}. The height of the common shaft is l_z. The expressions for induced forces and moments

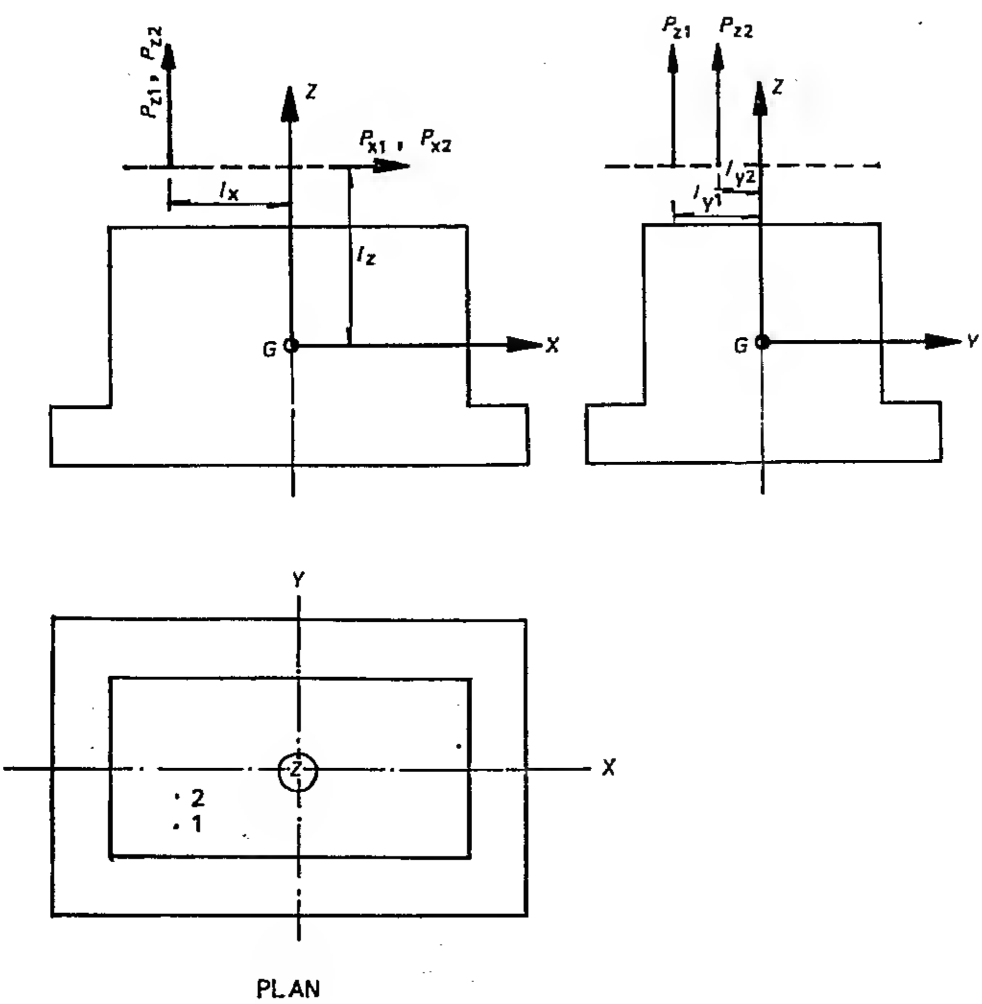


Fig. 4.23: Exciting Forces in a Two-Cylinder Reciprocating Engine.

on the foundation for the configuration shown in Fig. 4.23 are given below:

i. *Induced forces parallel to cylindrical axis (z axis):*

$$P_{z1} = r\,\omega_m^2\,(m_{rot} + m_{rec})\cos\omega_m t \tag{4.46a}$$

$$P_{z2} = r\,\omega_m^2\,(m_{rot} + m_{rec})\cos\left(\omega_m t + \frac{\pi}{2}\right) \tag{4.46b}$$

$$= -\,r\,\omega_m^2\,(m_{rot} + m_{rec})\sin\omega_m t$$

Total force $(P_z) = P_{z1} + P_{z2}$

$$= r\,\omega_m^2\,(m_{rot} + m_{rec})\,(\cos\omega_m t - \sin\omega_m t)$$

$$= \sqrt{2}\;r\,\omega_m^2\,(m_{rot} + m_{rec})\cos\left(\omega_m t + \frac{\pi}{4}\right) \tag{4.47}$$

ii. *Induced forces perpendicular to cylinder axis (x axis):*

$$P_{x1} = r\,\omega_m^2\,m_{rot}\sin\omega_m t \tag{4.48a}$$

$$P_{x2} = r\,\omega_m^2\,m_{rot}\sin\left(\omega_m t + \frac{\pi}{2}\right) \tag{4.48b}$$

Total force $(P_x) = P_{x_1} + P_{x_2}$

$$= r\,\omega_m^2\,m_{rot}\,(\sin\omega_m t + \cos\omega_m t)$$

$$= \sqrt{2}\;r\,\omega_m^2\,m_{rot}\sin\left(\omega_m t + \frac{\pi}{4}\right) \tag{4.49}$$

The total induced force in either direction is thus $\sqrt{2}$ times the force induced in each cylinder.

iii. *Induced moments:* Referring to Fig. 4.23 again, the induced moments about the axes of coordinates passing through the common centre of gravity of machine and foundation (G) can be written as

$$M_x = P_{z1}\,l_{y1} + P_{z2}\,l_{y2} \tag{4.50a}$$

$$M_y = (P_{x1} + P_{x2})\,l_z + (P_{z1} + P_{z2})\,l_x \tag{4.50b}$$

$$M_z = P_{x1}\,l_{y1} + P_{x2}\,l_{y2} \tag{4.50c}$$

The foregoing relations are valid only for engines having main cylinders and no auxiliaries (exhaust cylinders, etc.). If the engines have auxiliary cylinders, then in the computation of induced forces, the loads imposed by these auxiliaries should be added to those produced by main cylinders. The forces induced by the auxiliaries are generally very small and may be neglected in the computation of foundation vibrations.

b. Newcomb's Procedure for Design of Engine Foundations

Newcomb[C3.33] expressed the inertial forces (P) acting along the axis of the piston in the form

$$P = 0.0000284\,W r f_m^2\left(\cos\theta + \frac{r}{l}\cos 2\theta\right) \tag{4.51}$$

where P is the inertial force in pounds, r is the radius of crank in inches, W is weight of reciprocating parts in pounds, f_m is engine speed in rpm, and l is the length of connecting rod in inches and θ is the inclination of the crank to the piston axis.

The first term in the bracket represents the primary inertial force and the second term is the secondary inertial force.

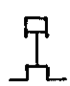
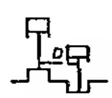
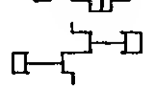
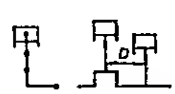
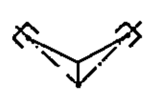
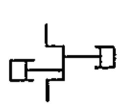
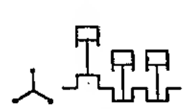
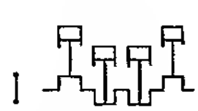
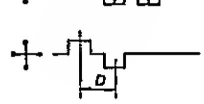
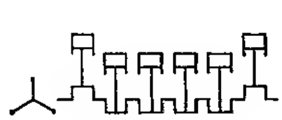
The maximum force corresponds to $\theta = 0$ and hence

$$P_{\max} = 0.0000284\ Wrf_m^2 \left(1 + \frac{r}{l}\right) \tag{4.52}$$

Table 4.7

UNBALANCED FORCES FOR MULTI-CYLINDER ENGINES

(after Newcomb, 1957)[C3.38]

Crank Arrangement		Forces Primary (P_1)	Forces Secondary (P_2)	Moments Primary (M_1)	Moments Secondary (M_2)
i. Single crank		P_1 without counter weights. $0.5P_1$ with counter weights.	P_2	Zero	Zero
ii. Two cranks at 180°					
a. In line cylinders		Zero	$2P_2$	P_1D without counter weights. $P_1D/2$ with counter weights.	Zero
b. Opposed cylinders		Zero	Zero	Zero	Zero
iii. Two cranks at 90°		1.41 P_1 without counter weights. 0.707 P_1 with counter weights.	Zero	1.41 P_1D without counter weights. 0.707 P_1D without counter weights.	P_2D
iv. Two cylinders on One crank—cylinders at 90°		P_1 without counter weights. zero with counter weights.	1.41 P_2	Zero	Zero
v. Two cylinders on One crank—opposed cylinders		2 P_1 without counter weights. P_1 with counter weights.	Zero	Zero	Zero
vi. Three cranks at 120°		Zero	Zero	3.46 P_1D without counter weights. 1.73 P_1D with counter weights.	3.46P_2D
vii. Four cylinders					
a. Cranks at 180°		Zero	Zero	Zero	Zero
b. Crank at 90°		Zero	Zero	1.41 P_1D without counter weights. 0.707 P_1D with counter weights.	4.0P_2D
viii. Six cylinders		Zero	Zero	Zero	Zero

Maximum primary force $(P_1) = 0.0000284\ Wrf_m^2$ (4.53a)

Maximum secondary force $(P_2) = P_1 \dfrac{r}{l}$ (4.53b)

Table 4.7 gives the unbalanced forces developed by multi-cylinder engines having identical cylinders. If the cylinders are not identical, this table should not be used. The unbalanced forces should then be computed for each cylinder separately and the results superimposed.

As stated in Section 1.5, the unbalanced forces for a particular machine should be available from the machine manufacturers, since these quantities would have been required for the original design of the machine. Where they are not supplied, the foundation designer should be furnished with all of the data necessary to compute them.

4.4.5 Forces Acting on the Foundation

For the structural design, the following forces which maintain the foundation in equilibrium should be considered:

1. Induced forces (and moments) multiplied by a fatigue factor
2. Inertial forces
3. Dynamic forces

The method of calculating the induced forces and moments for various types of engines has been given in the earlier section. The fatigue factor (ξ) may be taken as 3.

The expressions for inertial and dynamic forces (and moments) for various cases of exciting forces and moments are given in Table 4.8.

Table 4.8

FORCES ON THE FOUNDATION

Case		Inertial force (F_m) and moment (M_m)		Dynamic force (F_d) and moment (M_d)
1. Exciting force acting vertically and passing through the combined centre of gravity of machine and foundation (Fig. 4.24a)		$(F_m) = \xi m a \omega_m^2$		$(F_d)_z = \xi K_z a_z$
2. Exciting force acting vertically but eccentric to both axes (Fig. 4.24b)		$(F_m)_z = \xi m a_z \omega_m^2$ $(F_m)_x = \xi m a_x \omega_m^2$ $(F_m)_y = \xi m a_y \omega_m^2$ $(M_m)_x = \xi \varphi_x a_{\theta x} \omega_m^2$ $(M_m)_y = \xi \varphi_y a_{\theta y} \omega_m^2$		$(F_d)_z = \xi K_z a_z$ $(F_d)_x = \xi K_x (a_x - S a_{\theta x})$ $(F_d)_y = \xi K_y (a_y - S a_{\theta x})$ $(M_d)_x = \xi K_{\theta x} a_{\theta x}$ $(M_d)_y = \xi K_{\theta y} a_{\theta y}$
3. Exciting force acting horizontally in x (or y) direction at a certain height above centre of gravity (Fig. 4.24c)	(i) (ii) (iii)	$(F_m)_z = 0$ $(F_m)_x = \xi m a_x \omega_m^2$ $(F_m)_y = \xi m a_y \omega_m^2$ $(M_m)_x = \xi \varphi_x a_{\theta x} \omega_m^2$ $(M_m)_y = \xi \varphi_y a_{\theta y} \omega_m^2$	(i) (ii) (iii)	$(F_d)_z = 0$ $(F_d)_x = \xi K_x (a_x - S a_{\theta y})$ $(F_d)_y = \xi K_y (a_y - S a_{\theta y})$ $(M_d)_x = \xi K_{\theta x} a_{\theta x}$ $(M_d)_y = \xi K_{\theta y} a_{\theta y}$
4. Exciting moment (torsional) about z axis passing through centre of gravity (Fig. 4.24d)		$(F_m)_x = (F_m)_y = (F_m)_z = 0$ $(M_m)_x = (M_m)_y = 0$ $(M_m)_z = \xi \varphi_z a_\psi \omega_m^2$		$(F_d)_x = (F_d)_y = (F_d)_z = 0$ $(M_d)_x = (M_d)_y = 0$ $(M_d)_z = \xi K_\psi a_\psi$

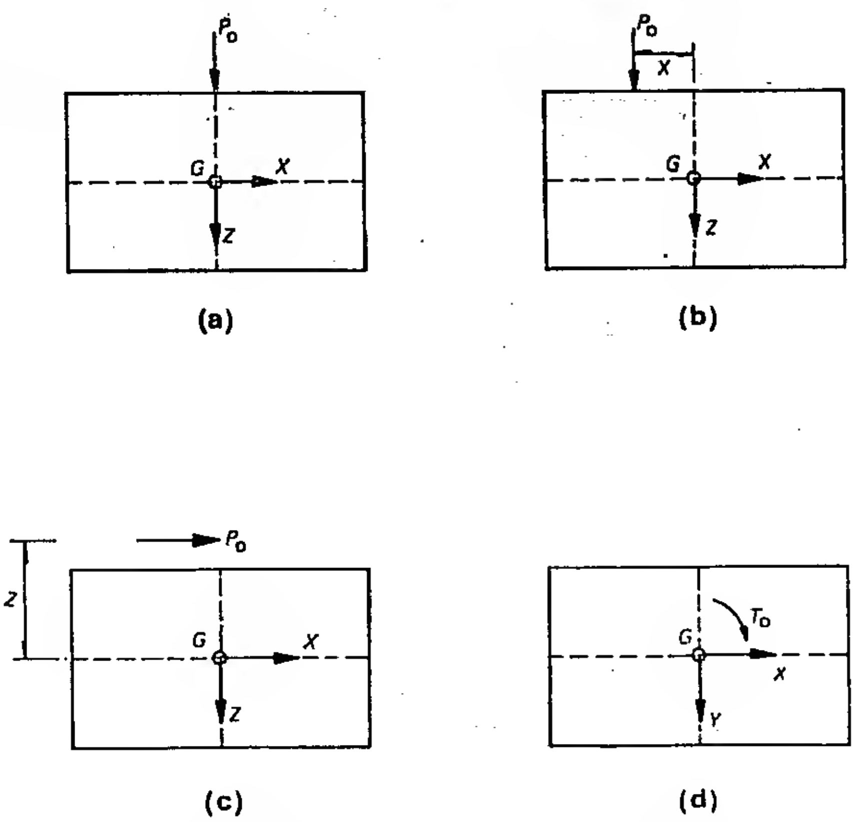


Fig. 4.24: Exciting Force in Different Directions.

The following notation is used in these expressions:

m	Mass of machine and foundation
ω_m	Operating circular frequency
K_x, K_y, K_z	Stiffnesses in x, y and z direction
a_x, a_y, a_z	Amplitudes of translation in respective directions
$a_{\theta x}$, $a_{\theta y}$, $a_{\theta z}$	Amplitudes of rotation about respective directions
$(F_m)_x$, $(F_m)_y$, $(F_m)_z$	Inertial forces in respective directions
$(F_d)_x$, $(F_d)_y$, $(F_d)_z$	Dynamic forces in respective directions
$(M_m)_x$, $(M_m)_y$, $(M_m)_z$	Inertial moments in respective directions
$(M_d)_x$, $(M_d)_y$, $(M_d)_z$	Dynamic moments in respective directions.

4.4.6 Distribution of Inertial Forces

The total inertial force (vertical and horizontal), as well as inertial moment acting on the foundation, may be evaluated from the expressions given in Table 4.8. How the inertial forces are shared by the machine and various parts of the foundation body is explained below.

Fig. 4.25 shows a foundation consisting of a number of rectangular portions $ABB'A'$, $CDEF$ and $C'D'F'E'$. Let us consider one of these portions, say $CDEF$. Considering the motion in the xz plane, let a_z and a_x denote the vertical and horizontal amplitudes and $a_{\theta y}$ the rotational amplitude of the foundation. The centre of rotation '0' is defined by the coordinates x_0 and z_0 (referred to the principal axes through the common centre of gravity G as coordinate axes).

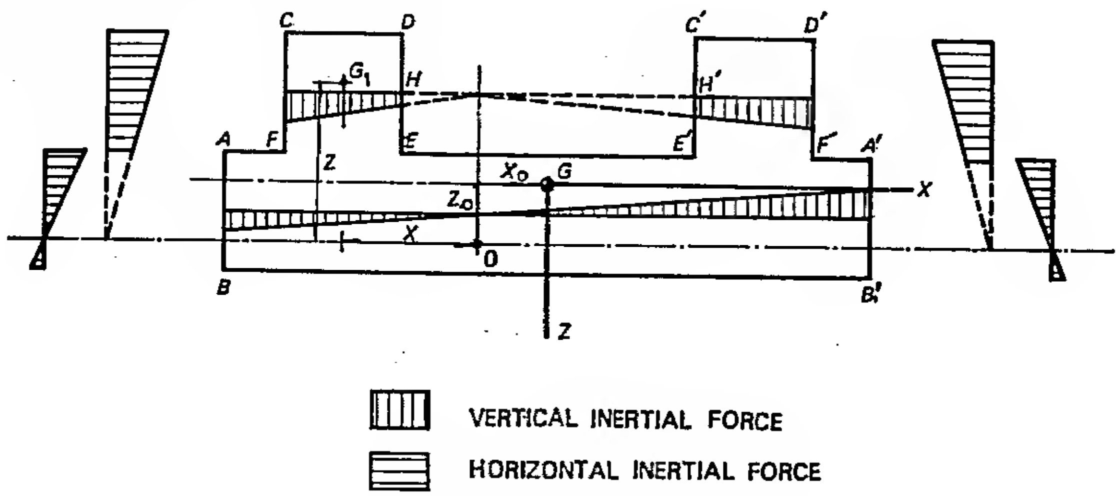


Fig. 4.25: Distribution of Inertial Forces.

Where

$$x_0 = -a_z / a_{\theta v} \tag{4.54a}$$

$$z_0 = -a_x / a_{\theta v} \tag{4.54b}$$

Let $(F_m)_z$, $(F_m)_x$ denote the total inertial force and $(M_m)_y$ the total inertial moment, the expressions for which are given in Table 4.8. Let W_1 be the weight of this part of the foundation and (x, z) are the coordinates of its centre of gravity (G_1) referred to the common centre of gravity G as origin. The inertial force shared by this portion of the foundation (having weight W_1) is given by the following relations.

Vertical inertial force $(F_m)_z$, shared by weight W_1

$$(F_{mz})_1 = W_1 \left[\frac{(F_m)_z}{W} \pm \frac{(M_m)_y}{\varphi_y} \frac{x}{g} \right] \tag{4.55a}$$

Horizontal inertial force (F_{mx}) shared by weight W_1

$$(F_{mx})_1 = W_1 \left[\frac{(F_m)_x}{W} \pm \frac{(M_m)_y}{\varphi_y} \frac{z}{g} \right] \tag{4.55b}$$

where W is the total weight of a machine and its foundation and φ_y is the mass moment of inertial about tbe axis of rotation.

Tbe inertia force distribution diagram may be in the form of a trapezium or a pair of triangles depending on the position of the centre of rotation (Fig. 4.25).

Eqs. 4.55a and 4.55b can be written in a more compact form as

$$(F_{mz})_1 = \frac{W_1}{\varphi_y} (M_m)_y \frac{X}{g} \tag{4.56a}$$

and

$$(F_{mx})_1 = \frac{W_1}{\varphi_y} (M_m)_y \frac{Z}{g} \tag{4.56b}$$

where

$$X = x - x_0 \tag{4.57a}$$

and

$$\mathcal{Z} = z - z_0 \tag{4.57b}$$

X and $\mathcal{Z}$ are the coordinates of G_1 referred to the centre of rotation (0) as origin.

Assuming a uniform mass distribution, the end ordinates of the inertial force distribution diagram for each rectangular part of the foundation can be evaluated from the following considerations:

i. The line joining the end ordinates of vertical inertial force shall meet the horizontal base line at a point which lies on the vertical line passing through the centre of rotation.

ii. The line joining the end ordinates of horizontal inertial force shall meet its base line (a vertical line in this case) at a point which lies on the horizontal line passing through the centre of rotation.

iii. The net area of the inertial force distribution diagram (full area in the case of a trapezium and difference of areas in case of a pair of triangles)—vertical or horizontal—should be equal to the magnitudes evaluated by Eqs. 4.56a or 4.56b, as the case may be.

The computation of inertial forces may be omitted if the natural frequency of foundation (f_n) is considerably greater than the operating frequency of machine (f_m) as they will be very insignificant in that case. The foundation may then be considered to be in equilibrium under the influence of (*i*) induced forces multiplied by the fatigue factor (ξ), and (*ii*) the dynamic forces. The latter occur in the form of reactive pressure of soil, spring forces, etc. Example (1) given in Section 4.4.8 is an illustration for this case.

On the other hand, if f_n is considerably smaller than f_m, the dynamic forces will be insignificant. The foundation may then be considered to be in equilibirum under the influence of (*i*) inertial force, and (*ii*) the induced forces multiplied by the fatigue factor (ξ). Example (2) given in Section 4.4.8 is an illustration for this case.

4.4.7 *Foundations on Vibration Absorbers*

In special cases due to environmental conditions it may become necessary to limit the amplitudes of vibration to much lower values than those usually adopted. It may not be practicable to achieve this requirement by proper selection of mass or base area of the foundation. In such cases, spring absorbers are recommended to be used under the foundation. Springs are relatively less expensive and are effective in decreasing the amplitudes of forced vibrations. The type of spring absorbers to be used and the method of using them will be explained in detail in Chapter 8. The theory of these absorbers as applied to foundations of reciprocating engines is given below.

The absorbers are usually placed on isolated pedestals or a thin slab called "sole plate", which rest on the soil. The absorbers support on their top the upper foundation block (where needed) to which the machine is anchored.

The use of absorbers under a block foundation results in a system of two masses supported on two springs. Although each mass (considered rigid) has in general six degrees of freedom—thus making a total of 12 degrees of freedom for the whole system—for practical purposes it may be considered that the vibrations in the vertical direction are independent of other modes. The system thus reduces to a two-degree spring mass system shown in Fig. 2.3. In this case m_1 is the mass of the foundation on which the springs are placed, m_2 is mass of the foundation (including that of machine mounted on it) above the springs, K_1 is the stiffness of supporting soil under the lower mass and K_2 is the stiffness of spring absorber assembly.

The amplitudes a_1 and a_2 of masses m_1 and m_2 under the influence of an induced oscillating force $P_0 \sin \omega_m t$ acting on mass m_2 are given in Chapter 2.

$$a_1 = \frac{\bar{\omega}_{n2}^2}{m_1 f(\omega_m^2)} P_0 \tag{2.31a}$$

and

$$a_2 = \frac{[(1+\alpha)\bar{\omega}_{n1}^2 + \alpha \bar{\omega}_{n2}^2 - \omega_m^2]}{m_2 f(\omega_m^2)} P_0 \tag{2.31b}$$

where

$$\bar{\omega}_{n2}^2 = \frac{K_2}{m_2} \tag{2.23a}$$

$$\bar{\omega}_{n1}^2 = \frac{K_1}{m_1 + m_2} \tag{2.23b}$$

$$f(\omega_m^2) = \omega_m^4 - (1+\alpha)(\bar{\omega}_{n1}^2 + \bar{\omega}_{n2}^2)\,\omega_m^2 + (1+\alpha)\,\bar{\omega}_{n1}^2\,\bar{\omega}_{n2}^2 \tag{2-32}$$

and

$$\alpha = m_2/m_1$$

Since the exciting force is proportional to square of the operating frequency of the engine (ω_m)

$$P_0 = \gamma\, \omega_m^2 \tag{4-58}$$

where γ is a factor which depends on the characteristics of engine.

Substituting Eq. 4.58 on the right-hand side of Eq. 2.31a and solving

$$a_1 = \frac{\gamma}{m_1} \frac{\eta_2^2}{1-(1+\alpha)(\eta_1^2 + \eta_2^2 - \eta_1^2 \eta_2^2)} \tag{4.59}$$

where

$$\eta_1 = \frac{\bar{\omega}_{n1}}{\omega_m} \tag{4.60}$$

and

$$\eta_2 = \frac{\bar{\omega}_{n2}}{\omega_m} \tag{4.61}$$

If no absorbers are used, neglecting damping in the system the amplitude a_z of the foundation is given by

$$a_z = \frac{P_0}{(m_1 + m_2)(\bar{\omega}_{n1}^2 - \omega_m^2)} \tag{4.62}$$

In terms of γ and α,

$$a_z = \frac{\gamma}{m_1} \frac{1}{(1+\alpha)} \frac{1}{(\eta_1^2 - 1)} \tag{4.63}$$

The degree of absorption β is defined as

$$\beta = \frac{a_z}{a_1} = \frac{1-(1+\alpha)(\eta_1^2 + \eta_2^2 - \eta_1^2 \eta_2^2)}{(1+\alpha)(\eta_1^2 - 1)\,\eta_2^2} \tag{4.64}$$

It can be verified from Eq. 4.64 that when $\eta_2 \to 0$, $\beta \to \infty$ and when $\eta_2 \to \infty$, $\beta \to 1.0$. Fig. 4.26 shows the variation of η_2 with β. It can be seen from the diagram that the absorber will be effective (i.e., $\beta < -1$) only when η_2 lies between zero and a value η_0 where η_0 can be expressed as

$$\eta_0 = \sqrt{\frac{(1 + \alpha)\,\eta_1^2 - 1}{2\,(1 + \alpha)\,(\eta_1^2 - 1)}} \tag{4.65}$$

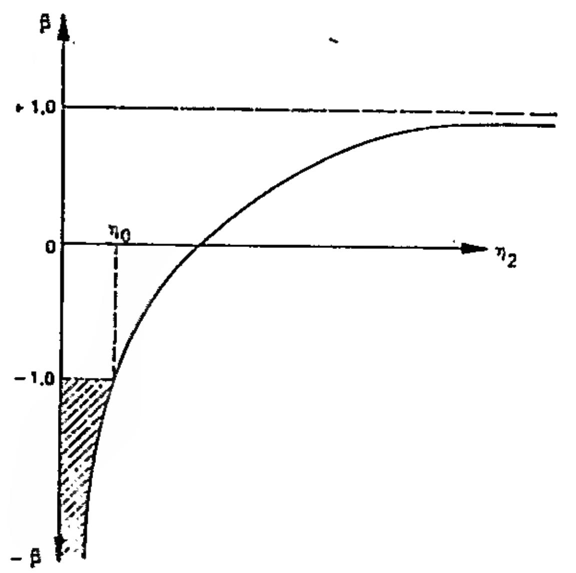


Fig. 4.26: Figure Illustrating Zone of Effectiveness of Absorbers (After Barkan, D. D., *Dynamics of Bases and Foundations*, McGraw-Hill, New York, 1962; with permission).

If the degree of absorption β is known, then from Eq. (4.64), η_2 can be obtained as

$$\eta_2^2 = \frac{1 - (1 + \alpha)\,\eta_1^2}{(1 + \alpha)\,(\beta - 1)\,(\eta_1^2 - 1)} \tag{4.66}$$

These relations are applied in Example (3) illustrated in Section 4.4.8 for the design of a block foundation for a vertical engine resting on spring absorbers.

4.4.8 *Numerical Examples*

1. Design of a Block Foundation for a Horizontal Compressor

The outline dimensions of the foundation for a horizontal compressor as suggested by the machine suppliers are shown in Fig. 4.27. The data are as follows:

a.	Machine data	
	i. Operating speed of engine (f_m)	150 rpm
	ii. Horizontal unbalanced force in the direction of the piston (P_x)	12 t
	iii. Weight of machine (W)	36 t
	iv. The horizontal unbalanced force acts at a height of 0.6 m above the top of the foundation (level +0.0)	
b.	Soil data	
	i. Nature of soil	sandy
	ii. Bearing capacity of soil	2 kg/cm²
	iii. Coefficient of elastic uniform shear (C_τ)	2.25 kg/cm³
c.	Grade of concrete	M-150

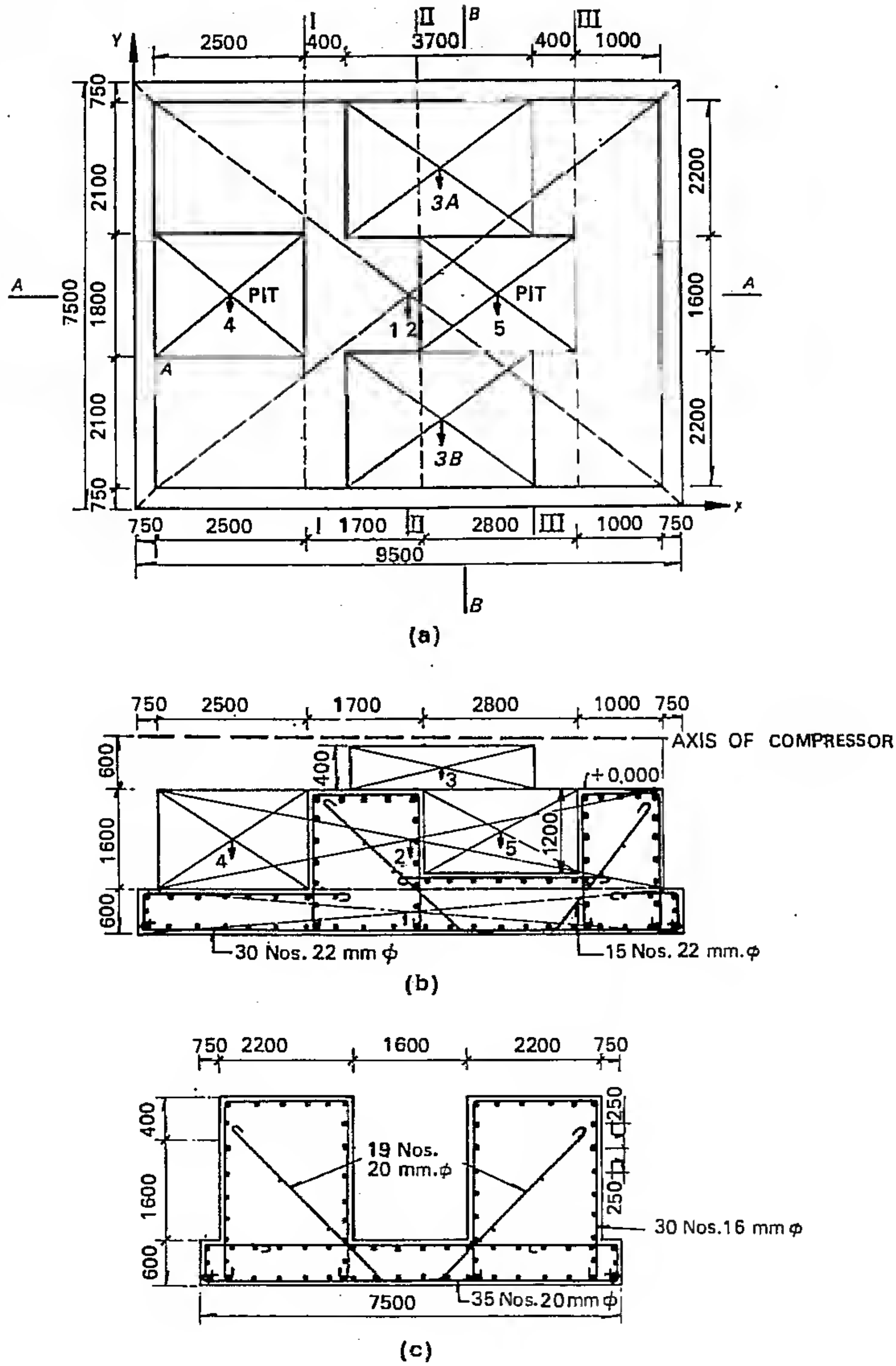


Fig. 4.27: Foundation for a Horizontal Compressor—**(a)** Plan at Shaft Level, **(b)** Section *A-A*, **(c)** Section *B-B*.

Table 4.9

COMPUTATIONS FOR CENTRE OF GRAVITY AND MASS MOMENTS OF INERTIA

Element (i) of the system (marked in Fig. 4.27)	Dimensions of elements			Weight*	Mass	Coordinates of centre of gravity of element			Static moment of mass			Moment of inertia of element about y axis passing through centre of gravity of the element			
	l_{xi} (m)	l_{yi} (m)	l_{zi} (m)	w_i (t)	m_i (t.sec²/m)	x_i (m)	y_i (m)	z_i (m)	$m_i x_i$	$m_i y_i$	$m_i z_i$	$\frac{m_i}{12}\left(l_{xi}^2 + l_{zi}^2\right)$	$x_{oi}=(\bar{x}-x_i)$	$z_{oi}=(\bar{z}-z_i)$	$m_i\left(x_{oi}^2+z_{oi}^2\right)$
Machine Foundation part No.	—	—	—	36.00	3.669	3.65	3.75	2.8	13.40	13.76	10.28	—	1.095	1.561	13.31
1	9.5	7.5	0.6	102.60	10.460	4.75	3.75	0.3	49.68	39.22	3.14	78.98	0.005	0.939	9.22
2	8.0	6.0	1.6	184.32	18.790	4.75	3.75	1.4	89.25	70.46	26.31	104.22	0.005	0.161	0.49
3A	3.7	2.2	0.4	7.81	0.796	5.5	5.65	2.4	4.38	4.50	1.91	0.92	0.755	1.161	1.53
3B	3.7	2.2	0.4	7.81	0.796	5.5	1.85	2.4	4.38	1.47	1.91	0.92	0.755	1.161	1.53
4	2.5	1.8	1.6	−17.28	−1.761	2.0	3.75	1.4	−3.52	−6.60	−2.47	−1.29	2.745	0.161	−13.32
5	2.8	1.6	1.2	−12.90	−1.315	6.35	3.75	1.6	−8.35	−4.93	−2.10	−1.02	1.605	0.361	− 3.56
				308.34	31.440				149.22	117.88	38.98	182.73			9.20

*Density of concrete is 2.4 t/m³.

It is required to check the dynamic stability of the foundation and to suitably design the same.

Stages in Computation

a. *Centre of gravity:* Referring to Table 4.9 and Fig. 4.27, the coordinates $(\bar{x}, \bar{y}, \bar{z})$ of the common centre of gravity of machine and foundation are given by

$$\bar{x} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{149.22}{31.44} = 4.745 \text{ m}$$

$$\bar{y} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{117.88}{31.44} = 3.718 \text{ m}$$

$$\bar{z} = \frac{\Sigma m_i z_i}{\Sigma m_i} = \frac{38.98}{31.44} = 1.239 \text{ m}$$

Eccentricity of common centre of gravity with respect to centroid of base area:

$$\text{Eccentricity in } x \text{ direction} = \frac{(4.75 \sim 4.745) \times 100}{9.5} = 0.053\%$$

$$\text{Eccentricity in } y \text{ direction} = \frac{(3.75 - 3.718) \times 100}{7.5} = 0.427\%$$

The eccentricities are within the permissible limit of five per cent of the length of foundation in either direction.

b. *Design parameters:*

i. Mass of the foundation (m) from Table 4.9 $= \Sigma m_i$
$= 31.44$ t-sec²/m

ii. Moment (M_y) caused by the horizontal exciting force (P_x) acting at a height of 0.6 m above the top of foundation (M_y) $= 12(2.2 + 0.6 - 1.239)$
$= 18.732$ t·m

iii. Operating frequency of the machine (f_m) $= 150$ rpm
Circular frequency (ω_m) $= 2\pi\ (150/60)$
$= 15.71\ \text{sec}^{-1}$

iv. The moment of inertia (I_y) of the base area about the axis passing through its centre of gravity and perpendicular to the plane of vibration

$$I_y = \frac{7.5 \times 9.5^3}{12}$$

$$= 535.86 \text{ m}^4$$

v. The mass moment of inertia (φ_y) of the whole system about the y axis passing through the common centre of gravity and perpendicular to the plane of vibration

$$\varphi_y = \frac{1}{12} \Sigma m_i (l_{xi}^2 + l_{zi}^2) + \Sigma m_i (x_{0i}^2 + z_{0i}^2)$$

$= 182.73 + 9.20$ (From Table 4.9)
$= 191.93$ t·m·sec^2

vi. The mass moment of inertia (φ_0) about the axis passing through the centroid of the base area and perpendicular to the plane of vibration

$$\varphi_{0y} = \varphi_y + m\bar{z}^2$$

$$= 191.93 + 31.44 \times (1.239)^2$$

$$= 240.16 \text{ t·m·sec}^2$$

vii. The ratio (α_y) is given by

$$\alpha_y = \varphi_y/\varphi_{0y} \qquad \text{(Eq. 4.31)}$$

$$= \frac{191.93}{240.16}$$

$$= 0.8$$

viii. Limiting frequencies

$$\omega_{\theta y}^2 = (C_\theta\, I_y - W\bar{z})/\varphi_{0y}$$

$$= \frac{9 \times 10^3 \times 535.86 - 308.34 \times 1.239}{240.16}$$

$$= 20.08 \times 10^3 \text{ sec}^{-2}$$

$$\omega_x^2 = \frac{C_\tau A_f}{m}$$

$$= \frac{2.25 \times 10^3 \times 9.5 \times 7.5}{31.44}$$

$$= 5.098 \times 10^3 \text{ sec}^{-2}$$

c. *Coupled natural frequencies:*

$$\omega_n^4 - \frac{(\omega_{\theta y}^2 + \omega_x^2)}{\alpha_y}\,\omega_n^2 + \frac{\omega_{\theta y}^2\, \omega_x^2}{\alpha_y} = 0 \qquad \text{(Eq. 4.30)}$$

Substituting the values and solving the roots ω_{n1} and ω_{n2} are obtained from Eq. 4.34 as

$$\omega_{n1}^2 = 26.676 \times 10^3 \text{ sec}^{-2}$$

$$\omega_{n2}^2 = 4.796 \times 10^3 \text{ sec}^{-2}$$

The corresponding natural frequencies are

$$f_1 = 26.00 \text{ cps}$$

$$f_2 = 11.03 \text{ cps}$$

The lower of the two natural frequencies is well above the operating speed. Hence, there is no possibility of resonance.

d. *Amplitudes:* The coefficient $f(\omega_m^2)$ from Eq. (4.37)

$$f(\omega_m^2) = m\varphi_v\ (\omega_{n1}^2 - \omega_m^2)\ (\omega_{n2}^2 - \omega_m^2)$$

Substituting the values we obtain

$f(\omega_m^2) = 72.55 \times 10^{10}$

Horizontal amplitude (a_x) given by Eq. (4.36a):

$$a_x = \left[(C_\theta\, I_v - WS + C_\tau\, A_f\, S^2 - \varphi_v\, \omega_m^2)\ P_x + (C_\tau\, A_f\, S)\ M_v \right] \frac{1}{f(\omega_m^2)}$$

Substituting $S = \bar{z} = 1.239$ and other values in the above expression, we obtain

$a_x = 0.0882$ mm

Rotational amplitude ($a_{\theta v}$) given by Eq. 4.36b is

$$a_{\theta v} = \frac{C_\tau A_f S}{f(\omega_m^2)} P_x + \frac{C_\tau A_f - m\omega_m^2}{f(\omega_m^2)} M_v$$

Substituting the values

$a_{\theta v} = 0.0072$

Net amplitude at base level

$= a_x - S a_{\theta v}$

$= 0.0882 - 1.239 \times 0.0072$

$= 0.0792$ mm

Net horizontal amplitude at top of the foundation (Eq. 4.38)

$= a_x + (H - S)\ a_{\theta v}$

$= 0.0882 + (2.2 - 1.239) \times 0.0072$

$= 0.0951$ mm

< 0.2 mm (permissible)

e. *Dynamic forces:* (see Table 4.8) Taking a fatigue factor of 3, horizontal dynamic force (F_d) $= 3 \times 2.25 \times 10^3 \times 9.5 \times 7.5 \times 0.0792 \times 10^{-3}$

$= 38.10$ t

Dynamic moment (M_d) $= 3 \times 9 \times 10^3 \times 535.86 \times 0.0072 \times 10^{-3}$

$= 104.52$ t·m

f. *Check for soil stress:* Static weight of machine and foundation

$W = 308.44$ t

Maximum and minimum stresses on soil are given by

$$\sigma = \frac{308.44}{9.5 \times 7.5} \pm \frac{104.52\ (9.5 - 4.745)}{535.86}$$

$$= 4.329 \pm 0.928$$
$$\sigma_{max} = 5.257 \text{ t/m}^2 \quad < 20 \text{ t/m}^2$$
$$\sigma_{min} = 3.401 \text{ t/m}^2$$

g. *Structural design—longitudinal direction:*

i. Static loads: The eccentricity of the centre of gravity of the machine and foundation with respect to centroid of base area being very small, the soil reaction due to dead loads may be assumed as uniform

$$\text{Intensity of soil reaction} = \frac{308.44}{9.5 \times 7.5}$$
$$= 4.329 \text{ t/m}^2$$

Table 4.10 contains the weights of various parts of the foundation block (Fig. 4.27). Following are the net bending moments induced at various sections under the influence of static loads and resulting soil pressure.

$$(M_{st})_{I} = 64.03 \text{ t·m}$$
$$(M_{st})_{II} = 80.71 \text{ t·m}$$
$$(M_{st})_{III} = 21.66 \text{ t·m}$$

Table 4.10

EVALUATION OF INERTIAL FORCES

Element (i) of the system	Weight of the element (W_i) (t)	x_i (m)	z_i (m)	$X_i=(x_i-x_0)$ (m)	$Z_i=(z_i-z_0)$ (m)	Inertial forces Vertical (t)	Inertial forces Horizontal (t)
Machine	36.00	−1.095	−1.561	−1.095	10.646	−0.02148	0.2089
Foundation part							
1	102.60	+0.005	+0.939	+0.005	13.146	0.00028	0.7351
2	184.32	+0.005	−0.161	+0.005	12.046	0.00050	1.2099
3A	7.81	+0.755	−1.161	+0.755	11.046	0.00321	0.0470
3B	7.81	+0.755	−1.161	+0.755	11.046	0.00321	0.0470
4	−17.28	−2.745	−0.161	−2.745	12.046	0.02585	−0.1134
5	−12.90	+1.605	−0.361	+1.605	11.846	−0.01128	−0.0833
						$\Sigma \simeq 0$	$\Sigma = 2.0512$

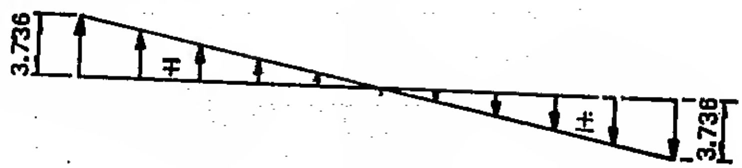
ii. Dynamic loads: The foundation is acted upon by the following dynamic loads:

a. Exciting moment multiplied by a fatigue factor of three which equals 56.196 t·m. The largest ordinate of the varying distributed loading (Fig. 4.28a) $= \dfrac{56.196 \times 6}{(9.5)^2}$

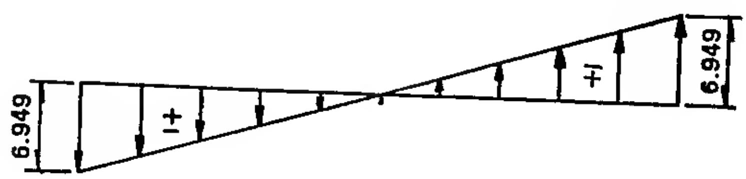
$$= 3.736 \text{ t/m}$$

b. Dynamic moment (104.52 t·m) which acts in the form of a varying distributed load, the largest ordinate being (Fig. 4.28b):

$$\frac{6 \times 104.52}{(9.5)^2} = 6.949 \text{ t/m}$$



(a)



(b)

Fig. 4.28: Varying Distributed Load Due to **(a)** Exciting Moment, **(b)** Dynamic Moment.

c. Inertial forces: The inertial forces may be neglected here, the frequency ratio $\left(\frac{f_m}{f_n}\right)$ being very small. However, for purpose of illustration, they are evaluated below to show that they are relatively very small.

Inertial force $(F_m)_x = \xi m \; a_x \; \omega_m^2$

$= 3 \times 31.44 \times 0.0882 \times 10^{-3} \times 15.71^2 = 2.053$ t

Inertial moment $(M_m)_y = \xi \varphi_y \; a_{\theta y} \; \omega_m^2$

$= 3 \times 191.93 \times 0.0072 \times 10^{-3} \times 15.71^2$

$= 1.026$ t-m

The coordinates of centre of rotation $O(X_0, Z_0)$ are obtained from Eqs. 4.54a and 4.54b, substituting $a_z = 0$

$$x_0 = 0$$

$$z_0 = \frac{-a_x}{a_{\theta y}} = \frac{0.0882}{0.0072} = -12.207 \text{ m}$$

Table 4.10 shows the inertial forces (vertical and horizontal) shared by various parts of the foundation body and the machine given by Eqs. 4.56a and 4.56b. In this table, (x_i, z_i) are the coordinates of the centre of gravity of part i of the foundation referred to the common centre of gravity of machine foundation as origin and (X_i, Z_i) are the coordinates of the same point referred to centre of rotation as origin.

In view of the small magnitudes of the inertial forces computed in Table 4.10, they may be neglected in the computation of net dynamic moments. Table 4.11 shows the net dynamic moments at the sections chosen.

iii. Net moments $(M_{st} \pm M_d)$

$M_I = 64.03 \mp 13.10 = 50.93;\ 77.13$ t-m

$M_{II} = 80.71 \pm 22.64 = 103.35;\ 58.07$ t-m

$M_{III} = 21.66 \pm 3.84 = 25.50;\ 17.82$ t-m

Table 4.11

DYNAMIC MOMENTS AT BASE LEVEL

Section (Fig. 4.27)	Moment due to dynamic forces t.m	Moment due to exciting forces t.m	Net dynamic moment t.m
I-I	∓ 28.33	± 15.23	∓ 13.10
II-II	± 48.97	∓ 26.33	± 22.64
III-III	± 8.31	∓ 4.47	± 3.84

Thirty numbers of 22 mm ϕ bars (at 25 cm spacing) are provided at sections I and II with alternate bars bent at section III (Fig. 4.27). A similar procedure is adopted for analysis in the y direction.

Fig. 4.27 shows the disposition of reinforcement in the foundation block. Nominal

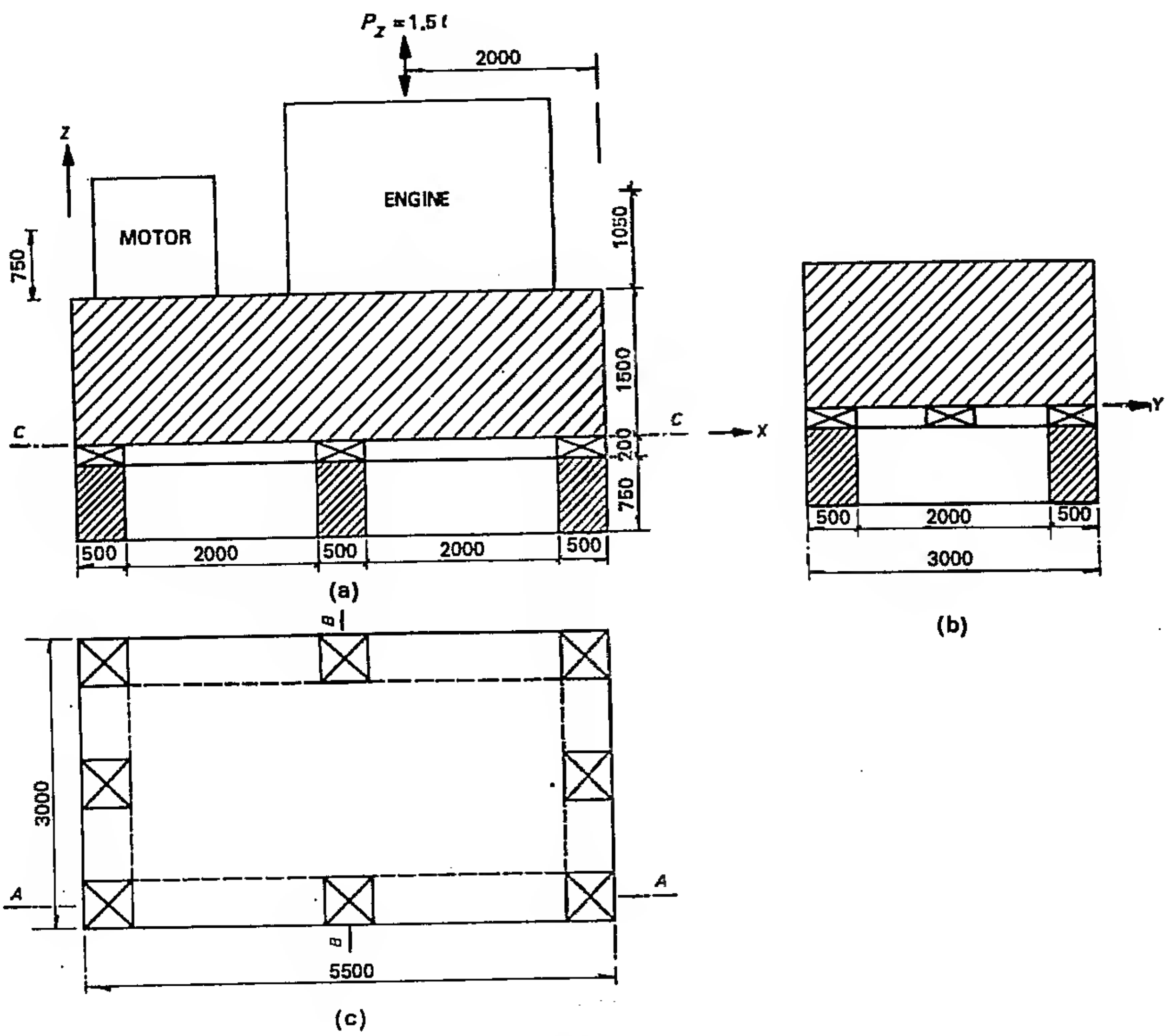


Fig. 4.29: Foundation for a Diesel Engine—(a) Section A-A, (b) Section B-B, (c) Plan at C-C.

reinforcement provided on other faces of the block (other than the tensile face) is according to the code of practice (IS 456-1964).

2. Design of a Block Foundation for a Diesel Engine

The schematio diagram of the proposed foundation for a four-cylinder diesel engine is shown in Fig. 4.29.

Data:	
i. Total weight of machine (W_m)	6 t
ii. Operating speed of machine (f_m)	1200 rpm
iii. Vertical exciting force (P_z)	± 1.5 t
iv. Permissible amplitude at floor level	0.04 mm
v. Nature of soil	stiff clay

Since the permissible amplitude (0.04 mm) is very low, spring absorbers shall be placed under the foundation block. The spring casings are placed on rigid pedestals and the foundation block is supported on them as shown in Fig. 4.29.

This example illustrates the principles of analysis and design of a block foundation resting on springs. Since the soil is stiff and the pedestals supporting the spring casings are short and rigid, the influence of soil may be ignored in design. The analysis is thus carried out for the foundation block resting on springs which are placed on a rigid base.

Stages in Computation

a. *Centre of gravity* $G(\bar{x}, \bar{z})$: Referring to Table 4.12

Table 4.12

COMPUTATIONS FOR CENTRE OF GRAVITY AND MASS MOMENTS OF INERTIA

Part	w_i	m_i	x_i	z_i	$w_i x_i$	$w_i z_i$	Mass moment of inertia about an axis parallel to the Y axis and passing through the common centre of gravity (φ_y)
	(t)	(t sec²/m)	(m)	(m)	(t.m)	(t.m)	(t.m.sec²)
Engine	4.0*	0.408	3.50*	2.55*	14.00	10.20	1.103
Motor	2.0*	0.204	1.50*	2.25*	3.00	4.50	0.368
Foundation block	59.4	6.055	2.75	0.75	163.35	44.55	16.549
Sum	65.4	6.667			180.35	59.25	18.020

* Figures based on data supplied.

$$\bar{x} = \frac{\Sigma W_i x_i}{\Sigma W_i}$$

$$= 180.3/65.4$$

$$= 2.757 \text{ m}$$

$$\bar{z} = \frac{\Sigma W_i z_i}{\Sigma W_i}$$

$$= 59.25/65.4$$

$$= 0.906 \text{ m}$$

b. *Mass moment of inertia* (φ): Table 4.12 contains the mass moment of inertia of the machine foundation about an axis parallel to the y axis and passing through the centre of gravity.

$$\varphi_y = 18.020 \text{ t. m.sec}^2$$

The moment of inertia (φ_{0y}) about the parallel axis (parallel to the y axis) through the base

$$\varphi_{0y} = \varphi_y + mS^2$$
$$= 18.020 + 6.667 \times 1.006^2 \quad (\text{since } S = \bar{z} + 0.1)$$
$$= 24.767 \text{ t·m·sec}^2$$

The factor α_y is given by Eq. (4.31)

$$\alpha_y = \varphi_y/\varphi_{0y}$$
$$= 0.728$$

c. *Dimensioning the springs:* The spring assembly consists of eight spring casings, each containing nine spiral springs.

Total number of springs = 72

Static load on each spring $(P_{St}) = \dfrac{65.4 \times 10^3}{72}$

$= 908.4$ kg

From Table 3.4, the springs having the following characteristics are chosen:

Diameter of coil (D) = 120 mm

Diameter of spring wire (d) = 20 mm

Number of turns (n) = 5

Spring coefficient (K_s) = 96.4/5 kg/mm

= 19.28 t/m

Load-carrying capacity of spring (P) (see Table 3.4) = 1270 kg

Moment of inertia of base resting on springs (Eq. 3.3b)

$$I_y = 6 \times 9 \times 2.5^2$$
$$= 337.5 \text{ m}^2$$

d. *Frequency calculations:*

i. Vertical vibration

The vertical spring coefficient $(K_z) = nK_s$

$= 72 \times 19.28$

$= 1388$ t/m

Mass of the whole system (m) $= 65.4/9.81$

$= 6.667$ t sec^2/m

The vertical natural frequency $\omega_{nz} = \sqrt{\dfrac{K_z}{m}}$

$$f_{nz} = \sqrt{1388/6.667}$$
$$= 14.43 \text{ sec}^{-1}$$
$$= 137.87 \text{ cpm}$$

ii. Vibrations in the xz plane
Length of spiral under static load (h) = 18 cm
Slenderness ratio $h/D = 180/120 = 1.5$
Compression of the spring under static load

$$\delta_z = \frac{1}{K_z} \cdot W$$
$$= 65.4/1388$$
$$= 0.04712 \text{ m} = 4.712 \text{ cm}$$
$$\frac{\delta_z}{h} = 4.712/18$$
$$= 0.262$$

The valuc of α takcn from Fig. 3.8 is 1.3.
The spring coefficient against horizontal translation (K_x)

$$= K_z \frac{1}{\alpha \times 0.385\left[1 + 0.77\left(\dfrac{h}{D}\right)^2\right]} \tag{3.18b}$$

$$= 1388 \frac{1}{1.3 \times 0.385\,[1 + 0.77 \times 1.5^2]}$$
$$= 1015 \text{ t/m}$$

The spring coefficient against rotation ($K_{\theta y}$)

$$= K_s I_y \tag{3.18}$$
$$= 19.28 \times 337.5 = 6507 \text{ t/m}$$

Limiting frequencies

$$\omega^2_{\theta y} \approx \frac{K_{\theta y}}{\varphi_{0y}} \text{ (Neglecting } WS \text{ in Eq. 4.32a)}$$
$$= 6507/24.767 = 262.7 \text{ sec}^{-2}$$
$$\omega_x^2 = \frac{K_x}{m} \cdot \tag{4.33b}$$
$$= 1015/6.667 = 152.2 \text{ sec}^{-2}$$

The equation for the coupled natural frequencies (Eq. 4.30) gives:

$$\omega_n^4 - \frac{\omega_{\theta y}^2 + \omega_x^2}{\alpha_y}\omega_n^2 + \frac{\omega_{\theta y}^2\,\omega_x^2}{\alpha_y} = 0$$

Substituting the values and solving,

$$\omega_{n1}^2 = 447.41 \text{ sec}^{-2}$$
$$\omega_{n1} = 21.15 \text{ sec}^{-1}$$
$$\omega_{n2}^2 = 122.82 \text{ sec}^{-2}$$
$$\omega_{n2} = 11.08 \text{ sec}^{-1}$$

$f_{n1} = 21.15/0.105 = 202$ cpm
$f_{n2} = 11.08/0.105 = 106$ cpm

e. *Amplitude calculations:*

i. The vertical amplitude

$$a_z = \frac{P_z}{m\,(\omega_{nz}^2 - \omega_m^2)} \tag{4.29}$$

where $\omega_m = 1200 \times (2\ \pi/60)\ \text{sec}^{-1}$
$= 125.66\ \text{sec}^{-1}$

$$a_z = \frac{1.5}{6.667[(14.43)^2-(125.66)^2]}$$
$= -0.143 \times 10^{-4}$ m
$= -0.0143$ mm <0.04 mm (permissible)

ii. Horizontal amplitude (a_x)

$$f\,(\omega_m^2) = m\varphi_y\,(\omega_{n1}^2 - \omega_m^2)\;(\omega_{n2}^2 - \omega_m^2) \tag{4.37}$$

$= 6.667 \times 18.020(447.41 - 15790.44)(122.82 - 15790.44) = 288.8 \times 10^8$

P_x being zero, amplitude (a_x) is given from Eq. 4.36a as

$$a_x = \frac{K_x\,S}{f\,(\omega_m^2)}\;M_y \tag{4.36a}$$

where $M_y = P_z \cdot e$
$= 1.5 \times 0.750 = 1.125$ t·m

Substituting the values

$$a_x = \frac{1015 \times 1.006 \times 1.125}{288.8 \times 10^8}$$

$= 3.978 \times 10^{-8}$ m (very small)

iii. Rotational amplitude ($a_{\theta y}$)

$$a_{\theta y} = \frac{K_x - m\,\omega_m^2}{f\,(\omega_m^2)}\;M_y \tag{4.36b}$$

$$= \frac{(1015 - 6.667 \times 15790)}{288.8 \times 10^8} \times 1.125$$

$= -0.406 \times 10^{-5}$

Net horizontal amplitude on the top face of foundation, neglecting the value of a_x which is small,

$a_{top} = (1.5 - 1.006)a_{\theta y}$
$= -0.494 \times 0.406 \times 10^{-5}$
$= -0.0020$ mm <0.04 mm (allowed)

f. *Dynamic forces:*

i. Vertical dynamic force

$V_d = \pm\, \xi\, K_z a_z$

$$= -3 \times 1388 \times 0.143 \times 10^{-4}$$
$$= -0.0595 \text{ t}$$

ii. Horizontal dynamic force

$$H_d = \pm \xi K_x (a_x - S a_{\theta y})$$
$$= \mp 3 \times 1015 \ (4.124 \times 10^{-6})$$
$$= \pm 0.0123 \text{ t}$$

iii. The dynamic moment in the vertical plane

$$M_d = \pm \xi K_{\theta y} \, a_{\theta y}$$
$$= \pm 3 \times 6507 \times 4.06 \times 10^{-6}$$
$$= 0.0793 \text{ t·m}$$

The dynamic forces are obviously very small, the frequency ratio $\left(\frac{\omega_m}{\omega_n}\right)$ being large.

g. *Dynamic loading on the springs:* In view of the transient resonance (f_n being less than f_m) twice the generating force may be considered as static equivalent force for purpose of checking the total load exerted on springs

$$F_z = \pm 2 \times 1.5$$
$$= \pm 3 \text{ t}$$

The associated moment (M_z) is given by

$$M_z = P_z . e$$
$$= 3 \times 0.75$$
$$= 2.25 \text{ t·m}$$

The dynamic load on the farthest spring is given by

$$P_d = \pm \left[\frac{P_z}{n} + \frac{M_z}{l n_1} \right]$$

where n is the number of springs

l is the distance between extreme spring casings

n_1 is the number of springs on one side.

Substituting the values

$$P_d = \pm \left[\frac{3000}{72} + \frac{225000}{5 \times 10^2 \times 27} \right]$$

$$= \pm 58.33 \text{ kg}$$

The maximum load on the farthest spring is given by

$$P_{max} = P_{st} + P_d$$
$$= 908.4 + 58.33$$
$$= 966.73 \text{ kg}$$
$$< 1270 \text{ kg (safe)}$$

h. *Inertial forces:* Coordinates of centre of rotation

$$x_0 = -\frac{a_z}{a_{\theta y}} = -3.522 \text{ m} \tag{4.54a}$$

$$z_0 = -\frac{a_x}{a_{\theta y}} = 0.0099 \text{ m} \tag{4.54b}$$

Since z_0 is very small, the horizontal inertial forces acting on the block may be neglected.

The foundation is in equilibrium under the action of thrice the exciting force (where factor three is the fatigue coefficient) the inertial forces and the dynamic forces. For structural design the dynamic forces may be neglected in this case since their magnitudes are very small.

Fig. 4.30 shows the equilibrating forces acting on the foundation.

i. Thrice the exciting force (4.5 t) acting from above and
ii. Inertial forces.

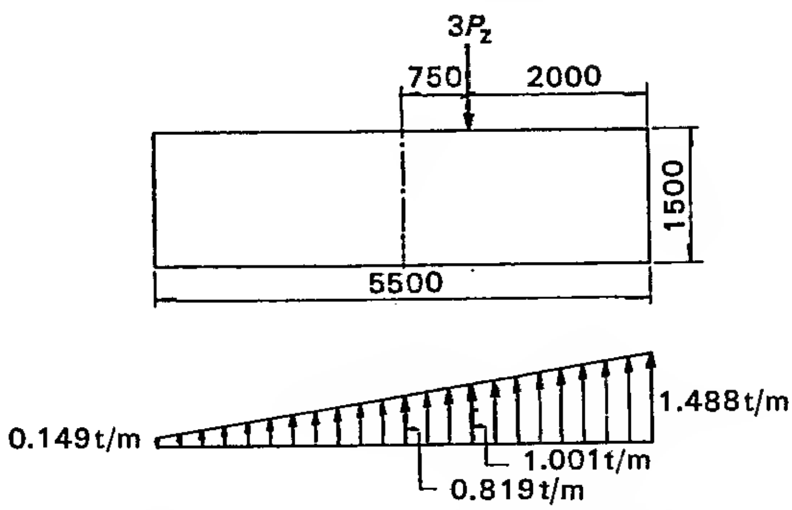


Fig. 4.30: Equilibrating Forces on the Foundation.

For equilibrium the end ordinates of the vertical inertial force distribution diagram and the pressure distribution diagram due to the eccentric load of 4.5 t should have equal magnitudes. Their lines of action should, however, be opposite to one another. The end ordinates (Fig. 4.30) are given by

$$= \frac{3P_z}{L}\left[1 \pm \frac{6e}{L}\right]$$

$$= \frac{4.5}{5.5}\left(1 \pm \frac{6 \times 0.75}{5.5}\right)$$

$$= 1.488 \text{ t/m}: 0.149 \text{ t/m}$$

From Fig. (4.30), the following moments may be calculated

Bending moment at the section where the load of 4.5 t acts	2.051 t.m
Bending moment at the middle section	1.408 t.m

These moments being small, nominal reinforcement as per constructional practice (see Section 8.2) may be provided.

3. Design of a Block Foundation for a Vertical Compressor on Spring Absorbers

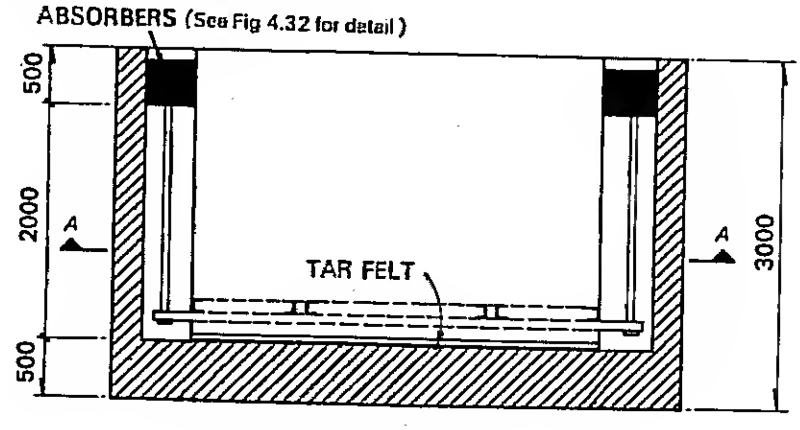
The foundation for a compressor is to be located close to a precision machine shop. Design a suitable foundation given the following data:

i. Output of engine $= 100$ kW
ii. Operating speed of machine $= 360$ rpm
iii. Exciting force in vertical direction (P_z) $= 3$ t
iv. Permissible amplitude of foundation (a_z) $= 0.04$ mm
v. Coefficient of elastic uniform compression of soil (C_z) $= 2 \times 10^3$ t/m^3

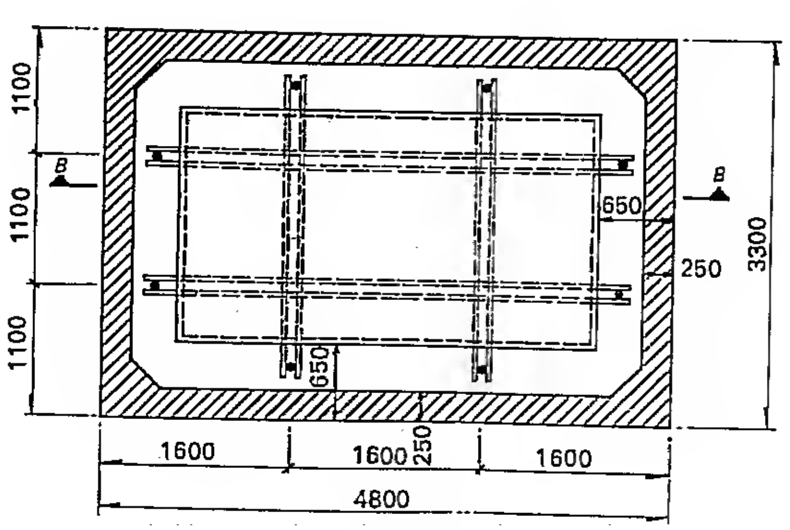
This example illustrates the application of suspended spring absorbers. The upper foundation block supporting the machine is suspended by springs which are provided in an enclosure around the foundation as shown in Fig. 4.31. The system is considered here as having two degrees of freedom and the analysis is carried out on the lines explained in Section 4.4.7.

The selcted dimensions of the foundation as shown in Fig. 4.31 give the following data for computations:

Foundation area in contact with soil 15.84 m^2
Wcight of foundation below the springs 41.81 t



(a)



(b)

Fig. 4.31: Compressor Foundation Suspended from Springs—(**a**) Section *B-B*, (**b**) Section *A-A*.

Weight of foundation above the springs $= 42$ t

Coefficient of rigidity of base (K_1) $= C_z A$

$= 2 \times 10^3 \times 15.84$

$= 31.68 \times 10^3$ t/m

Mass (m_1) of foundation below springs $= 41.81/9.81$

$= 4.261$ t sec²/m

Mass (m_2) of foundation above springs $= 42/9.81$

$= 4.281$ t sec²/m

Limiting natural frequency ($\bar{\omega}_{n1}$) is given by Eq. 2.23b

$$\bar{\omega}_{n1}^2 = \frac{K_1}{m_1 + m_2}$$

$$= \frac{31.680 \times 10^3}{4.261 + 4.281}$$

$$= 3.708 \times 10^3 \text{ sec}^{-2}$$

The coefficient (η_1) according to Eq. 4.60 is given by

$$\eta_1^2 = \frac{3.708 \times 10^3}{(12\,\pi)^2}$$

$$= 2.609$$

Mass ratio (α)

$$= \frac{4.281}{4.261} = 1.005$$

For the chosen dimensions of the foundation, the amplitude of vertical vibrations when there are no absorbers should be according to Eq. 4.62

$$a_z = \frac{P_z}{(m_1 + m_2)\,(\omega_{n1}^2 - \omega_m^2)}$$

$$= \frac{3}{(4.261 + 4.281)\,(3.708 - 1.421)\,10^3}$$

$$= 0.154 \text{ mm}$$

Permissible amplitude (a_1) of the foundation from the data given $= 0.04$ mm

The efficiency of absorber to be designed (β) $= \dfrac{0.154}{0.04}$

$= 3.84$

From Fig. 4.26 adopting a value of $\beta = -5$, and using Eq. 4.66 the required value of η_2 is obtained as

$$\eta_2^2 = \frac{1 - (1 + 1.005)\,2.609}{(1 + 1.005)\,(-5 - 1)\,(2.609^2 - 1)}$$

$$= 0.219$$

From Eq. 4.61, $\bar{\omega}^2_{n2} = \eta^2_2 \times \omega^2_m$
$= 0.219 \times (12\pi)^2$
$= 310.63$

Also $$\bar{\omega}^2_{n2} = \frac{K_2}{m_2}$$

∴ The rigidity of absorbers (K_2) is given by
$K_2 = 310.63 \times 4.281$
$= 1329.8$ t/m

a. *Natural frequencies:* The natural frequencies of the system are given by the roots of Eq. 2.22

$$\omega^4_n - (1 + 1.005)\ (3708 + 310.63)\ \omega^2_n + (1 + 1.005)\ 3708 \times 310.63 = 0$$

This gives

$\omega^2_1 = 7.760 \times 10^3\ \text{sec}^{-2}$
and $\omega^2_2 = 0.298 \times 10^3\ \text{sec}^{-2}$
so $f_1 = 14.02$ cps
and $f_2 = 2.75$ cps

The two frequencies are well away from the operating frequency of 6 cps.

b. *Design of spring absorber assembly:* Let n_1 be the number of springs in each casing and n_2 the number of casings.

The rigidity of each spring $(K_S) = \dfrac{K_2}{n_1 n_2}$

Assuming $n_1 = 2$ and $n_2 = 8$, $K_S = \dfrac{1329.8}{16}$
$= 83.11$ t/m

From Table 3.4 adopting springs having coil diameter (D) equal to 80 mm, and wire diameter (d) equal to 24 mm, the stiffness of each spring having eight turns is equal to 672/8 = 84 t/m.

From Table 3.4, the permissible load on the spring (P) = 2.75 t. Actual load on spring $= K_2\,(a_2 \sim a_1)$ where a_2 and a_1 are the amplitudes of masses m_2 and m_1.

c. *Amplitudes:* From Eq. 2.32:

$$f(\omega^2_m)^2 = \omega^4_m - (1 + \alpha)\ (\bar{\omega}^2_{n2} + \bar{\omega}^2_{n1})\ \omega^2_m + (1 + \alpha)\ \bar{\omega}^2_{n1}\ \bar{\omega}^2_{n2}$$

$$= (12\pi)^4 - (1 + 1.005)\ (0.3106 + 3.708)10^3\ (12\pi)^2 + (1 + 1.005)\ (0.3106 \times 3.708)10^6$$
$$= 7.122 \times 10^6$$

From Eq. 2.31b

$$a_2 = \frac{[(1+\alpha)\ \bar{\omega}^2_{n1} + \alpha\,\bar{\omega}^2_{n2} - \omega^2_m]\ P_0}{m_2 f\,(\omega^2_m)}$$

$$= \frac{[(1+1.005)\ 3.708 \times 10^3 + 1.005 \times 0.3106 \times 10^3 - (12\,\pi)^2]\ 3.0}{4.281 \times -7.122 \times 10^6}$$

$= -\ 0.6224$ mm

From (Eq. 4.64):

$$a_1 = \frac{a_2}{\beta} = \frac{0.154}{-5.0} = -0.0307$$

Dynamic force on the spring absorbers $(F_d) = K_2(a_2 \sim a_1)$

$$(F_d) = 1329.8(-0.6224 + 0.0307) \times 10^{-3}$$
$$= 0.8685 \text{ t}$$

Total load on springs $= W + F_d$

$$= 42 + 0.8685$$

$$\text{Load on each spring} = \frac{42.8685}{16} = 2.679 \text{ t}$$

This is less than 2.75 t which is the permissible load on each spring.
Fig. 4.32 shows the mounting details of the suspended type of absorbers.

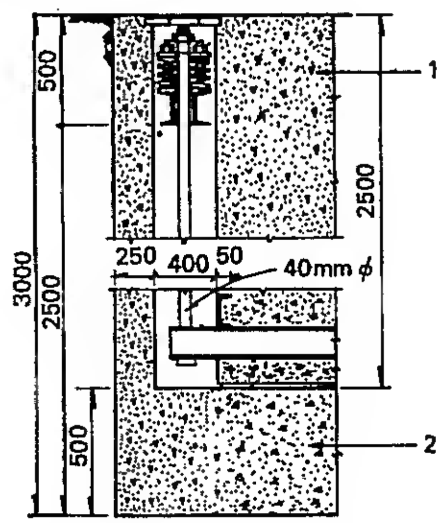


Fig. 4.32: Detail of Spring Absorbers in Suspended System—(*1*) Upper Foundation Block (*2*) Lower Foundation Block.

4.5 Foundations Subject to Impact-Type Forces (Example: Hammers)

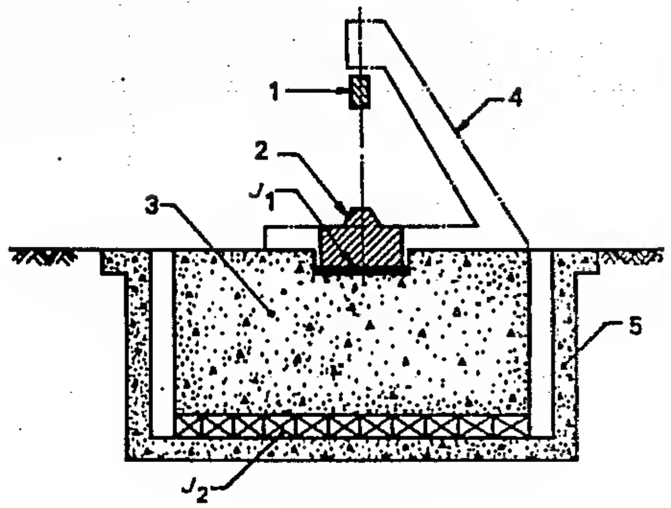
Hammers are typical examples of impact-type machines. From the designer's point of view, two types of hammer are considered. In one of them the anvil is fixed while in the other, it also moves similar to the falling tup. The latter type of hammer is termed "counter-blow" hammer.

Hammer foundations are generally of reinforced concrete block type of construction (Fig. 4.33). The anvil (2) on which the tup (1) falls repeatedly is usually placed on an elastic layer (J_1) which may be in the form of timber grillage, cork, etc. The foundation (3) may be placed either directly on soil as in Fig. 4.33(b) or on a suitable elastic layer (J_2) as in Fig. 4.33a. These elastic underlayers below the foundation serve the purpose of providing isolation and protecting the environment from the harmful effects of vibration caused by the impacts.

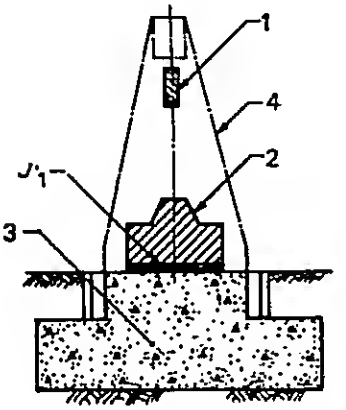
The frame of the hammer (4) may either rest directly on the foundation (as shown in Fig. 4.33) or it may be supported from outside as per convenience.

4.5.1 Special Considerations In Planning

a. The foundation should be so laid that the centre line of anvil and the centroid of the



(a)



(b)

Fig. 4.33: Typical Hammer Foundation—(**a**) Resting on Elastic Supports, (**b**) Resting on Soil, (*1*) Tup, (*2*) Anvil, (*3*) Foundation, (*4*) Frame, (*5*) RC Trough (From IS: 2974-1966; with permission of the Indian Standards Institution, New Delhi).

base area lie on the vertical line passing through the common centre of gravity of the machine and its foundation.

b. Where elastic underlayers are used under the anvil and the foundation base, care should be taken to ensure uniform distribution of loading and protection of these materials against water, oil, etc., which may cause progressive deterioration of their elastic properties. It is recommended that the foundation be laid in a reinforced concrete trough formed by retaining walls on all of the sides. The foundation and the side walls may be separated by means of an air gap or a cavity filled with some elastic material.

c. If timber is used as elastic support under the anvil, the timber beams should be laid horizontally in the form of a grillage. The beams must be impregnated with preservative for protection against moisture.

d. The thickness of the elastic layers provided is governed by permissible stresses in the respective materials. Table 4.13 gives guidance for the thickness of pads under the anvil.

Table 4.13

THICKNESS OF TIMBER PADS UNDER ANVIL

(After Major, 1962)

Type of hammer	Thickness of pad for a falling weight of		
	upto 1 t (m)	1-3 t (m)	>3 t (m)
Double acting drop hammers	0.2	0.2 to 0.6	0.6 to 1.2
Single acting drop hammers	0.1	0.1 to 0.4	0.4 to 0.9
Forging hammers	0.2	0.2 to 0.6	0.6 to 1.00

e. When two neighbouring foundations are laid at different depths, the straight line connecting the adjacent edges should form an angle not exceeding 25° to the horizontal (Fig. 4.34). However, if the foundations are too close, they may be laid to the same depth and a common mat provided as base.

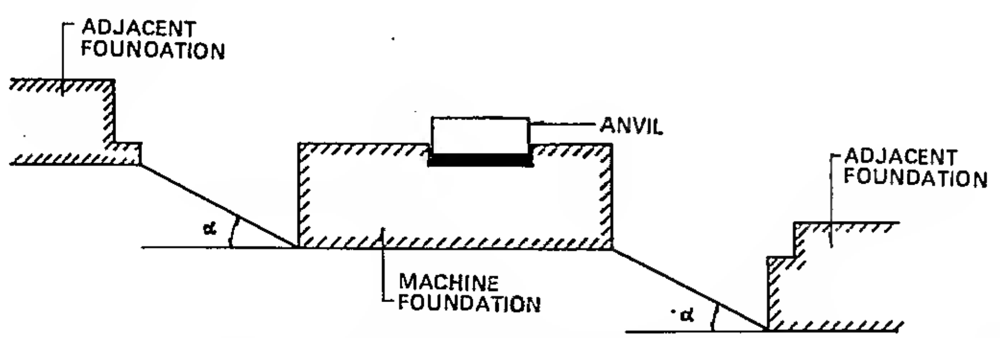


Fig. 4.34: Criteria for Location of Neighbouring Foundations (From IS: 2974, Pt. II-1966; with permission of Indian Standards Institution, New Delhi).

4.5.2 Design Data

The following data are required to be supplied to the designer:

a. Type of hammer
b. Weight of falling tup (W_t)
c. Weight of anvil (W_a)
d. Weight of hammer stand supported on foundation (W_{st})
e. Dimensions of base area of anvil ($L_a \times B_a$)
f. Maximum stroke or fall of hammer (h)
g. Effective working pressure (p) on piston and area of piston (A)
h. Outline of the foundation showing the position of anchor bolts, level of the operating floor, position of adjacent foundations, etc.

4.5.3 Design Criteria

a. The amplitudes of foundation block and anvil should not exceed the permissible values given below:

i. For the foundation block (a_f):

The maximum vertical vibration amplitude of foundation should not exceed 1.2 mm. In the case of foundations resting on sand below ground water table, the permissible amplitude limit should be 0.8 mm.

ii. For the anvil (a_a):

The permissible amplitudes of anvil which depend on the weight of falling tup are as follows:

Weight of tup (W_t)	Upto 1 t	2 t	3 t
Maximum permissible amplitude	1 mm	2 mm	3–4 mm

b. The maximum stresses on soil and other elastic layers shall be less than the permissible limits for the respective materials.

4.5.4 Foundations Resting on Soil : Principal Stages in Design Calculations

a. Minimum Weight of Foundations and Base Area Required

The minimum weight (W_f) of the foundation is based on the requirement that the amplitude of vibration is less than the permissible limit of 1 mm. This gives the expression

$$W_{min} = W_t\left[8(1+k)v - \frac{W_a + W_{st}}{W_t}\right] \tag{4.67}$$

where k is coefficient of impact (k=0.5 for stamping hammers and 0.25 for forging hammers), and v is the initial velocity of hammer head. The term W_{st} should be used in Eq. 4.67 only if the hammer stand is directly resting on foundation.

The minimum base area of the foundation is decided on the requirement that the stress on the soil should be within the permissible limit (σ_p). This gives the expression

$$A_{min} = \frac{20(1+k)}{\sigma_p} v \cdot W_t \tag{4.68}$$

Table 4.14 gives the minimum thickness of foundation below the anvil base for different weights of hammer head.

Table 4.14

MINIMUM THICKNESS OF FOUNDATION

(After Major, 1962)

Weight of hammer head (t)	Minimum thickness of foundation under Anvil (m)
Upto 1.0	1.0
2.0	1.25
4.0	1.75
6.0	2.25
$>$6.0	$>$2.25

b. Analysis for Vertical Vibrations

Hammer foundations are analysed essentially for vertical vibrations. The principal steps involved in the dynamic analysis for vertical vibrations are explained below:

The following notation is used :

Mass of tup	$m_t = W_t/g$ (t.sec^2/m)
Mass of anvil	$m_a = W_a/g$ (t.sec^2/m)
Mass of foundation	$m_f = W_f/g$ (t.sec^2/m)
Mass of hammer stand resting on the foundation	$m_{st} = W_{st}/g$ (t.sec^2/m)

Area of foundation base	$= L \times B$ (m²)
Area of anvil base	$= L_a \times B_a$ (m²)
Thickness of elastic pad under anvil	$= t_a$ (m)
Modulus of elasticity of the elastic pad under anvil	$= E_a$ t/m²
Initial velocity of impact	$= v$ m/sec

For the analysis of vertical vibrations, the two-mass spring system shown in Fig. 4.35 shall be adopted. The analysis of such a system subjected to free vibrations has been given in Section 2.3.

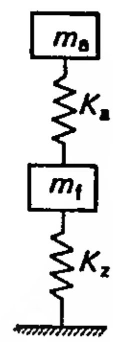


Fig. 4.35: Model System for Dynamic Analysis—Anvil and Foundation.

i. *Masses:*

$$m_f = (W_f + W_{at})/g \tag{4.69a}$$

$$m_a = W_a/g \tag{4.69b}$$

ii. *Stiffnesses of spring layers:* The vertical stiffness (K_z) of soil is given by

$$K_z = C_z' A_f \tag{4.70}$$

where
$$C_z' = \alpha C_z \tag{4.70a}$$

C_z is the coefficient of elastic uniform compression of soil corresponding to the actual base area of the foundation and α an experimentally determined factor which may be taken as three for hammer foundations.

Stiffness of pad under anvil (K_a) is given by

$$K_a = E_a A_a / t_a \tag{4.71}$$

iii. *Limiting frequencies* (ω_a, ω_z): The square of the limiting frequency (ω_a) defined as the frequency of natural vibration of the anvil assuming the soil to be rigid ($K_z = \infty$) is given by

$$\omega_a^2 = \frac{K_a}{m_a} \tag{4.72}$$

The square of the other limiting frequency (ω_z) of the entire system assuming $K_a = \infty$ is given by

$$\omega_z^2 = \frac{K_z}{m_f + m_a + m_{at}} \tag{4.73}$$

m_{st} should be added in the denominator only if the hammer stand is directly resting on the foundation.

iv. *Natural frequencies* (ω_{n1}), (ω_{n2}): From the analysis of a two-degree system (Fig. 4.35) subjected to free vibrations as explained in Sec. 2.3, the two circular natural frequencies ω_{n1}, ω_{n2} shall be determined as roots of the following quadratic equation in ω_n^2

$$\omega_n^4 - (\omega_a^2 + \omega_l^2)(1+\alpha)\omega_n^2 + (1+\alpha)\omega_a^2\,\omega_l^2 \tag{4.74}$$

where

$$\alpha = \frac{m_a}{m_l + m_{st}} \tag{4.75}$$

v. *Velocity of tup (v) before impact:*
(a) For a free-fall hammer

$$v = \alpha\sqrt{2gh_0} \tag{4.76}$$

where h_0 is height of fall
α is correction factor which characterizes the resistance of exhaust steam ($\alpha \approx 1$ for well-adjusted hammers).
(b) For a double-acting hammer

$$v = \alpha\sqrt{\frac{2g(W_t + pA)l}{W_t}} \tag{4.77}$$

where p is the mean pressure on piston
a is the area of piston
and l is length of stroke
α varies from 0.5 to 0.8. An average value of 0.65 may be taken.
If the energy of impact (E_0) is given by the manufacturers, then

$$h_0 = E_0/W_t \tag{4.78a}$$

and

$$v = \sqrt{2gh_0} \tag{4.78b}$$

vi. *Velocity after impact* (V): For a central blow, the velocity (V) with which the system would move after the impact is given by

$$V = \frac{(1+k)}{\left(1 + \dfrac{W}{W_t}\right)}\,v \tag{4.79}$$

where W is the weight of the system receiving impact. (Here, $W = W_a$).
For an eccentric blow, the initial velocity (V) and initial angular velocity $(\dot{\theta})$ of the moving system after impact are given by the following relations.

$$V = \frac{(1+k)}{1 + \dfrac{W}{W_t} + \dfrac{e^2}{i^2}} \cdot v \tag{4.80a}$$

and

$$\dot{\theta}=\frac{(1+k)e}{i^2\left(1+\frac{W}{W_t}\right)+e^2}\cdot v \tag{4.80b}$$

where $i^2=\varphi/m$, φ being the mass moment of inertia of the moving system about the axis of rotation m is its mass and e is eccentricity of impact.

vii. *Amplitudes (a):* The amplitude of foundation (a_f) is given by

$$a_f=\frac{-(\omega_a^2-\omega_{n2}^2)(\omega_a^2-\omega_{n1}^2)V}{\omega_a^2(\omega_{n1}^2-\omega_{n2}^2)\omega_n}. \tag{4.81a}$$

The amplitude of anvil (a_a) is given by

$$a_a=\frac{-(\omega_a^2-\omega_{n1}^2)V}{(\omega_{n1}^2-\omega_{n2}^2)\omega_n} \tag{4.81b}$$

The above relations are thc same as Eqs. (2.29a and b) with appropriate changes in notation.

c. Computation of Dyoamic Forces (F_d)

i. The dynamic force under the foundation $(F_d)_f$ is givcn by

$$(F_d)_f=\xi K_z a_f \tag{4.82a}$$

where ξ is the fatigue factor which may be assumed as three.

ii. The dynamic force under anvil $(F_d)_a$ is given by

$$(F_d)_a=\xi(a_f-a_a)K_a \tag{4.82b}$$

d. Checks on Design

i. The amplitudes calculatcd from Eqs. 4.81a and 4.81b should be within the permissible values given under design criteria (Section 4.5.3).

ii. The stress on soil (σ_s) assuming uniform distribution of load is given by

$$\sigma_s=\frac{W+(F_d)_f}{A_f} \tag{4.83}$$

where W is total weight of machine and foundation. The stress (σ_a) on the elastic layer used under the anvil is given by

$$\sigma_a=\frac{W_a+(F_d)_a}{A_a} \tag{4.84}$$

The stresses (σ_s and σ_a) shall be within the permissible limits for the respective materials.

e. Structural Design

Bending moments due to dynamic loads: Referring to Fig. 4.36, let $(F_d)_a$ bc the dynamic force on the anvil base and $(F_d)_f$ be the dynamic force under the foundation. The foundation block is in equilibrium under the action of these two dynamic forces and the inertia

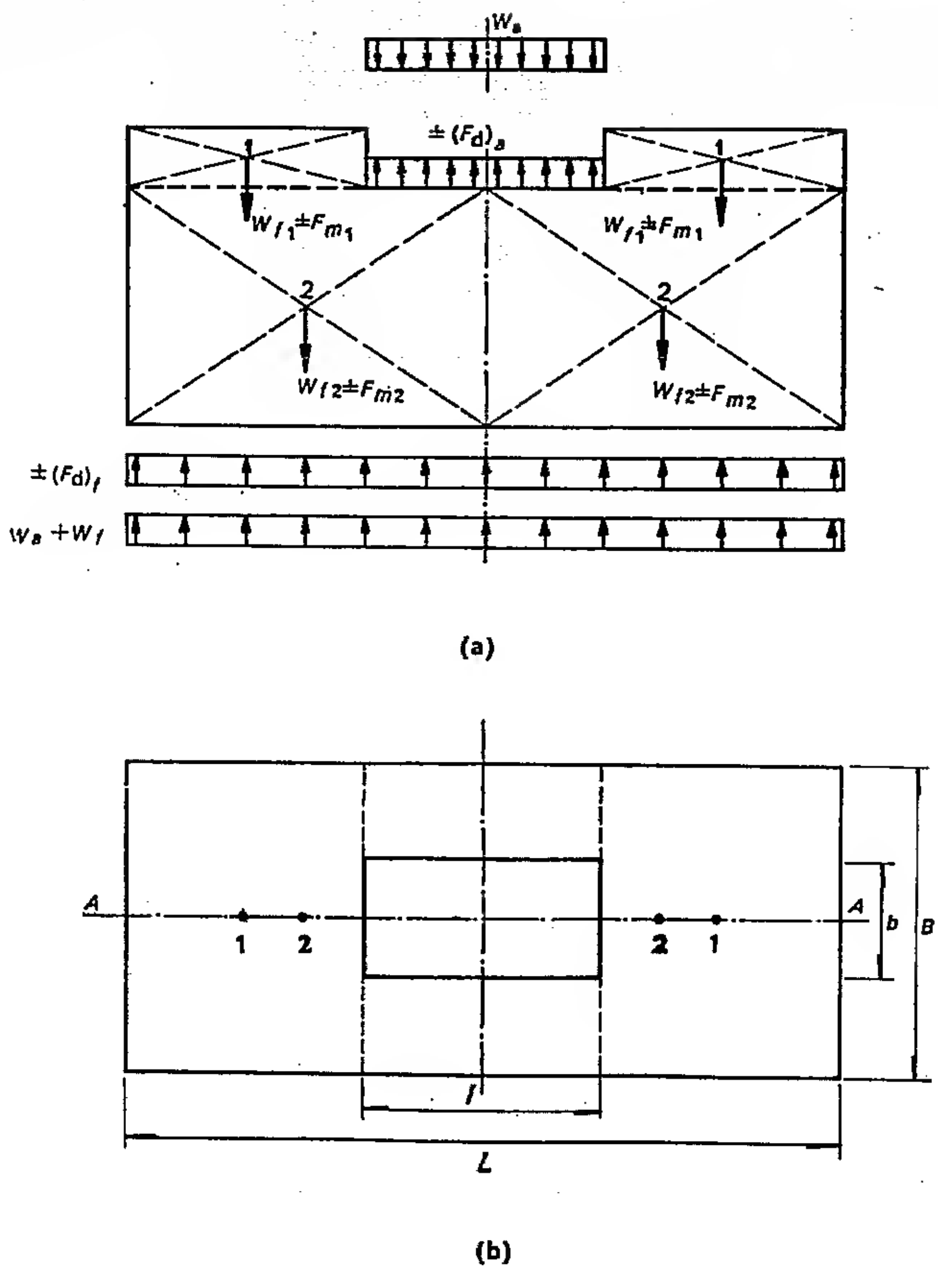


Fig. 4.36: Forces Acting on a Hammer Foundation—**(a)** Section *A-A*, **(b)** Plan.

force (F_m). Assuming the amplitudes a_f and a_a to be both positive, for equilibrium the following relation shall be satisfied.

$$F_m \downarrow = (F_d)_a \uparrow + (F_d)_f \uparrow \tag{4.85}$$

The arrows indicate the direction of forces.

If the foundation consists of a number of rectangular blocks (1, 2, etc.), as shown in Fig. 4.36, the total inertial force F_m is apportioned to these various parts of the foundation body by the relation

$$(F_m)_i = \frac{W_i}{\Sigma W_i} F_m \tag{4.86}$$

Inertial forces associated with parts of the machine (e.g., hammer stand) resting on foundation should be considered in the same manner.

The bending moments (M_d) and shear (Q_d) at any section due to dynamic loads can be calculated from Fig. 4.36 considering all forces acting to the left or right of the particular section.

The net moments and shears for design are

$$M = M_{st} \pm M_d \tag{4.87}$$

and

$$Q = Q_{st} \pm Q_d \tag{4.88}$$

where M_{st} and Q_{st} are respectively the moments and shears at the section due to static loads.

The area of reinforcing steel calculated on the basis of the bending moments is generally small in hammer foundations. A certain minimum reinforcement is therefore prescribed in practice.

The minimum reinforcement under the anvil consists of at least two layers of horizontal steel grillages formed by 14–20 mm diameter bars at 20–30 cm spacing in both directions.

Besides this, three-directional reinforcement amounting to a minimum of 25 kg/m^3 of concrete should be provided in the foundation block as a whole.[C4.4]

Fig. 4.37 shows the typical disposition of reinforcement in a hammer foundation. The analysis and design of a hammer foundation resting on soil is illustrated in example 1 in Sec. 4.5.7.

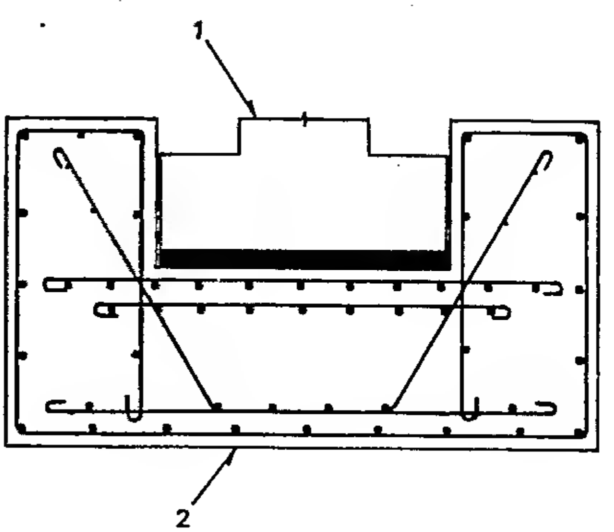


Fig. 4.37: Typical Reinforcement in a Hammer Foundation—(*1*) Anvil Block, (*2*) R. C. Foundation.

4.5.5 Foundations on Vibration Absorbers

In order to reduce the amplitudes of vibration of a hammer foundation resting on soil the mass of the foundation and its base area should be increased. This, however, may not always be possible due to practical difficulties, such as limitations of space, etc. Moreover, the environmental conditions may sometimes necessitate the vibration amplitudes under the foundation to be reduced to a value much lower than the usually accepted value of 1 mm. In such cases, use of vibration absorbers is recommended. Spring-type absorbers are commonly used for this purpose. The spring casings are interposed between two portions of the foundation block, as shown in Fig. 4.38.

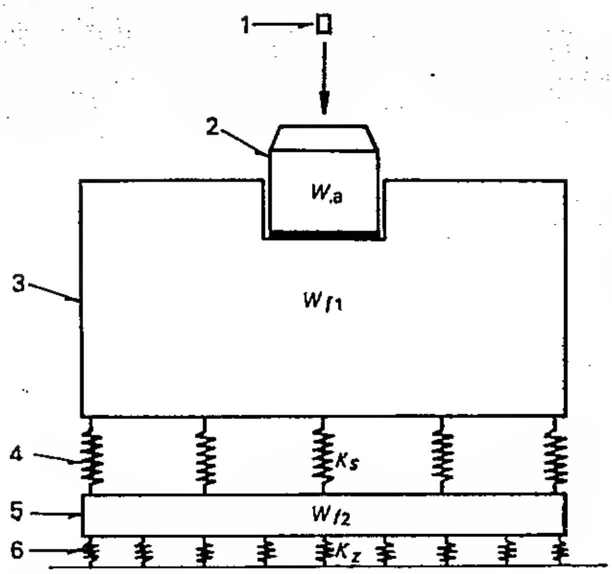


Fig: 4.38: Hammer Foundation on Springs—(*1*) Tup, (*2*) Anvil, (*3*) Upper Foundation Block, (*4*) Spring Layer, (*5*) Lower Foundation Block, (*6*) Soil Spring Layer.

a. Data Required

Besides the data mentioned in the earlier section, it is necessary to obtain the permissible amplitudes of the anvil block and the foundation. The latter is prescribed taking into account the environmental conditions.

b. Analysis of Vertical Vibrations

As a first approximation, considering the vibrating system as having one degree of freedom (neglecting the elasticity of any cushioning layer used under the anvil block), the amplitude of vibration of the upper portion of the foundation block can be expressed as

$$a_a = \frac{(1+k)W_t V}{W\omega_{na}} \tag{4.89}$$

where W, the weight of sprung part, is equal to the weight of anvil W_a plus the weight of upper foundation block (W_{f_1}). Substituting $\omega_{na} = \sqrt{\frac{K_s g}{W}}$ where K_s is the stiffness of spring absorber, Eq. 4.89 becomes

$$a_a = \frac{(1+k)W_t V}{\sqrt{W}\sqrt{K_s g}} = \frac{\alpha}{\sqrt{K_s W}} \tag{4.90}$$

where

$$\alpha = (1+k)W_t \frac{V}{\sqrt{g}} \tag{4.91}$$

The static settlement (δ) of the part of foundation above the springs is given by

$$\delta = W/K_s \tag{4.92}$$

The value of δ may initially be assumed as 0.01 to 0.02 m.[01.12] From Eqs. 4.90 and 4.92 the following relations which define the total weight of the foundation above the springs and the necessary stiffness of the spring assembly to be used are obtained:

$$W = \frac{\alpha}{a_a}\sqrt{\delta} \tag{4.93}$$

$$K_s = \frac{W}{\delta} \tag{4.94}$$

The effect of elastic layer, if any, placed under the anvil is neglected in this approach. The justification for this assumption is that the stiffness of elastic layer under the anvil is generally very large compared to the stiffness of the spring assembly (Eq. 4.94) used under the foundation.

c. The Principal Stages in Design Calculations

i. The value of α is computed from Eq. (4.91).

ii. Using the assumed values of δ(0.01 to 0.02 m) and the known value of the permissible amplitude of the anvil (a_a), the total weight W above the absorbers is computed from Eq. 4.93.

iii. The weight of the upper foundation block (W_{f1}) is equal to the total weight W minus the weight of anvil (W_a) and other parts of machinery such as hammer frame (W_{st}) resting on the foundation.

iv. The spring coefficient of the absorbers (K_s) is obtained from Eq. 4.94.

v. Considering the system shown in Fig. 4.38 for the analysis of vertical vibrations, the limiting circular frequencies (ω_a, ω_z) in this case are

$$\omega_a = \sqrt{\frac{K_s g}{W_{f1} + W_a + W_{st}}} \tag{4.95}$$

$$\omega_z = \sqrt{\frac{K_z g}{W_{f1} + W_{f2} + W_a + W_{st}}} \tag{4.96}$$

vi. The coupled natural frequencies are obtained from Eq. 4.74.

vii. The amplitudes of lower foundation block (a_{f2}) and upper foundation block (a_{f1}) may be obtained from Eqs. 4.81a and 4.81b respectively. The velocity (V) is obtained from Eq. (4.79) with W_a replaced by ($W_{f1} + W_a + W_{st}$)

d. Design of Spring Absorbers

Let n_1 be the number of spring casings in the absorber assembly and n_2 be the number of spring coils in each casing. Required rigidity of each spring

$$K_i = \frac{K_s}{n_1\, n_2} \tag{4.97}$$

The actual rigidity of each spring K_i is given by

$$K_i = \frac{1}{8n}\frac{d^4}{D^3}G \tag{4.98}$$

where n is number of turns in the spring coil
d is the diameter of spring wire
D is the diameter of spring coil
and G is the shear modulus of material of spring.

Table 3.4 contains the values of K_t for $n=1$ and $G=8.3\times10^5$ kg/cm². From Eqs. 4.97 and 4.98

$$n_1 n_2 = \frac{8n K_s}{G} \frac{D^3}{d^4} \tag{4.99}$$

The total number of spring casings (n_1) and the required number of springs (n_2) in each casing can be suitably selected from the above expression.

e. Checks on Design

i. The total load on each spring should be less than the permissible load (Table 3.4) for the spring used.

ii. The stress on soil shall not exceed the allowable limit.

f. Structural Design

Let F_d be the dynamic force on springs and W_{f_1} be the weight of upper foundation block $W_{f_1}=\Sigma W_{f_i}$). If the elastic supports under the foundation are closely spaced, as in the example 2 illustrated in Sec. 4.5.7, the foundation may be considered to be supported evenly on the bearing area shown in section lines in Fig. 4.39. Fig. 4.39 also shows the forces which maintain the foundation in equilibrium. The total inertial force F_m in

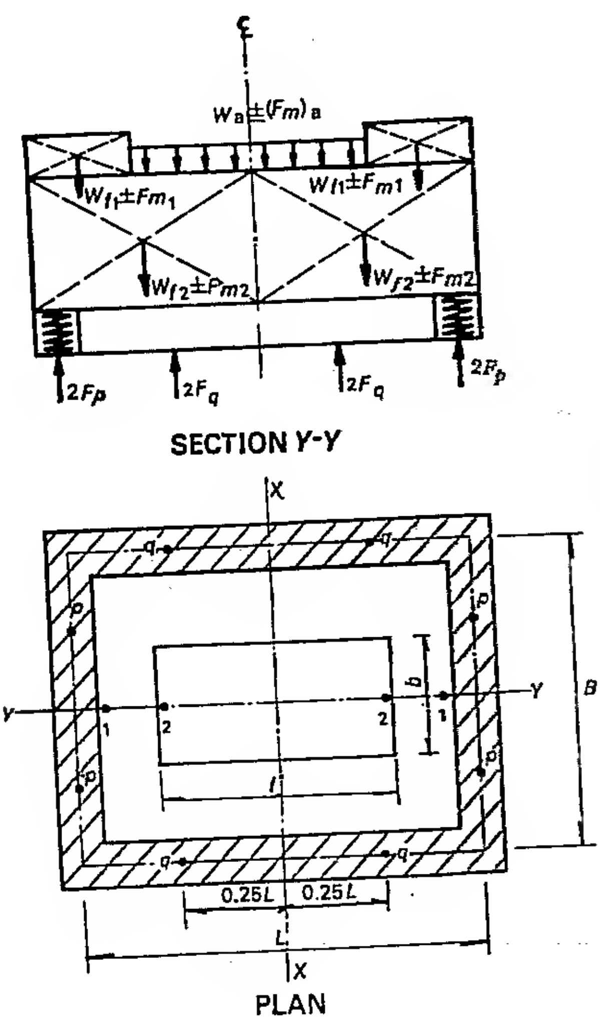


Fig. 4.39: Forces Acting on a Foundation Resting on Spring Supports.

this case is equal to F_d and its distribution to various parts of the foundation and machine is done in the same way as explained in the previous case (according to Eq. 4.86). The distributed reactive forces offered by the springs F_p and F_q (shown in Fig. 4.39) can be expressed as follows:

$$F_p = (F_p)_{\text{static}} \pm (F_p)_{\text{dynamic}}$$

$$= \left[W_a + W_f \pm F_d \right] \frac{B}{4(L+B)} \tag{4.100}$$

and

$$F_q = (F_q)_{\text{static}} \pm (F_q)_{\text{dynamic}}$$

$$= \left[W_a + W_f \pm F_d \right] \frac{L}{4(L+B)} \tag{4.101}$$

where L and B are as shown in Fig. 4.39 and W_f is the weight of foundation including parts of the machine (e.g., hammer stand) directly resting on it.

The positions of these forces are shown in plan (Fig. 4.39). The bending moments and shears at the middle section and at the anvil edge can be calculated as before, and the requirement of steel computed accordingly.

The design of a hammer foundation resting on springs is illustrated in example 2 in Section 4.5.7.

4.5.6 Foundations for Counter-Blow Hommers

Counter-blow hammers are characterized by the fact that the impact is caused by the striking of the upper and lower sliding assemblies moving towards each other. They are operated by compressed air or steam. The upper and lower moving parts are generally of unequal weights, the lower one being heavier. Compared to the hammers with fixed anvil (for which the design criteria were given in the preceding sections), counter-blow hammers transmit relatively less impact energy to their foundations. Fig. 4.40 shows the working system of such a hammer.

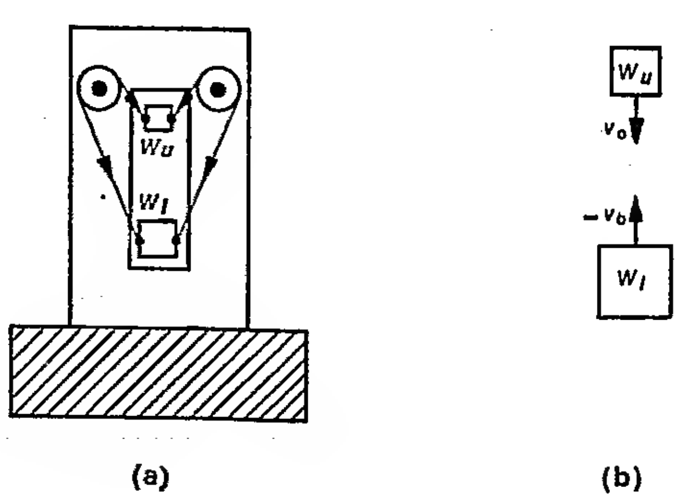


Fig. 4.40: Working System of a Counter-blow Hammer.

a. Data Required for Design

Total weight of machine	W_m (t)
Weight of upper tup	W_u (t)
Weight of lower tup	W_l (t)
Terminal velocity of the striking parts before impact	v (m/sec)
Length of travel of each tup	h (m)
Operating frequency of hammer	f_m (rpm)

b. Principal Stages in Design Calculations

The working process of the hammer involves two successive impacts. For design purpose, their cumulative effect shall be considered. The procedure given below is based on the approach suggested by Rausch.[CI.15]

i. *First impact:* The effect of first impact may be considered as if a tup of weight $W_d=(W_l-W_u)$ strikes with a velocity v. The impact momentum (S_1) is given by

$$S_1 = \frac{W_d\, v}{g} \tag{4.102}$$

Acceleration with which the tups strike ($\ddot{z}$) $= \dfrac{v^2}{2h}$ (4.103)

Impact force (P) $= \dfrac{W_d \ddot{z}}{g}$ (4.104)

Period of impact (T) $= \dfrac{2h}{v}$ (4.105)

The natural period (T_n) of foundation assuming a single-degree freedom system $= 2\pi\sqrt{\dfrac{m}{K_z}}$ (4.106)

where K_z is the stiffness of elastic layer (or soil) under the foundation and m is the total mass of machine and foundation. Assuming the load-time relationship of the impact pulse to be of rectangular form, the maximum dynamic factor (μ) may be obtained from Fig. 2.10. The theoretical basis for this case has been explained in Chapter 2. The dynamic force (F_1) due to first impact is given by

$$F_1 = \xi \mu P \tag{4.107}$$

where ξ is a fatigue factor which may be assumed as 3.

ii. *Second impact:* After the first impact, the upper tup goes away with a relative velocity (V) given by

$$V = \frac{4\, W_d\, v}{W_u + W_l} \tag{4.108}$$

Due to gravity, however, the upper tup falls back and strikes again with the same velocity, thus causing a second impact. Since the two impacts occur one after another, it is necessary to consider their combined effect for the foundation design.

For the calculation of the dynamic force due to the second impact, it is necessary to know the stiffness (K_B) of the band springs which control the movement of the tups. The impact force induced in the band springs (P_2) is given by

$$P_2 = V\sqrt{\frac{K_B W_u}{g}} \tag{4.109}$$

Assuming that the second impact is caused by the lower tup (which is heavier), the expression for dynamic force due to second impact may be obtained from

$$F_2 = 7.5\ V\sqrt{\frac{K_B W_u}{g}}\ \frac{W_1}{W_u} \tag{4.110}$$

The factor 7.5 is product of a fatigue factor equal to 3, a dynamic factor 2 and a correction factor of 1.25 which accounts for possible uncertainties in the evaluation of stiffness of band springs.

Total dynamic force

$$F_d = F_1 + F_2 \tag{4.111}$$

c. Alternative Design Procedure

The following simplified procedure[C1.14] may be adopted when the natural frequency of the foundation is low ($\frac{f_m}{f_n} \geqslant 2$), as in the case when the foundation is supported on spring absorbers. For foundations resting directly on soil, this method over-estimates the dynamic forces on the foundation and is therefore too conservative.

Impact momentum (S_1) due to first impact $\Big\}\ = \frac{W_d}{g}\ v$ (4.112)

Impact momentum (S_2) due to second impact $\Big\}\ = 1.25\frac{W_1}{g}\ V$ (4.113)

where V is given by Eq. 4.108 and the multiplier 1.25 is the correction factor which accounts for possible uncertainties in evaluating the stiffness of the band springs.

Total impact momentum $(S) = (S_1 + S_2)(1 + k)$ (4.114)

where k is the coefficient of the impact which may be taken as 0.6.

If ω_n is the natural frequency of the foundation, the total dynamic force

$$(F_d) = \xi\ \omega_n\ S \tag{4.115}$$

where ξ is fatigue factor equal to 3.

d. Checks on Design

i. For foundations resting on soils, the maximum stress on soil (σ_s) is given by

$$\sigma_s = (W_m + W_f + F_d)/A_f \text{ which shall be } < \sigma_p \tag{4.116}$$

where W_m, W_f, and A_f denote the weight of machine, weight of foundation and area of foundation base respectively.

ii. For foundations resting on springs, the total load on one spring should be less than the permissible load (Table 3.4).

e. Structural Design

Fig. 4.41 shows a rectangular block foundation for a counter-blow hammer. $(L \times B)$ are the base dimensions of the foundation, and $(l \times b)$ are the dimensions of the base plate of machine. W_m is the weight of the machine and F_d is the dynamic force induced by its working.

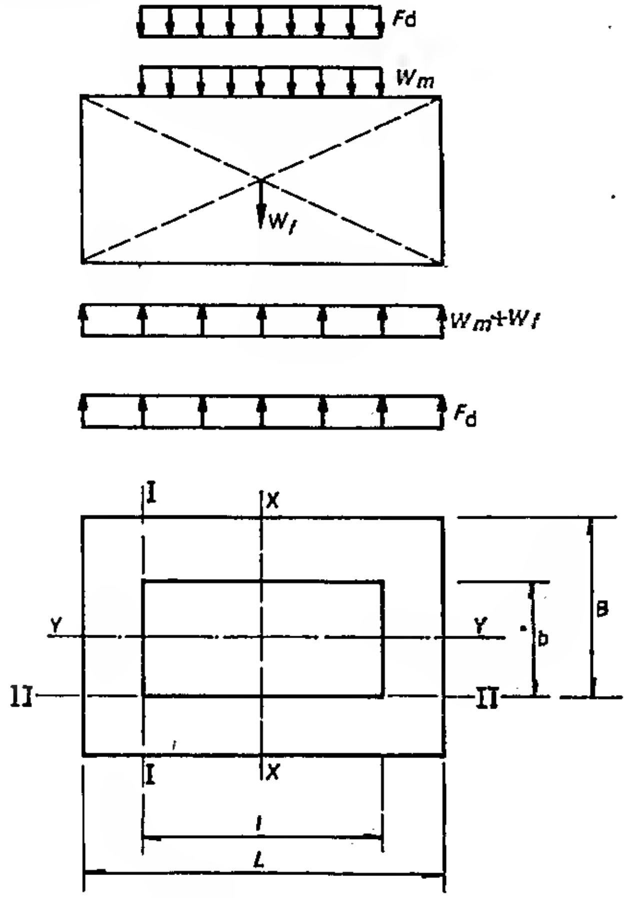


Fig. 4.41: A Rectangular Foundation Resting on Soil

From Fig. 4.41, the bending moment at mid-section XX and YY can be written as

$$M_{xx} = \left[\frac{W_m \pm F_d}{8}\right]\left[L - l\right] \tag{4.117}$$

and

$$M_{yy} = \left[\frac{W_m \pm F_d}{8}\right]\left[B - b\right] \tag{4.118}$$

The shear at the edge of the base plate (Q) is given by

$$Q_{I-I} = \left[\frac{W_m \pm F_d}{L}\right]\left[\frac{L-l}{2}\right] \tag{4.119}$$

$$Q_{\text{II-II}} = \left[\frac{W_{\text{m}} \pm F_{\text{d}}}{B}\right]\left[\frac{B-b}{2}\right] \tag{4.120}$$

Foundations resting on springs (or other elastic supports) arranged along the perimeter of the base (Fig. 4.42) may be considered in the same way as explained in Sec. 4.5.5.

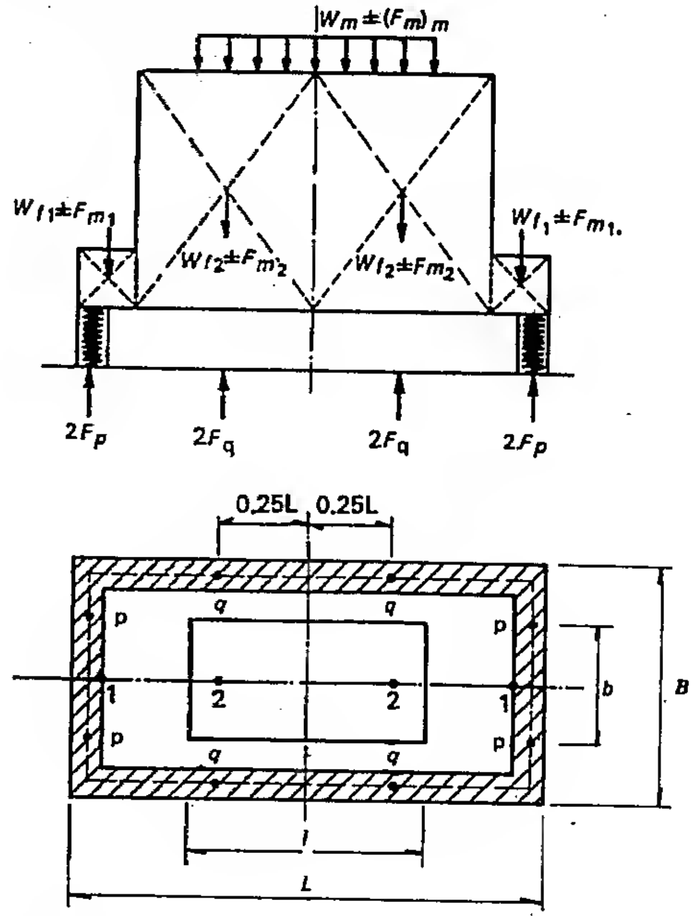


Fig. 4.42: A Foundation Resting on Spring Supports.

4.5.7 Numerical Examples

1. Design of a Hammer Foundation Resting on Soil

a. DATA

Weight of tup (W_t)	3,400 kg
Weight of anvil (W_a)	75,000 kg
Weight of frame (W_{Bt}) resting on foundation block	38,350 kg
Area of anvil base (A_a)	8.32×10^4 cm^2
Thickness of timber layer (t_a) under anvil	60 cm
Elasticity of timber layer (E_a)	13×10^3 kg/cm^2
Velocity of fall of tup (v)	600 cm/sec
Coefficient of restitution (k)	0.25
Bearing capacity of soil (σ_p)	3.5 kg/cm^2
Coefficient of elastic compression of soil (C_z)	3.8 kg/cm^3

b. PRELIMINARY DIMENSIONING

i. From Table 4.14 minimum thickness of foundation to be provided under anvil (by interpolation) 1.60 m

Thickness provided — 1.60 m

ii. From Table 4.13 thickness of elastic pad to be provided under the anvil — 0.6 to 1.2 m

Thickness provided (t_a) — 60 cm

iii. Minimum weight of foundation (Eq. 4.67)

$$W_f \min = W_t\left[8(1+k)\,v - \frac{W_a + W_{st}}{W_t}\right]$$

$$= 3.4\left[8(1+0.25)\,6.0 - \frac{75+38.35}{3.4}\right]$$

$$= 90.65 \text{ t}$$

Weight of foundation provided (Fig. 4.43)

Gross weight $= (7.1\times 4.7\times 2.62)2.4 = 209.8$ t

Deducting for

Anvil pit $= -(3.2\times 2.6\times 1.02)2.4 = -20.36$

Edge strips (2 Nos.) $= -2(0.55\times 1.4\times 7.1\times 2.4) = -26.24$

Net weight $(W_f) = 209.8 - 20.36 - 26.24 = 163.2$ t

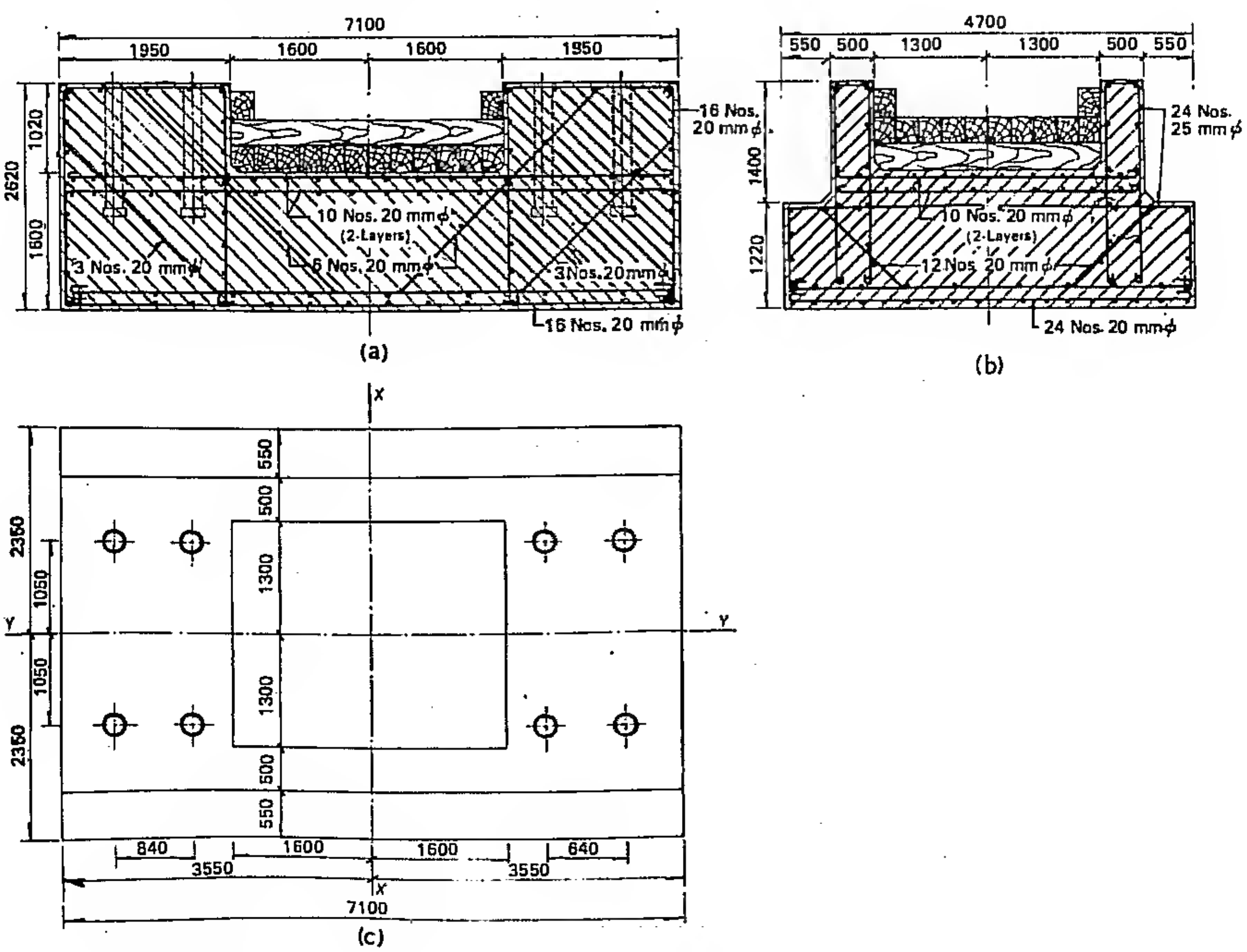


Fig. 4.43: Foundation for a 3.4 t Forge Hammer—**(a)** Section y-y, **(b)** Section x-x, **(c)** Plan.

iv. Minimum base area required (Eq. 4.68)

$$A_{min} = \frac{20\,(1+k)\,v\,W_t}{\sigma_p}$$

$$= \frac{20 \times 1.25 \times 6.0 \times 3.4}{35}$$

$$= 14.57 \text{ m}^2$$

Area provided (Fig. 4.43) $=7.1 \times 4.7$
$=33.37 \text{ m}^2$

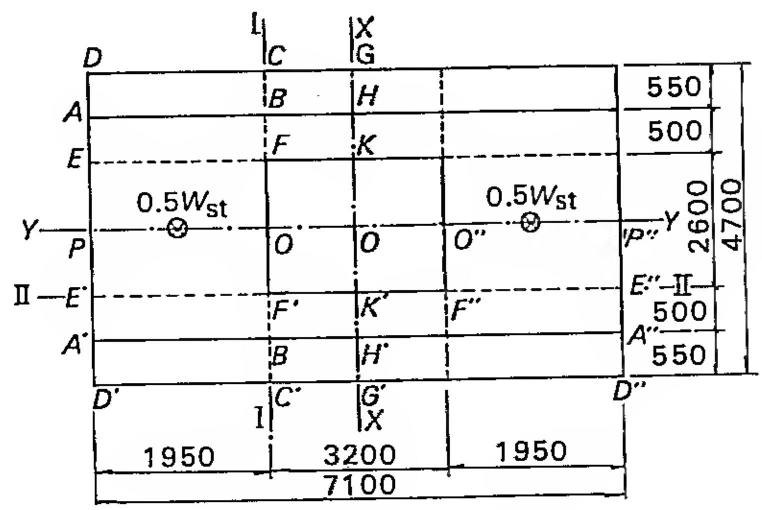


Fig. 4.44: Plan of the Foundation.

c. DYNAMIC ANALYSIS

Coefficient of rigidity (K_a) of the pad under the anvil (Eq. 4.71)

$$= \frac{E_a A_a}{t_a}$$

$$= \frac{13 \times 10^3 \times 8.32 \times 10^4}{60}$$

$$= 18.02 \times 10^6 \text{ kg/cm}$$

Mass of anvil (m_a)

$$= \frac{W_a}{g} = \frac{75 \times 10^3}{981}$$

$$= 76.45 \text{ kg·sec}^2\text{/cm}$$

Square of the limiting frequency (ω_a^2) of the anvil (Eq. 4.72)

$$= \frac{K_a}{m_a}$$

$$= \frac{18.02 \times 10^6}{76.45}$$

$$= 23.57 \times 10^4 \text{sec}^{-2}$$

The coefficient of rigidity (K_z) of base under foundation (Eq. 4.70)

$$= C'_z A_f$$

$$= 3 \times 3.8 \times 33.37 \times 10^4$$

$$= 380.42 \times 10^4 \text{ kg/cm}$$

Total mass of foundation (m_f+m_{st})

$$= \frac{W_f+W_{st}}{g} = \frac{163200+38350}{981}$$
$$= 205.5 \text{ kg sec}^2/\text{cm}$$

The square of the limiting frequency (ω_z^2) of the whole system (Eq. 4.73)

$$= \frac{K_z}{m_a+m_f+m_{st}}$$
$$= \frac{380.42\times 10^4}{76.45+205.5}$$
$$=1.349\times 10^4 \text{ sec}^{-2}$$

The ratio (α) of the mass of anvil to the mass of foundation (Eq. 4.75)

$$= 76.45/205.5$$
$$= 0.372$$

Velocity (V) after impact (Eq. 4.79) $= \dfrac{1.25 \times 600}{1 + 75/3.4}$

$$=32.53 \text{ cm/sec}$$

The frequency equation (Eq. 4.74) gives

$$\omega_n^4 - (\omega_a^2 + \omega_z^2)\ (1+\alpha)\ \omega_n^2 + (1+\alpha)\ \omega_a^2\ \omega_z^2 = 0$$

Substituting and solving, we obtain

$$\omega_{n1}^2 = 32.861 \times 10^4 \text{ sec}^{-2} \quad ; \quad \omega_{n1} = 573.3 \text{ sec}^{-1}$$

$$\omega_{n2}^2 = 1.328 \times 10^4 \text{ sec}^{-2} \quad ; \quad \omega_{n2} = 115.22 \text{ sec}^{-1}$$

Amplitude (a_f) of the foundation (Eq. 4.81a) $= \dfrac{-(\omega_a^2-\omega_{n2}^2)\ (\omega_a^2-\omega_{n1}^2)\ V}{\omega_a^2\ (\omega_{n1}^2-\omega_{n2}^2)\ \omega_{n2}}$

$$= \frac{-(23.57-1.328)10^4\ (23.57-32.861)10^4 \times 32.53}{23.57 \times 10^4\ (32.861-1.328)10^4 \times 115.22}$$
$$= 0.785 \text{ mm}$$

Amplitude (a_a) of the anvil (Eq. 4.81b) $= -\dfrac{(\omega_a^2-\omega_{n1}^2)\ V}{(\omega_{n1}^2-\omega_{n2}^2)\ \omega_{n2}}$

$$= \frac{-(23.57-32.861)10^4 \times 32.53}{(32.861-1.328)10^4\times 115.22}$$
$$= 0.832 \text{ mm}$$

The amplitudes are within the acceptable limit of 1 mm.
Dynamic force $(F_d)_a$ under the anvil (Eq. 4.82b) $= \xi K_a (a_f - a_a) \uparrow$

$$= 3 \times 180.2 \times 10^4 (0.7851 - 0.8328)10^{-3} \uparrow$$
$$= 253.54 \text{ t} \downarrow$$

The arrow indicates the direction of force.
Dynamic force $(F_d)_f$ under the foundation (Eq. 4.82a)

$$= \xi K_z a_f \uparrow$$
$$= 3 \times 38.042 \times 10^4 \times 0.7851 \times 10^{-3}$$
$$= 896 \text{ t} \uparrow$$

Stress (σ_a) on the timber pad (Eq. 4.84) $= \dfrac{(253.54 + 75) \times 10^3}{8.32 \times 10^4}$

$$= 3.95 \text{ kg/cm}^2$$

Stress (σ_s) on the soil (Eq. 4.83) $= \dfrac{(896 + 163.2 + 75+38.35)10^3}{33.37 \times 10^4}$

$$= 3.5 \text{ kg/cm}^2$$

d. STRUCTURAL DESIGN

Considering the YY direction, the bending moments due to static and dynamic loads are calculated as follows:

i. *Static loads* $(+)$: Referring to Fig. 4.44 and Table 4.15 and substituting $W_a = 75$ t; $W_f = \sum_i W_{fi} + W_{st} = 163.2 + 38.35 = 201.55$t; $L = 7.1$ m; $l = 3.2$ m and $\Sigma W_i l_i = 204.19$
the bending moments at middle section and anvil edge are obtained as follows:

M_{st} at middle section: $\left(\dfrac{201.55 + 75}{2}\right)\dfrac{7.1}{4} - \dfrac{75}{2}\,\dfrac{3.2}{4} - 204.19$

$$= 10.65 \text{ t.m}$$

M_{st} at the edge of the anvil $= \left(\dfrac{201.55 + 75}{7.1 \times 4.7}\right) 4.7 \times 1.95 \times \dfrac{1.95}{2} - 67.87$

$$= 6.19 \text{ t.m}$$

ii. *Dynamic loads* $(\pm)$: Referring to Fig. 4.44 and Table 4.15 and substituting, the bending moments (M_d) are obtainded thus:

M_d at middle section: $\dfrac{896}{2}\left(\dfrac{7.1}{4}\right) - \dfrac{253.54}{2} \times \dfrac{3.2}{4} - 650.73$

$$= 43.11 \text{ t.m}$$

Table 4.15

BENDING MOMENTS ABOUT TRANSVERSE AXIS ($X-X$)

Dynamic force under anvil $(F_d)_a = K_a(a_f - a_a) = 253.54\downarrow$
Dynamic force under foundation $(F_d)_f = \xi\ K_z a_f = 896$ t$\uparrow$
Inertial force $(F_m) = (F_d)_f + (F_d)_a = 642.46$ t

Foundation part	Weight (W_i) of part i of the foundation (Fig. 4.44) (t)	Inertial force (F_{mi}) $= \frac{W_i}{\Sigma W_i}\left(\frac{F_m}{2}\right)$ (t)	Distance of centre of gravity of each part: From anvil edge (m)	Distance of centre of gravity of each part: From middle section (m)	Moment due to Self-weight: At anvil edge (t.m)	Moment due to Self-weight: At middle section (t.m)	Moment due to Inertial forces: At anvil edge (t.m)	Moment due to Inertial forces: At middle section (t.m)
$ABB'A'$	(1.95 × 3.6 × 1.4 × 2.4) ≐ 23.57	75.18	0.975	2.575	23.0	60.74	73.3	193.59
$DCC'D'$	(1.95 × 4.7 × 1.22 × 2.4) = 26.84	85.54	0.975	2.575	26.17	69.11	83.4	220.27
$CGG'C'$	(1.6 × 4.7 × 1.22 × 2.4) = 22.02	70.18	—	0.800	—	17.62	—	56.14
$BHKF$ $B'H'K'F'$	2(1.6 × 0.5 × 1.4 × 2.4) = 5.376	17.13	—	0.800	—	4.30	—	13.70
$FKK'F'$	(1.6 × 2.6 × 0.38 × 2.4) = 3.794	12.09	—	0.800	—	3.04	—	9.67
Machine frame	½ (38.35) = 19.175	61.11	0.975	2.575	18.70	49.38	59.58	157.36
Sum	≃ 100.80	321.23			67.87	204.19	216.28	650.73

$$M_d \text{ at the edge of the anvil} = \frac{896}{4.7 \times 7.1} \times 4.7 \times \frac{1.95^2}{2} - 216.28$$
$$= 23.65 \text{ t.m}$$

iii. *Net moments:* Net moment at middle section (M_{xx}) $= 10.95 \pm 43.11$
$= 54.06;\ -32.1$ t.m

Net moment at anvil edge (M_n) $= 6.19 \pm 23.65$
$= 29.84$ t.m; -17.47 t.m

The steel requirement for the bending moments calculated above is small. Nominal reinforcement comprising of 20 mm diameter bars at 30 cm spacing is provided at the bottom of the foundation.

$$\text{Net shear at anvil edge} = \frac{(201.55+75)}{7.1 \times 4.7}\ 4.7 \times 1.95 - (23.59 + 26.84 + 19.175)$$
$$\pm 896 \times \frac{4.7 \times 1.95}{4.7 \times 7.1} \mp (75.18 + 85.54 + 61.11)$$
$$= 30.6;\ -17.9 \text{ t}$$

$$\text{Inclined steel requirement for shear} = \frac{30.6}{1.4 \times \sqrt{2}}$$
$$= 15.46 \text{ cm}^2$$

Six numbers of 20 mm diameter inclined bars are provided at the anvil edge. An

additional three numbers of 20 mm diameter inclined bars are provided at a farther distance from centre, as shown in Fig. 4.43. A similar procedure is adopted for arriving at the steel requirement in the other direction. The disposition of reinforcement in the foundation is shown in Fig. 4.43.

2. Design of Hammer Foundation Resting on Spring Absorbers

Design a suitable foundation for the forge hammer data for which is given in the previous

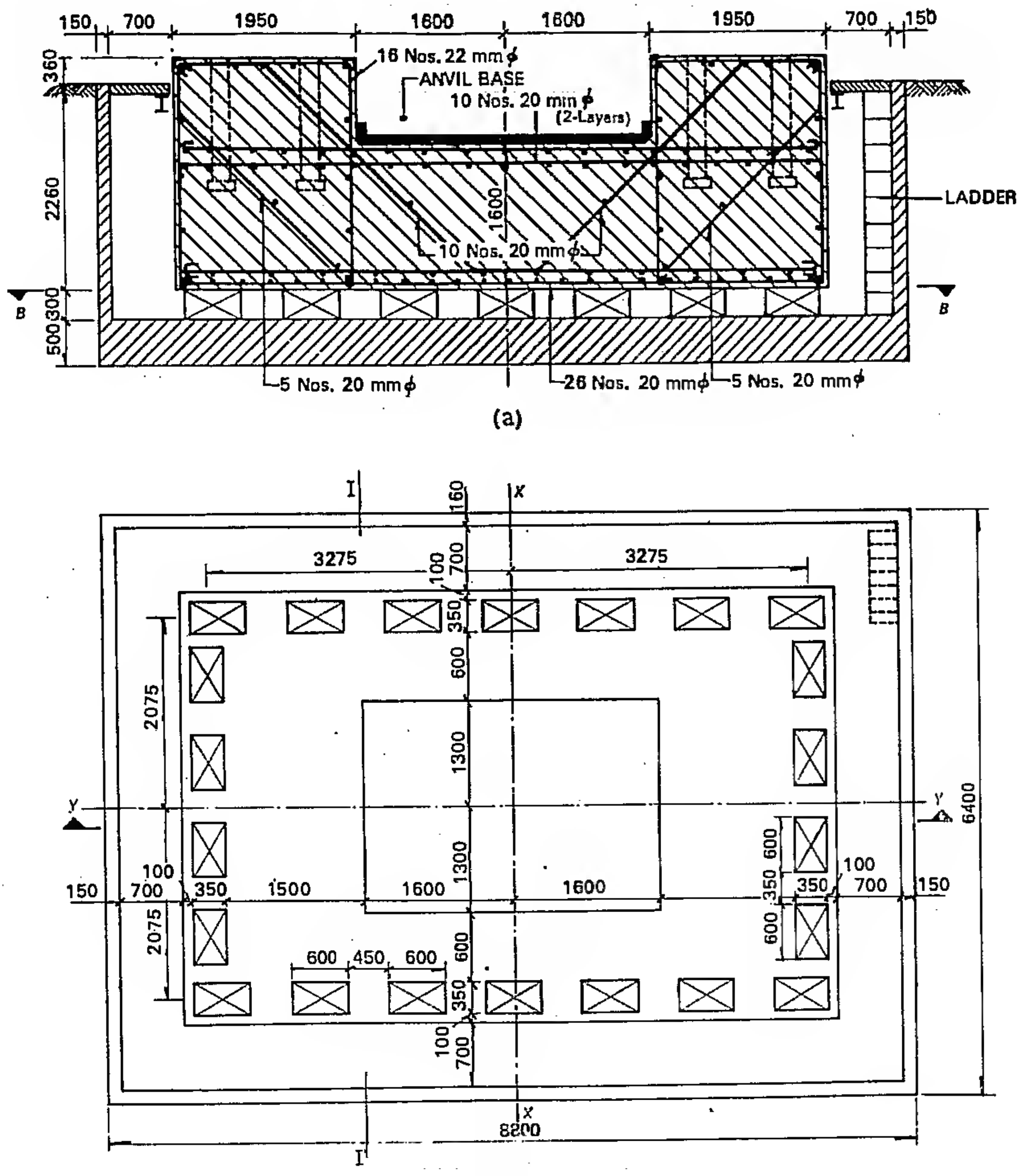


Fig. 4.45: Foundation for a 3.4 t Hammer on Springs—(a) Section y-y, (b) Plan at B-B.

example if the amplitude of movement of the soil under the foundation shall be limited to 0.2 mm and that of the anvil to 3 mm. The bearing capacity of soil is 1.5 kg/cm^2 and the coefficient of elastic uniform compression is 3.8 kg/cm^3. Grade of concrete used for the foundation is M-150.

Spring absorbers are proposed to be used in this case. The configuration of the foundation is as shown in Fig. 4.45.

a. PRELIMINARY DIMENSIONING

From Eq. 4.91, using $k = 0.25$

$$\text{Factor } \alpha = \frac{(1+k)\ W_t\ V}{\sqrt{g}} = \frac{1.25 \times 3.4 \times 6.0}{\sqrt{9.81}} = 8.14$$

Required weight of the foundation (W) (including machine) above the springs from Eq. 4.93 $= \dfrac{\alpha\sqrt{\delta}}{a_a}$

Assuming $\delta = 0.01$ m and substituting $= \dfrac{8.14\sqrt{0.01}}{0.003}$

$= 271.33$ t

Weight of the concrete foundation above the springs (W_{f_1}) $= 271.33 - (\text{weight of anvil} + \text{frame})$

$= 271.33 - (75 + 38.35)$

$= 158$ t

Choosing the dimensions of the upper foundation block same as that adopted in the previous example,

The actual weight of the foundation block above the springs (W_{f_1}) $= 163.2$ t

Total weight above the springs as provided $= 163.2 + 75 + 38.35$

$= 276.55$ t

The thickness of foundation slab below the springs is chosen as 0.5 m and that of side retaining wall as 0.15 m.

b. DYNAMIC ANALYSIS

i. *Stiffness Factors*

Required rigidity of spring absorbers (K_s) from (Eq. 4.94) $= \dfrac{276.55}{10^{-2}}$

$= 2.766 \times 10^4$ t/m

Stiffness of wooden layer under the anvil (K_a) as calculated in the previous example $= 180.2 \times 10^4$ t/m

Stiffness of soil spring under the lower foundation block (K_z) $= 3 \times 3.8 \times 10^3 \times 8.8 \times 6.4$

$= 64.21 \times 10^4$ t/m

The dynamic analysis is carried out considering a two-degree system formed by the upper and lower foundation blocks, the anvil being considered to be rigidly attached to the

former. This assumption is justified in this case since the wooden layer under the anvil is relatively very stiff.

ii. *Masses*

Upper mass (m_2) $= (W_a + W_{st} + W_{f1})/g$

$$= \frac{75 + 38.35 + 163.2}{9.81}$$

$= 28.190 \text{ t sec}^2/\text{m}$

Lower mass (m_1) including side walls

$= 95.05/9.81 = 9.689 \text{ t sec}^2/\text{m}$

iii. *Limiting Frequencies* (ω_a, ω_z)

$$\omega_a^2 = K_s/m_2$$
$$= 2.766 \times 10^4/28.190 = 9.812 \times 10^2 \ (\text{sec}^{-2})$$
$$\omega_z^2 = K_z/(m_1 + m_2)$$
$$= 64.21 \times 10^4/37.879 = 169.5 \times 10^2 \ (\text{sec}^{-2})$$

Mass ratio (α) $= m_2/m_1$

$= 28.190/9.689 = 2.909$

iv. *Coupled Frequencies* $(\omega_{n1}, \omega_{n2})$

$$\omega_n^4 - (\omega_a^2 + \omega_z^2)\ (1 + \alpha)\ \omega_n^2 + (1 + \alpha)\ \omega_a^2\ \omega_z^2 = 0 \qquad \text{(Eq. 4.74)}$$

$$\omega_n^4 - (9.812 + 169.5)\ 10^2\ (1 + 2.909)\ \omega_n^2 + (1 + 2.909)\ 9.812 \times 169.5 \times 10^4 = 0$$

Solving

$$\omega_{n1}^2 = 691.530 \times 10^2 \ (\text{sec}^{-2})$$
$$\omega_{n2}^2 = 9.401 \times 10^2 \ (\text{sec}^{-2})$$
$$\omega_{n1} = 262.97 \text{ sec}^{-1}$$
$$\omega_{n2} = 30.66 \text{ sec}^{-1}$$

v. *Velocity after Impact* (V)

$$V = (1 + k)\ W_t\, v/(W_t + W_a + W_{st} + W_{f1})$$
$$= (1.25 \times 3.4 \times 6)/(3.4 + 75 + 38.35 + 163.2) = 0.0911 \text{ m/sec}$$

vi. *Amplitudes*

Upper foundation block (with anvil)

$$a_{f_2} = \frac{-(\omega_a^2 - \omega_{n1}^2)\ V}{(\omega_{n1}^2 - \omega_{n2}^2)\ \omega_{n2}}$$

$$= \frac{-(9.812 - 691.530)10^2 \times 0.0911}{(691.530 - 9.401)10^2 \times 30.66} = +2.97 \text{ mm} < 3 \text{ mm}$$

Lower foundation block (or soil)

$$a_{f_1} = \frac{-(\omega_a^2 - \omega_{n2}^2)\,(\omega_a^2 - \omega_{n1}^2)\,V}{\omega_a^2\ (\omega_{n1}^2 - \omega_{n2}^2)\ \omega_{n2}}$$

$$= \frac{-(9.812-9.401)10^2\,(9.812-691.530)\times 10^2\times 0.0911}{9.812\times 10^2\,(691.530-9.401)10^2\times 30.66} = 0.124\ \text{mm} < 0.2\ \text{mm (acceptable)}$$

c. DESIGN OF SPRINO ABSORBERS

The springs having the following dimensions are adopted:

Diameter of coil (D) $= 100$ mm

Diameter of spring wire (d) $= 32$ mm

Number of windings (n) $= 5$

From Table 3.4, Stiffness of each spring $= \dfrac{10{,}880}{5} = 2176$ kg/cm

and permissible load on each spring $= 5060$ kg

Number of springs required $= \dfrac{2.766\times 10^4}{217.6} = 127$

Provide 22 spring casings each containing six spiral springs

Permissible compression (δ) of spring $= \dfrac{5060}{2176} = 2.326$ cm

Actual compression $= \delta + a_{t_2}$

$= 1 + 0.297 = 1.297$ cm

< 2.326 cm

d. DYNAMIC FORCES (F_{d1} and F_{d2})

Dynamic force (F_{d2}) on springs $= \xi\, K_s\,(a_{t2} - a_{t1})$

$= 3 \times 2.766 \times 10^4\,(2.97 - 0.124)10^{-3}$

$= 236.2$ t

Dynamic force on soil (F_{d1}) $= \xi\, K_z\, a_{t1}$

$= 3 \times 64.21 \times 10^4 \times 0.124 \times 10^{-3}$

$= 238.9$ t

e. INERTIAL FORCES (F_m)

For dynamic equilibrium (F_m) $\downarrow = (F_d)_2 \uparrow$

The total inertial force in the upper foundation block $= 236.2$ t

This is shared by the various portions of the foundation block in proportion to their masses. The inertial force shared by mass m_i is $\dfrac{m_i}{\Sigma m_i}\Sigma F_{mi}$. The computations are shown in Table 4.16.

f. CHECKS ON DESIGN

(i) Total load on springs (Static + dynamic) $= 276.55 + 236.2$

$= 512.75$ t

Load per spring $= 512.75/132$

$= 3.884$ t

< 5.06 t (safe)

(ii) Total load on soil (Static+dynamic) $= 276.55 + 95.05 + 238.9$
$= 610.5$ t

Soil stress $= 610.5/(8.8 \times 6.4)$
$= 10.84$ t/m^2
< 15 t/m^2 (safe)

g. STRUCTURAL DESIGN

Considering bending in the YY direction, the bending moments and shears are computed as shown below.

The superimposed static and dynamic loads on the upper foundation block are carried uniformly along the perimeter (21.4 m) of the spring assembly (Fig. 4.45)

i. Static loads

Referring to Fig. 4.39 reactive force per metre length of the spring support $= \dfrac{276.55}{21.4} = 12.92$ t.m

$(F_p)_s = 12.92 \times 4.15 = 53.62$ t

$(F_q)_s = 12.92 \times 3.275 = 42.32$ t

From Fig. 4.39 and Table 4.16

Table 4.16

DESIGN OF UPPER FOUNDATION BLOCK

Dynamic force on springs $(F_{d_2}) = 236.2$ t ↑
Total inertial force $\Sigma(F_m)_i = (F_{d_2}) = 236.2$ t ↓

Foundation Part i	Weight W_i (t)	Inertial force $(F_m)_i$ $\frac{W_i}{\Sigma W_i} \times \left(\frac{F_m}{2}\right)$ (t)	Distance of centre of gravity (m): From anvil edge X_i (m)	From middle section Y_i (m)	Static moment: At anvil edge W_iX_i (t.m)	At middle section W_iY_i (t.m)	Dynamic moment: At anvil edge $F_{mi}X_i$ (t.m)	At middle section $F_{mi}Y_i$ (t.m)
$ABB'A'$	23.59	20.14	0.975	2.575	23.00	60.74	19.64	51.86
$DCC'D'$	26.84	22.92	0.975	2.575	26.17	69.11	22.35	59.02
$CGG'C'$	22.02	18.80	—	0.800	—	17.62	—	15.04
$BHKF$	5.376	4.59	—	0.800	—	4.30	—	3.67
$B'H'K'F'$								
$FKK'F'$	3.794	3.24	—	0.800	—	3.04	—	2.59
Machine frame	19.175	16.37	0.975	2.575	18.70	49.38	15.96	42.15
Anvil	37.500	32.02	—	0.800	—	30.00	—	25.62
Sum	138.3 $\approx \frac{1}{2} \times 235.7$	118.08			67.87	234.19	57.95	199.95

Static bending moment at mid-section XX

$$(M_s)_{xx} = [(F_p)_s + (F_q)_s]\ 3.275 - \Sigma W_i y_i$$
$$= (53.62 + 42.32)\ 3.275 - 234.19$$
$$= 80.02 \text{ t.m}$$

Bending moment at anvil edge

$$(M_s)_{II} = [53.62 + (12.92 \times 1.675)]\ 1.675 - 67.87$$
$$= 58.2 \text{ t.m}$$

ii. Dynamic loads

Referring to Fig. 4.39 and Table 4.16 again

Dynamic load per metre length of spring assembly $= 236.2/21.4$
$= 11.04$ t/m

$(F_p)_d$ $= 11.04 \times 4.15$
$= 45.81$ t

$(F_q)_d$ $= 11.04 \times 3.275$
$= 36.16$ t

Bending moment at middle section $(M_d)_{cc} = [(F_p)_d + (F_q)_d]\ 3.275 - \Sigma F_{mi}\, y_i$
$= (45.81 + 36.16)3.275 - 199.95$
$= 68.50$ t.m

Bending moment at anvil edge $= (45.81 + 11.04 \times 1.675)\ 1.675 - 57.95$
$(M_d)_{II} = 49.56$ t.m

iii. Net moments $(M_{st} \pm M_d)$

At the middle section $= 80.02 \pm 68.50$
$= 148.52$ t.m; 11.52 t.m

At the anvil edge $= 58.2 \pm 49.56$
$= 107.76$ t.m; 8.64 t.m

iv. Shear

Shear force at the anvil edge $= 12.92\ (4.15 + 2 \times 1.675) - (23.59 + 26.84 + 19.175)$
$\pm 11.04\ (4.15 + 2 \times 1.675) \mp (20.14 + 22.92 + 16.37)$
$= 53.66$ t; 3.92 t

v. Reinforcement

Steel required at middle section $= \dfrac{148.52 \times 10^5}{1400 \times 0.865 \times 155} = 79.12 \text{ cm}^2$

26 numbers of 20 mm diameter bars are provided at bottom.

Inclined steel required at the edge of the anvil $= \dfrac{53.66}{1.4\sqrt{2}} = 27.18 \text{ cm}^2$

Ten numbers of 20 mm diameter inclined bars are provided. An additional five numbers of inclined bars are provided at a farther distance from centre as shown in Fig. 4.45.

Two layers each consisting of 10 numbers of 20 mm diameter bars are provided under the anvil.

Reinforcement provided on other faces is nominal.

The complete disposition of reinforcement is shown in Fig. 4.45.

3. Design of Foundation for a Counter-Blow Hammer

MACHINE DATA

Weight of whole machine (W_m)	=170 t
Weight of upper tup (W_u)	=31.5 t
Weight of lower tup (W_l)	=34.5 t
Stroke of the tups (h)	=0.7 m
Terminal velocity of the tups (v)	=3.0 m/sec

Operating speed (f_m) $=10$–60 blows/min
Stiffness of band springs used $=20.5 \times 10^2$ t/m

SOIL DATA

The safe bearing capacity of soil (σ_p) $=30$ t/m^2
The coefficient of elastic uniform compression (C_z) $=6$ kg/cm^3

The dimensions of the foundation satisfying the mechanical requirements are shown in Fig. 4.46.

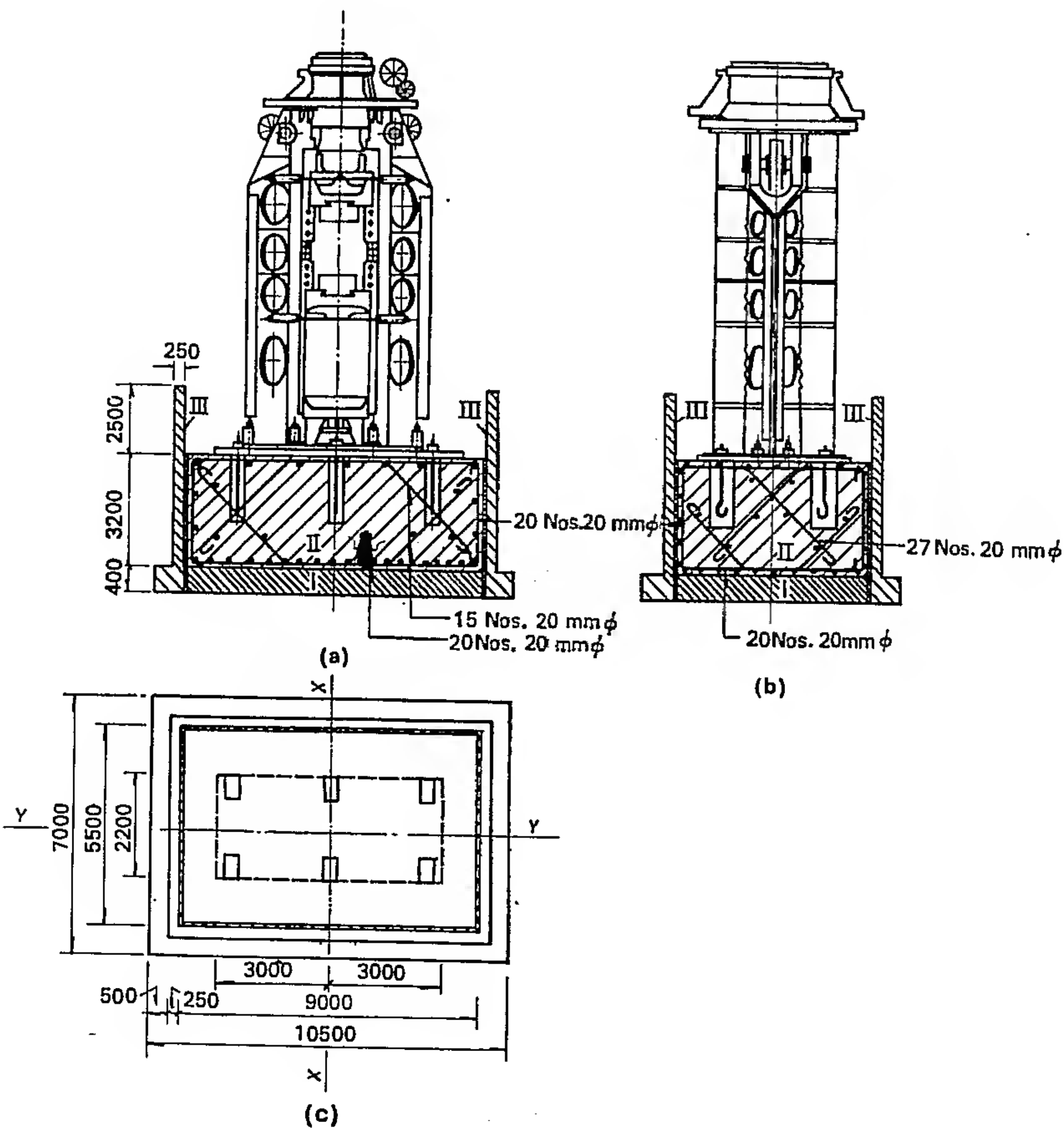


Fig. 4.46: Foundation of a Counter-blow Hammer—**(a)** Section y-y, **(b)** Section x-x, **(c)** Plan.

STAGES IN COMPUTATION

a. *Weight of foundation:* (Fig. 4.46)

Weight of portion I=$9\times5.5\times0.4\times2.4$	=47.520 t
Weight of portion II=$9\times5.5\times3.2\times2.4$	=380.160
Deduct for holes=$6\times0.3\times0.3\times1.425\times2.4$	=(—)1.85 t
Net weight of foundation	=47.52+380.16—1.85
	=425.83 t
Total weight of machine plus foundation	=425.83+170
	=595.83 t

b. *Dynamic analysis:*

Velocity of tups before impact (v) =3 m/sec

Difference in weights of tups (W_d) =3 t

i. First impact

Acceleration ($\ddot{z}$) of the tups (Eq. 4.103)

$$=\frac{v^2}{2h}$$

$$=\frac{3\times3}{2\times0.7}=6.43\,\text{m/sec}^2$$

Impact force P_1 (Eq. 4.104)

$$=\frac{W_d\,\ddot{z}}{g}$$

$$=\frac{3}{9.81}\times6.43=1.97\,\text{t}$$

Period of the blow (T) (Eq. 4.105)

$$=\frac{2h}{v}$$

$$=\frac{2\times0.7}{3}=0.466\text{ sec}$$

Natural period (T_n) of foundation soil system (f_n) (Eq. 4.106) $= 2\pi\sqrt{\dfrac{m}{K_s}}$

$$= 2\pi\sqrt{\frac{595.83}{6000\times9\times5.5\times9.81}}$$

$$= 0.09\text{ sec}$$

Period ratio (T/T_n) $= 4.96$

From Fig. 2.10 the corresponding value of maximum dynamic factor (μ) is 2.0

Dynamic force (F_1) due to first impact (Eq. 4.107)

$$=\xi\times\mu\times P_1$$

$$=3\times2.0\times1.97$$

$$=11.82\text{ t}$$

ii. Second impact

Velocity (V) with which the upper tup falls on the lower one causing the second impact (Eq. 4.108)

$$=\frac{4\,W_d\,v}{W_u\,W_2}$$

$$=\frac{4\times3\times3}{31.5+34.5}$$

$$=0.545\text{ m/sec}$$

Stiffness (K_B) of the and springs used $= 20.5\times10^2$ t/m

Dynamic force (F_2) induced on the foundation due to second impact (Eq. 4. 110)

$$= 7.5\times0.545\sqrt{\frac{31.5}{9.81}\times20.5\times10^2}\,\frac{34.5}{31.5}$$

$$=363.22\text{ t}$$

Total dynamic force (F_d) (Eq. 4.111)

$$= F_1 + F_2$$
$$= 11.92 + 363.22$$
$$= 375.14 \text{ t}$$

iii. Aliter:

Using the alternative design procedure given in Section 4.5.6(c)

Impact momentum (S_1) associated with the first impact (Eq. 4.112) $= \dfrac{W_d\, v}{g}$

$$= \frac{3 \times 3}{9.81}$$
$$= 0.92 \text{ t.sec.}$$

Impact momentum (S_2) associated with second impact (Eq. 4.113)

$$= 1.25 \times \frac{34.5}{9.81} \times 0.545$$
$$= 2.4 \text{ t.sec}$$

Total impact momentum (S) (Eq. 4.114)

$$= (0.92 + 2.4)(1 + 0.6)$$
$$= 5.31 \text{ t.sec}$$

Total dynamic force (Eq. 4.115)

$$= 3 \times 2\pi \times \frac{668}{60} \times 5.31$$
$$= 1113.78 \text{ t}$$

This valuc is more than thrcc times the value obtained in the preceding section. As statcd in Section 4.5.6, this procedure over-estimates considerably the dynamic force on the foundation, since thc natural frequency is quite large compared to the operating speed.

c. *Check for soil stress:*

Maximum stress on soil $= \dfrac{170 + 425.83 + 375.14}{9 \times 5.5} = 19.66 < 30 \text{ t/m}^2$ (safe)

d. *Structural design:*

From Figs. 4.41 and 4.46 and Eq. 4.117 the bending moments are calculated as follows:

i. Longitudinal direction

Moment at middle section $= \dfrac{(170 \pm 375.14)(9-6)}{8} = 204.43$ t.m.

Steel required (A_{st}) $= \dfrac{204.43 \times 10^5}{1400 \times 0.865 \times 310} = 54.4 \text{ cm}^2$

20 numbers of 20 mm diameter bars are provided.

Shear force (Q) at the edge of base plate $= \dfrac{(375.14 + 170)}{2}\ \dfrac{(9-6)}{2} = 90.86$ t

Inclined steel required $= \dfrac{90.86 \times 10^3}{1400\sqrt{2}} = 45.89 \text{ cm}^2$

15 numbers of 20 mm diameter bars are provided as shown in Fig. 4.46.

ii. Cross direction

Moment at middle section (y-y) $= (170 + 375.14)\,(5.5 - 2.2)/8$
$= 224.9$ t.m

Steel required (A_{st}) $= \dfrac{224.9 \times 10^5}{1400 \times 0.865 \times 310}$

$= 60.0 \text{ cm}^2$

Provide 20 nos. 20 mm diameter bars

Shear force (Q) at the edge of base plate

$$= \frac{(375.14 + 170)}{5.5} \times \frac{(5.5 - 2.2)}{2}$$

$$= 163.5 \text{ t}$$

$$A_{st} = \frac{163.5 \times 10^3}{1400\sqrt{2}}$$

$$= 82.6 \text{ cm}^2$$

27 nos. of 20 mm diameter bars are provided.

The disposition of reinforcement is shown in Fig. 4.46. Reinforcement provided on the side faces is nominal. The side walls may be designed as retaining walls.

CHAPTER FIVE

Analysis and Design of Framed Foundations for High-Speed Machinery

HIGH-SPEED machines such as turbo-generators are generally mounted on framed-type foundations (Fig. 5.1). The turbo-generator foundation is a vital and expensive part in a power plant complex. It is, therefore, essential that the foundation is designed adequately for all possible combinations of static and dynamic loads. While the mechanical engineers usually furnish the layout diagram—showing the broad outlines of the foundation and clearances required for piping, linkages, etc. and also the loading diagram—it is the task of the structural designer to check the adequacy of the foundation under static and dynamic conditions. At times, it may become necessary to suitably alter the dimensions of the foundation as suggested by the machine manufacturers so as to satisfy the design requirements. Any alterations thus found necessary must, however, have the prior concurrence of the mechanical engineers to ensure that these changes do not affect the erection or operation of the machine. It is desirable, therefore, to have a close coordination between the foundation designers and the erection staff (of the mechanical and electrical installations) from the early planning stage until the foundation is completed and the machine installed.

In the early stages of development, turbo-generators were mounted on the so-called "wall-type foundations" consisting of a pair of walls on which were seated the turbine and generator (Fig. 1.1c). With the increase in the size and ouptut of the machinery, more sophisticated types of foundation had to be devised for functional reasons. Framed foundations are now popular for supporting high-speed machinery, on account of their many advantages, such as saving in space, saving in materials, easy accessibility to all machine parts for inspection and less liability to cracking due to settlement and temperature changes. The common materials of construction used for these foundations are reinforced concrete and steel. Reinforced concrete foundations are common both in India

and abroad.

The conventional framed foundation consists of (Fig. 5.1) a heavy foundation slab (called "sole plate") which is supported from underneath by soil (or piles) and which supports on its top a series of columns. The columns are connected at their top ends by longitudinal

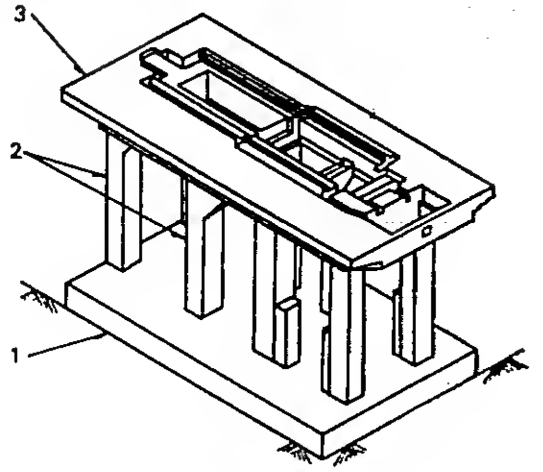


Fig. 5.1: Typical Framed Foundation—(*1*) Lower Slab, (*2*) Column, (*3*) Upper Slab.

and transverse beams forming a rigid table (called "upper plate" or "table plate") on which rest the turbine and generator. The condensers rest generally on independent supports below the turbine portion on the foundation slab.

5.1 Design Data

a. Machine Data

i. The data required include a detailed loading diagram showing the magnitude and position of all loads (static and rotating loads separately) acting on the foundation. The loading diagram should contain not only the loads but also the area over which the loads will be distributed on the foundation. A typical loading diagram is shown in Fig. 5.2.

ii. The rated capacity of the machine.

iii. Operating speed of machine.

iv. The lay-out of auxiliary equipment and platforms at the floor level of the machine hall.

v. The distribution of pipe lines and the temperature of their outer surfaces.

vi. A detailed drawing showing the sizes and location of all anchor bolts, pipe lines, chases, pockets, inserts, etc.

b. Soil Data

i. Soil profile and characteristics of soil upto at least thrice the width of the turbine foundation or till hard strata are reached.

ii. The relative positions of the ground water table in different seasons of the year.

5.2 Special Considerations in Planning

a. The foundation should be completely separated from the main building or other neighbouring foundations by providing a clear gap all around. This avoids transfer of vibrations to the surroundings.

b. All the beams and columns of the foundation should be provided with adequate

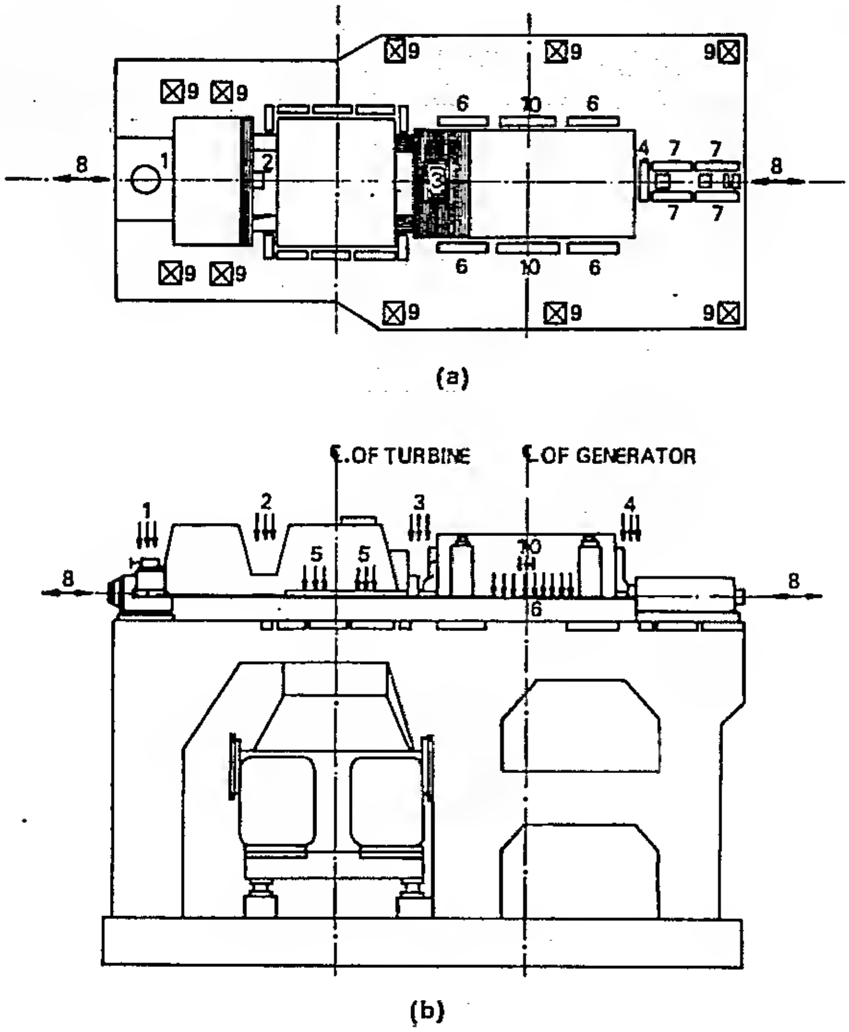


Fig. 5.2: Loading Diagram for a Turbo-generator Foundation—**(a)** Plan, **(b)** Elevation. *(1,2,3)* Weight Due to Cylinder, Turbine Rotor and Foundation Plates, *(3,4)* Weight Due to Cylinder, Bearing Plate and Rotors of Generator, *(5)* Load Due to Depression in Condenser, *(6)* Load Due to Generator Stator, Gas Coolers, Shields and Foundation Plate, *(7)* Load Due to Exciter and Foundation Plate, *(8)* Horizontal Load in the Longitudinal Direction, *(9)* Miscellaneous Loads Due to Platform Adjoining the Turbo set, Emergency Stop valves, Oil tank, etc., *(10)* Load Due to Short-circuit Moment.

haunches to ensure rigidity of joints and to avoid large concentration of stresses.

c. As far as possible, it is desirable to avoid overhanging cantilevered projections. Where they are unavoidable, they should be so designed as to ensure adequate rigidity against vibrations.

d. The base slab should be rigid to prevent non-uniform settlement of soil. According to the prevailing standard codes on the subject, the effective thickness of the base slab should be at least one-tenth of its length or the minimum width of column, whichever is more. Further, the codes stipulate that the weight of the base slab should be not less than the total weight of machine plus the weight of the foundation excluding the base slab. The minimum thickness of the base should be 2 m.

e. As far as possible, the transverse frames should be located directly under the bearings and the eccentric loading on the transverse girders be avoided.

f. As far as possible, the foundation should be so dimensioned that the resultant force due to the weight of the machine and foundation including the upper deck, intermediate slabs, if any, the base slab and the columns passes through the centre of gravity of the base area in contact with the soil.

5.3 Principal Design Criteria

a. From the point of view of vibration, the natural frequencies of foundation system should be far away from the operating speed of machine as well as the critical speeds of the rotor. A clear separating margin of at least 20 per cent should be ensured in design.

b. The amplitudes of vibration should be within the permissible limits. The permissible limits specified at the bearing level of the machine are stated as under:

i. For machines with operating speeds of 3000 rpm or more:

Vertical vibrations	0.02 mm
Horizontal vibrations	0.04 mm

ii. For machines with operating speeds less than 3000 rpm:

Vertical vibrations	0.04 mm
Horizontal vibrations	0.07 mm

iii. The static calculation should be carried out separately for each loading case—dead loads, dynamic (or equivalent static) forces in vertical or horizontal directions, short-circuit force, and forces due to thermal and shrinkage effects. The bending moments obtained for the most unfavourable combination of these loading cases should be considered for design. The effect of vertical and horizontal dynamic forces should not be added since they do not occur simultaneously.

iv. The soil stress below the base slab of the foundation should not exceed the allowable bearing pressure on soil. For the computation of the total load on the soil, only half the vertical dynamic force need be considered.

v. The torsional moments in frame girders caused by eccentric loading of the machine should be accounted for in structural design.

5.4 Dynamic Analysis

For the dynamic analysis of framed foundations, three methods are available at present, namely, the "resonance method" developed by Rausch in Germany, the "amplitude method" proposed by Barkan in the U.S.S.R. and the "combined method" proposed by Major in Hungary. The brief outlines of these three methods are given below along with their relative advantages and disadvantages.

According to the resonance method, the primary requirement is that the foundation should be "out of tune" with the machine. This means that the natural frequency of the foundation should differ by at least 20 per cent from the operating speed of the machine.

It has been realized subsequently that the resonance method is not complete in itself and it suffers from many drawbacks, some of which are explained below.

a. A check on resonance does not guarantee adequate design if, for example, the natural frequency of the foundation is considerably lower than the operating speed (i.e., if the foundation is under-tuned). Actual observations have shown that in case of under-tuned foundations, even though the natural frequency is well away from the operating speed (which means that the resonance conditions would be satisfied) excessive vibration

is still noticed every time the machine speed passes through the natural frequency value during acceleration and deceleration stages.

b. It has also been found in some cases that although the natural frequency of the foundation is close to the operating speed of machine (which means that there is theoretically a possibility of resonance), no damage is caused to the foundation. In such cases, although resonance might have occurred, the resulting amplitudes are so small that it does not damage the structure.

c. For the analysis of frequencies, a single spring-mass system is suggested. This is an over-simplification of the real system.

The basic objection to the resonance method is that it does not predict the extent of damage to the foundation, since the determination of amplitudes is omitted.

This has led to the adoption of the amplitude method developed by Barkan. According to this method, the fundamental requirement is that the amplitude of foundation under forced vibrations should not exceed a certain permissible value. Based on various investigations, different permissible amplitudes are prescribed for different machines. The method is based on a system with two degrees of freedom, which is an obvious improvement over the resonance method. The method, however, neglects the fact that the amplitudes increase during acceleration and deceleration stages (in case of under-tuned foundations). Actual observations showed that the amplitudes computed by this method are smaller than those actually measured on under-tuned foundations.

In fact, the resonance method and the amplitude method are complimentary. This gave rise to the third method, known as "combined method", also termed the "extended resonance method". According to this method, while the possibility of resonance is investigated, the amplitudes should also be determined. In the case of under-tuned foundations, the maximum dynamic effects that occur during acceleration and deceleration stages are also considered in design.

All the methods mentioned above indicate that for purpose of dynamic analysis, each cross-frame of the foundation may be considered independently. It has been found from practical observations that the resonant range of the first natural frequency is wide and this gradually narrows down for the second and higher-order frequencies. This led to the belief that dangerous resonance at higher frequencies is remote and that only the fundamental natural frequency may be considered for checking the occurrence of resonance and for the determination of amplitudes.

A summary of the various stages in computation based on the three methods mentioned above is given below:

5.4.1 Resonance Method

a. Determination of the Natural Frequencies

The natural frequency in the vertical direction is the average of the frequencies of individual cross-frames. For the calculation of vertical natural frequencies of cross-frames, the self-weights and super-imposed loads on longitudinal girders are considered as concentrated loads over columns (Fig. 5.3). For the horizontal frequencies, the bottom slab is assumed to be infinitely rigid. In both cases, a single-degree freedom system is assumed for analysis. As a further check, the vertical frequencies may also be computed on the basis of a two-degree system with the soil under the base slab acting as an elastic spring (Fig. 5.4).

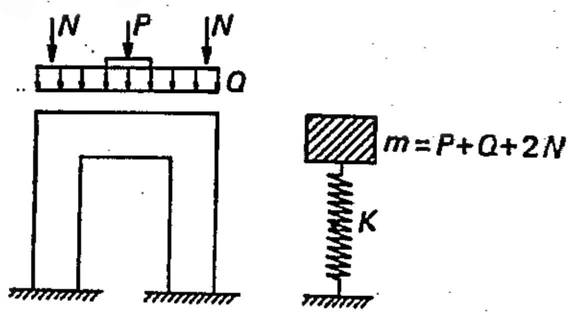


Fig. 5.3: Model System for a Cross-frame (Resonance and Combined Methods).

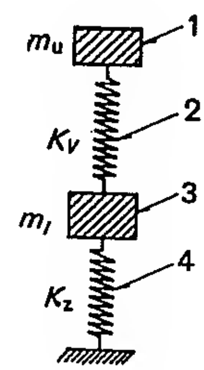


Fig. 5.4: Coupled System for Vertical Vibrations—(*1*) Upper Slab, (*2*) Frame, (*3*) Lower Slab, (*4*) Soil.

i. *Vertical Frequency*

$$f_v = \frac{\Sigma f_i}{n} \tag{5.1}$$

where f_i is the frequency of the ith cross-frame. (Total number of frames is n.) The natural frequency is obtained on the assumption of a single-degree freedom system (Fig. 2.1).

The circular frequency (ω_n) is given by

$$\omega_n = \sqrt{\frac{Kg}{W}} \text{ Sec}^{-1} \tag{2.7}$$

Expressing the frequency in rpm and substituting $\frac{W}{K} = \delta_{st}$ where δ_{st} is the static displacement, it can be easily deduced that

$$f_n = \frac{30}{\sqrt{\delta_{st}}} \text{ (cpm)}$$

where δ_{st} is in metres.

The vertical frequency of the cross-frame is then expressed as

$$\omega_n = 30/\sqrt{\delta_v} \text{ (cpm)} \tag{5.2}$$

where δ_v is the total vertical deflection in metres at the mid-point of cross-beam.

$$\delta_v = \delta_1 + \delta_2 + \delta_3 + \delta_4 \tag{5.3}$$

where δ_1, δ_2, δ_3 and δ_4 are given by Eq. 5.4.

The following notations are used

P = Concentrated load of machine
q = Self-weight per unit length of cross-beam
N = Concentrated load on columns
A_b = Area of cross-section of beam
A_c = Area of cross-section of column
I_b = Moment of inertia of beam
I_c = Moment of inertia of column
E = Modulus of elasticity of foundation material
h = Effective height of column
l = Effective span of beam
$K = \frac{I_b}{I_c}\frac{h}{l}$

Deflection* due to concentrated load (P)

$$\delta_1 = \frac{Pl^3}{96\,EI_b}\;\frac{2\,K+1}{K+2} \tag{5.4a}$$

Deflection* due to uniform distributed load ($Q = ql$)

$$\delta_2 = \frac{Q\,l^3}{384\,EI_b}\;\frac{5\,K+2}{K+2} \tag{5.4b}$$

Deflection* due to shear

$$\delta_3 = \frac{3}{5}\frac{l}{EA_b}\left(P+\frac{Q}{2}\right) \tag{5.4c}$$

Compression of column due to axial load* (N) transferred from longitudinal girders

$$\delta_4 = \frac{h}{EA_c}\left(N+\frac{P+Q}{2}\right) \tag{5.4d}$$

Fig. 5.3 shows the loads, P, Q and N acting on a typical cross-frame.

ii. *Horizontal Frequency*

Assuming a single-degree system

$$(f_h) = 30\sqrt{\frac{K_{h_1}+K_{h_2}+\ldots+K_{h_n}}{W}} \tag{5.5}$$

W = Total load of machine and upper table (t)
$K_{h_1}, K_{h_2}, \ldots$ = Lateral stiffnesses of individual cross-frames (t/m)

The lateral stiffness of frame "i" neglecting joint rotation is given by

$$K_{hi} = \frac{12\,EI_c}{h^3}\left(\frac{6\,K+1}{3\,K+2}\right) \tag{5.6}$$

* Kleinlogel: "Rigid Frame Formulae" (1964), Frederick Ungar Publication Co., N.Y.

b. Dynamic Forces

According to DIN 4024,[C4.7] which is based on the resonance method, the dynamic (or equivalent static) force (F) on the foundation of a rotating machine having operating speed f_m (in rpm) is given by

$$F = 1.5 \frac{\eta^2}{|\eta^2 - 1|} \frac{R f_m}{3000} \tag{5.7}$$

where η is ratio of the natural frequency (f_n) to the operating frequency (f_m) of machine and R is weight of rotor. The value of F shall be limited to a maximum value given by

$$F_{max} = 15\, R \frac{f_m}{3000} \tag{5.8}$$

DIN 4024[C4.7] further suggests that a 10 per cent more unfavourable value for 'η' may be adopted in calculation so as to get a higher dynamic force. For under-tuned foundation ($\eta < 1$) a 10 per cent addition (i.e., $1.1\, f_n$) is to be made for the computed natural frequency, and for over-tuned cases ($\eta > 1$) a 10 per cent reduction is made ($0.9\, f_n$).

5.4.2 *Amplitude Method*

As in the resonance method, the vibration analysis is carried out for each cross-frame independently. For the computation of vertical and horizontal frequencies, however, a two-degree system is adopted (Fig. 5.5). The main criterion for design is that the amplitudes due to forced vibrations are within the permissible limit.

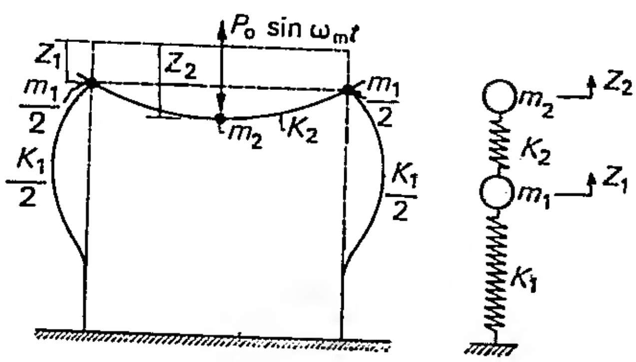


Fig. 5.5: Model System for Vertical Vibrations (Amplitude Method).

i. *Unbalanced Forces*

From simple dynamics, it can be shown that the centrifugal force of an unbalanced rotating shaft is given by

$$C_f = \frac{R}{g}\, e\, \omega_m^2 \tag{5.9}$$

where e is the eccentricity of rotor, which may be assumed as 0.05 mm for a 3000 rpm machine, 0.2 mm for a 1500 rpm machine and 0.35 to 0.8 mm for a 750 rpm machine. R is weight of rotor and ω_m is angular frequency of rotation.

The vertical (P_v) and horizontal (P_h) components of the exciting force are given by

$$P_v = C_f \sin \omega_m t \tag{5.10a}$$

$$P_h = C_f \cos \omega_m t \tag{5.10b}$$

ii. *Vertical Natural Frequency*

For the vertical frequency a two-degree spring-mass system shown in Fig. 5.5 is adopted. The equivalent weight (m_2) assumed to be concentrated at centre of transverse beam is given by

$$m_2 = m_0 + 0.45\, m_b \tag{5.11}$$

where $m_0 \left(= \dfrac{P}{g} \right)$ is the concentrated mass of the machine carried by the girder and m_b is the mass of the cross-girder.

The mass m_1 lumped over the columns is given by

$$m_1 = m_a + 0.255\, m_b + 0.35\, m_c \tag{5.12}$$

where m_a is the mass transferred from the longitudinal girders. This includes the own mass of the longitudinal girders and mass due to loads carried by them and m_c is the mass of columns.

The stiffness (K_1) of the frame columns is given by

$$K_1 = \frac{2\, EA_c}{h} \tag{5.13}$$

The stiffness (K_2) of frame beam is obtained from

$$K_2 = 1/\delta_v \tag{5.14}$$

where

$$\delta_v = \frac{l^3\,(1 + 2\,K)}{96\, EI_b\,(2 + K)} + \frac{3l}{8\,GA_b} \tag{5.15}$$

G is the modulus of rigidity which is generally taken as half of E. The two natural circular frequencies of the system are obtained by solving Eq. 2.32 (see Chapter 2).

iii. *Vertical Amplitude*

The equations of motion for forced vibration of the system (Fig. 5.5) having two degrees of freedom are given by Eqs. 2.30a and 2.30b.

The amplitudes of motion a_1 and a_2 of masses m_1 and m_2 are obtained from Eqs. 2.31a and b.

iv. *Horizontal Natural Frequency*

The upper and lower foundation slabs are assumed to be infinitely rigid and the columns act as leaf-springs, the stiffness of which is equal to the lateral stiffness of individual cross-frames. The elasticity of soil is not considered in this approach.

Fig. 5.6 shows the model system used for the analysis of horizontal vibrations. The springs represent the cross-frames and the stiffness of the spring is the lateral stiffness of the particular cross-frame.

The equivalent mass (m_i) lumped over the spring i (representing frame i) is given by

$$m_i = m_{0i} + m_{bi} + 0.3\, m_{ci} + m_{ai} \tag{5.16}$$

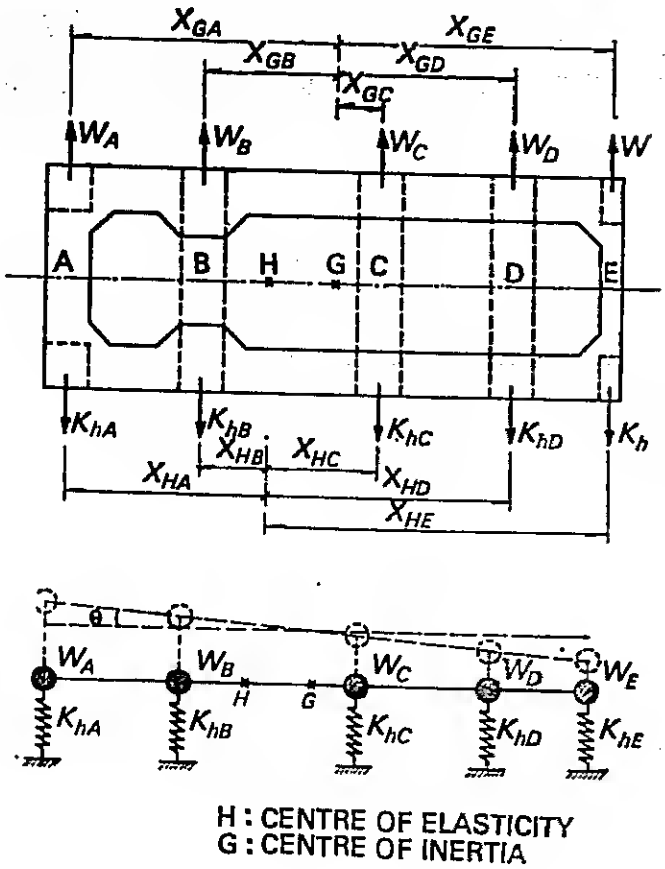


Fig. 5.6: Model System for Horizontal Vibrations (Amplitude and Combined Methods).

where

m_{0i} = mass of machine resting on cross-beam of frame i
m_{bi} = mass of cross-beam i
m_{ci} = mass of columns of frame i
m_{ai} = mass transferred from longitudinal girders on either side

Considering the upper slab as a rigid body resting on springs, the equations of motion for lateral translation (x) and rotation (θ) in the horizontal plane are given by

$$m\ddot{x} + Kx + Ke\theta = C \sin \omega_m t \tag{5.17}$$

$$\varphi\ddot{\theta} + Kex + (Ke^2 + \gamma)\,\theta = M_0 \sin \omega_m t \tag{5.18}$$

where

m is the total mass on the upper slab ($\sum m_i$) (5.19)
K is the sum of the lateral stiffnesses of cross-frames ($\sum K_{hi}$) (5.20)

$$\gamma = \Sigma K_{hi} X_{Hi}^2 \tag{5.21}$$

$$\varphi = \Sigma m_i X_{Gi}^2 \tag{5.22}$$

C is the total centrifugal force calculated from Eq. 5.9 ($\sum C_i$) (5.23)

$$M_0 = \sum (C_i X_{Gi}) \tag{5.24}$$

e is the distance between the centre of inertia G and centre of rigidity H (Fig. 5.6)

i refers to one of the cross-frames of the foundation and the summation Σ is over the total number of cross-frames. The coupled natural frequencies of horizontal motion ω_{h1} and ω_{h2} of the system are obtained from the roots of the following quadratic equation in ω_n^2 .

$$f(\omega_m^2) = \omega_n^4 - (\alpha\omega_x^2 + \omega_\theta^2)\, \omega_n^2 + \omega_x^2\omega_\theta^2 = 0 \tag{5.25}$$

where

$$\omega_x = \sqrt{\frac{K}{m}} \tag{5.26}$$

$$\omega_\theta = \sqrt{\frac{r}{\varphi}} \tag{5.27}$$

$$\alpha = 1 + e^2/r^2 \tag{5.28}$$

and

$$r^2 = \varphi/m \tag{5.29}$$

v. *Horizontal Amplitudes*

Solving Eqs. 5.17 and 5.18 the following expressions for amplitudes a_x and a_θ can be derived:

$$a_x = \frac{\left(\dfrac{e^2}{r^2}\omega_x^2 + \omega_\theta^2 - \omega_m^2\right)\dfrac{C}{m} - e\,\omega_x^2\,\dfrac{M_0}{\varphi}}{f(\omega_m^2)} \tag{5.30a}$$

and

$$a_\theta = \frac{-\dfrac{e^2}{r^2}\omega_x^2\dfrac{C}{m} - (\omega_x^2 - \omega_m^2)\,\dfrac{M_0}{\varphi}}{f(\omega_m^2)} \tag{5.30b}$$

The net amplitude of horizontal vibration is

$$a_h = a_x + a_\theta\, x' \tag{5.31}$$

where x' is the distance of the farthest point of the foundation from the centre of gravity G.

vi. *Dynamic Forces*

According to the U.S.S.R. specifications[C4.9] the vertical dynamic force (P_v) is given by:

$$P_v = 7.5 \cdot K_v\, a_v \tag{5.32a}$$

where K_v is the stiffness factor of frame beam or column (K_1 or K_2) and a_v is the corresponding vertical amplitude.

The horizontal dynamic force on each cross-frame is given by

$$P_h = 7.5\, K_h\, a_h \tag{5.32b}$$

where K_h is the lateral stiffness of the cross-frame and a_h is the net amplitude given by Eq. 5.31

5.4.3 Combined Method

i. *Vertical Frequencies*

The vertical natural frequencies of individual cross-frames are calculated as in the case of the resonance method using Eqs. 5.1 to 5.4.

ii. *Horizontal Frequencies*

The horizontal natural frequencies are obtained from the expression

$$(f_n)_h = 30\sqrt{\alpha_0 \pm \sqrt{\alpha_0^2 - \frac{\Sigma K_{hi}}{\Sigma W_i}\frac{I_H}{I_G}}} \qquad (5.33)$$

where (see Fig. 5.6)

K_{hi} = lateral stiffness of the cross-frame i (t/m)

W_i = total weight on frame i, including machine weight, weight of transverse beam and weight transmitted by longitudinal girders

$I_G = \sum_i W_i X_{Gi}^2 = W_A X_{GA}^2 + W_B X_{GB}^2 + \ldots$

X_G being the distance of weight W from the vertical axis through centre of gravity.

$$I_H = \Sigma K_{hi} X_{hi}^2 = K_{hA} X_{HA}^2 + K_{hB} X_{HB}^2 + \ldots$$

X_H is the distance of each frame from the centre of rigidity (H)

and

$$\alpha_0 = \frac{1}{2}\left[e^2 \frac{\Sigma K_{hi}}{I_G} + \frac{\Sigma K_{hi}}{\Sigma W_i} + \frac{I_H}{I_G}\right] \qquad (5.34)$$

iii. *Amplitudes*

The following steps lead to the evaluation of amplitudes in vertical and horizontal directions:

STEP 1: The dynamic factor (μ) is computed from the following expression

$$\mu = \frac{1}{\sqrt{\left(1 - \frac{f_m^2}{f_n^2}\right) + \left(\frac{\Delta}{\pi}\right)^2 \cdot \frac{f_m^2}{f_n^2}}} \qquad (5.35)$$

where Δ is logarithmic decrement of damping, which may be assumed as 0.4 for concrete foundations. For under-tuned foundations, i.e., $f_n < f_m$, $f_n = f_m$ should be used in the above formula.

Then

$$\mu = \frac{\pi}{\Delta} \text{ and for } \Delta = 0.4,\ \mu = 7.85$$

STEP 2: The centrifugal force (C) caused by the rotation of unbalanced shaft is obtained for under-tuned foundations ($f_n < f_m$) as

$$C_i = \alpha R\left[\frac{f_n}{f_m}\right]^2 \qquad (5.36)$$

where α is as given below for different machine speeds.

$$\alpha = 0.2 \text{ for } N \geqslant 3000 \text{ rpm} \qquad (5.36a)$$

$$= 0.16 \text{ for } N = 1500 \text{ rpm} \tag{5.36b}$$
$$= 0.10 \text{ for } N = 750 \text{ rpm} \tag{5.36c}$$

The above values of α correspond to the normal balanced state of the rotor. For over-tuned cases (i.e., $f_n > f_m$)

$$C_i = \alpha \cdot R \tag{5.37}$$

STEP 3(a): Vertical amplitudes (a_v)

$$a_v = \mu \delta_v \tag{5.38}$$

where

$$\delta_v = \frac{C}{E}\left\{\frac{l^3}{96 I_b}\frac{2K+1}{K+2} + \frac{3}{5}\frac{l}{A_b} + \frac{1}{2}\frac{h}{A_c}\right\} \tag{5.39}$$

μ and C are obtained from steps (1) and (2) above. For under-tuned foundations, μ is taken as π/Δ or 7.85, which is the maximum value.

STEP 3(b): Horizontal amplitudes (a_h)

For computing the horizontal amplitudes, the horizontal centrifugal forces acting on individual frames are determined in the following manner. The horizontal centrifugal force obtained from Eqs. 5.36 or 5.37 (as the case may be) is distributed to various frames in proportion to their lateral rigidities.

$$C_i = C\frac{K_{hi}}{\Sigma K_{hi}} + e_1 \frac{C K_{hi} X_{Hi}}{I_H} \tag{5.40}$$

where

$$I_H = \Sigma K_{hi} X_{Hi}^2$$

C is the total centrifugal force (ΣC_i)

e_1 is eccentricity of the resultant of centrifugal forces from the centre of elasticity $= X_H - X_C$, where X_H is the centre of elasticity (centre of gravity of forces K_h) and X_C is the centre of gravity of centrifugal forces or the rotating weights.

ΣK_{hi} is the sum of lateral rigidities of cross-frames

X_H is the distance of each frame from the centre of elasticity.

If C_i is the centrifugal force on the frame i (Eq. 5.36 or 5.37), the lateral deflection δ_{hi} under the static influence of C_i is given by

$$\delta_{hi} = \frac{C_i}{K_{hi}} \tag{5.41}$$

The horizontal amplitude a_{hi} of cross-frame i is given by

$$a_{hi} = \mu \delta_{hi} \tag{5.42}$$

where μ is given by Eq. (5.35).

Since the horizontal frequency is generally very low, as compared to operating frequency, the maximum value of μ equal to 7.85 may be taken for the computation of horizontal amplitudes.

iv. *Dynamic Forces*

To account for possible uncertainties in the calculation of natural frequencies, the computed natural frequency should be modified by multiplying it by a term

$(1 \pm \alpha)$ such that $f'_n = f(1 \pm \alpha)$ (5.43)

where α is a correction factor which may be taken as 0.2. The plus sign may be taken when $f_n < f_m$ and minus sign when $f_n > f_m$. If f_n lies between $\frac{f_m}{1+\alpha}$ and $\frac{f_m}{1-\alpha}$, then $f'_n = f_m$ may be used.

The following expressions may be used for the calculation of dynamic forces:

Case (a): $f'_n < f_m$

$$\text{when } f_m = 3000 \text{ rpm}, \quad F = 16R\left(\frac{f'_n}{f_m}\right)^2 \tag{5.44a}$$

$$\text{when } f_m = 1500 \text{ rpm} \quad F = 12R\left(\frac{f'_n}{f_m}\right)^2 \tag{5.44b}$$

$$\text{when } f_m = 750 \text{ rpm}, \quad F = 8R\left(\frac{f'_n}{f_m}\right)^2 \tag{5.44c}$$

where R is the rotating weight on the frame.

The maximum values of F should be as follows:

$$\text{when } f_m = 3000 \text{ rpm}, \quad F_{max} = 16R \tag{5.45a}$$

$$f_m = 1500 \text{ rpm}, \quad F_{max} = 12R \tag{5.45b}$$

$$f_m = 750 \text{ rpm}, \quad F_{max} = 8R \tag{5.45c}$$

Case (b): $f'_n > f_m$

$$F = \frac{2F_{max}}{\sqrt{\left(1 - f_m^2/f'^2_n\right)^2 + \left(\frac{\Delta}{\pi}\right)^2\left(f_m^2/f'^2_n\right)^2}} \tag{5.46}$$

$$\text{where } F_{max} = 1.0R \text{ for 3000 rpm machine} \tag{5.46a}$$

$$= 0.8R \text{ for 1500 rpm machine} \tag{5.46b}$$

$$= 0.5R \text{ for 750 rpm machine} \tag{5.46c}$$

Case (c): If f_n lies between $\frac{f_m}{1-\alpha}$ and $\frac{f_m}{1+\alpha}$, then $\mu = \frac{\pi}{\Delta} \approx 8.0$.

From Eq. (5.46),

$$F = 16R \text{ for } f_m = 3000 \tag{5.47a}$$

$$F = 12R \text{ for } f_m = 1500 \tag{5.47b}$$

$$F = 8R \text{ for } f_m = 750 \tag{5.47c}$$

The coefficients 1, 0.8 and 0.5 in the Eq. 5.46 correspond to the worst possible unbalanced state of rotor at which the amplitudes built up are five times the corresponding values under normal balanced state equations (5.36 a, b, c).

To compute the vertical dynamic force which acts at the mid-point of cross-beam, the rotating weight on cross-beam (R_b) alone should be substituted for R in the above expressions and for the calculation of horizontal dynamic force which acts transversely at the beam level, the total rotating weight ($R_b + R_c$), including the rotating weight transferred from longitudinal girders on to the columns, should be considered.

5.4.4 Commentary on the Various Methods

Of the three methods described above for the vibration analysis of framed foundations, the combined method recommended by Major is most popular in design offices, because of the following advantages:

a. It combines the advantages of the resonance and amplitude methods in that it checks for occurrence of resonance as well as for the limitation of amplitudes.

b. It accounts (although conservatively) for the occurrence of transient resonance which inevitably occurs in under-tuned foundations.

5.5 Structural Design

Having evaluated the equivalent static loads occurring during the operation of machine, the structural design is done as in the static case. The design consists of calculating the bending moments, shears, etc., in the frame members, considering the worst combination of the various loading cases given in the next section. Fig. 5.7 shows the diagramatic view of the various loading cases to be considered in design.

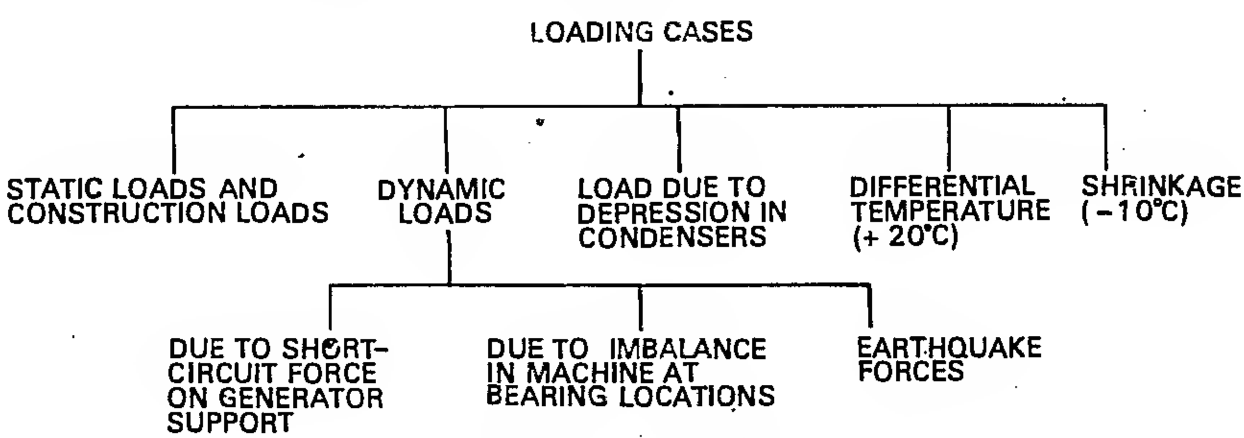


Fig. 5.7: Loading Cases.

5.5.1 Loading Cases

The loading diagram furnished by mechanical engineers gives the magnitude, point of application and direction of all loads—both stationary and rotating.

a. Construction Loads

Construction loads occur only when the machine is being erected. As such they are not to be considered as acting simultaneously with the dynamic loads which occur only during the operation of the machine. The construction loads are generally taken as a uniformly distributed load varying from 1000 kg/m^2 to 3000 kg/m^2, depending on the size of the machine unit.

b. Dynamic Loads

The dynamic loads exerted on the foundation are of two types:

i. Centrifugal forces (equal to $me\omega^2$, where m is the unbalanced mass, e the eccentrictiy and ω the circular rotating frequency) caused by the rotation of imperfectly balanced rotors. This is a periodic load.

ii. Loads which are impulsive (shock-like) in nature and which act irregularly.

The dynamic effects caused by faulty balancing of the machine have been considered in Chapter 2.

The impulsive loads are caused by the mutual magnetic effect between the stator and rotor. The shock, which is in the form of a couple known as "short-circuit moment," tends to break the stator off the foundation and this imposes vertical loads on the longitudinal beam supporting the generator stator. The machine suppliers usually furnish information leading to the determination of this impulsive load. If accurate data is not available, the short-circuit moment (M) may be taken empirically as four times the rated capacity (in MW) of the turbo-generator unit. The following formula may also be used

$$M = \frac{RDN}{3000} \tag{5.48}$$

where R is weight of rotor (in t), D is diameter of generator casing (in m) and N is frequency of rotation (in rpm).

The Russian practice is to take seven times the normal rated torque as the short-circuit moment.

c. Loads Due to Depression in Condensers

Owing to the depression in the condenser, a suction effect occurs between the condenser and the turbine. The magnitude of this load depends on the nature of coupling between them. If the coupling tie is elastic, this causes an increased load on the turbine and consequently on the foundation. The condenser supports are correspondingly relieved of the same amount of load. However, if the condenser connection is rigid, the depression has no effect on the foundation. The load due to depression in condenser, if not supplied by the manufacturer, can be evaluated from the following formula

$$P = A(p_1 - p_2)$$

where A is the cross-sectional area of the connecting tie between the condenser and turbine and $(p_1 - p_2)$ is the difference between internal pressure in condenser and the external pressure. This difference in pressure may be taken as 10 t/m^2.

The load due to the condenser being a force without mass should not be considered in the computation of natural frequencies.

d. Effects of Temperature and Shrinkage

When designing framed foundations, the effect of differential thermal expansion and shrinkage should also be considered. Where exact data is not available, a differential temperature of 20°C may be assumed between the upper and lower slabs. Besides, a differential temperature of 20°C may be assumed between the inner and outer faces of the upper slab. The upper slab should be treated as a horizontal closed frame and analysed for the induced moments due to differential temperature.

To account for the shrinkage of the upper slab relative to the base slab, a temperature fall of 10°C may be assumed. If the frames are concreted more than two months after the base slab is cast, the temperature fall of 15°C may be assumed.

e. Forces Due to Earthquake Effects

The force due to earthquake is generally considered as a lateral force acting on the individual cross frames of the foundation. The evaluation of this force depends on the seismic zone in which the foundation is located and has been illustrated in the example worked out in Section 5.6. For foundations located in power stations, the Indian Standard Code IS: 1893-1970 ("Criteria for Earthquake Resistant Design of Structures") recommends a 50 per cent increase in the seismic coefficient considered for normal buildings. This is in view of the post-earthquake importance of these structures. When earthquake forces are considered in elastic design, the permissible stresses in materials and the allowable bearing pressure on soils may be increased by a suitable proportion as suggested in the above Code.

5.5.2 Design of Cross-Frames

The bending moments, shears, etc. in the frame members are evaluated separately for each of the above loading cases. The worst combination of the effects of these various loading cases should be considered for design. The bending moments and shears in the cross-frames may be evaluated by the use of expressions given by Kleinlogel.

Where haunches are provided at the beam-column junctions as shown in Fig. 5.8, the effective span (l) and height (h) are calculated from

$$l = l_o - 2\,X\alpha \tag{5.49a}$$

$$h = h_o - Z\alpha \tag{5.49b}$$

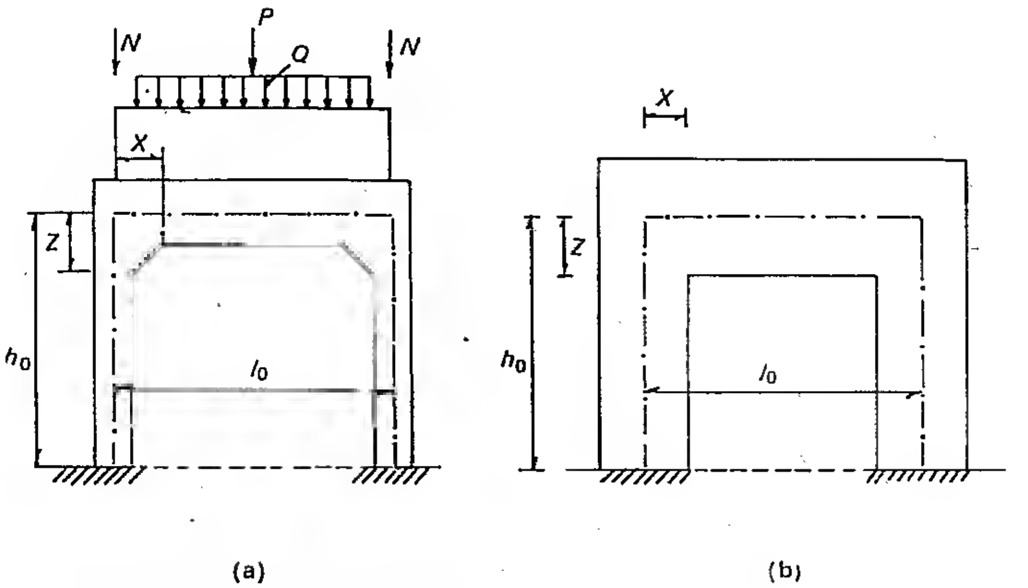


Fig. 5.8: A Typical Cross-Frame—**(a)** With Haunches, **(b)** Without Haunches.

where X and Z are as marked in Fig. 5.8 and α is a factor to be determined from Fig. 5.9.

It may be found that the sections of the foundation members as given in the drawings supplied by the manufacturers are too massive for the induced moments and shears in these members. According to the standard codes on the subject,[C4.3, C4.7] atleast 50 kg of steel per m³ of concrete section should be provided in all of the foundation members.

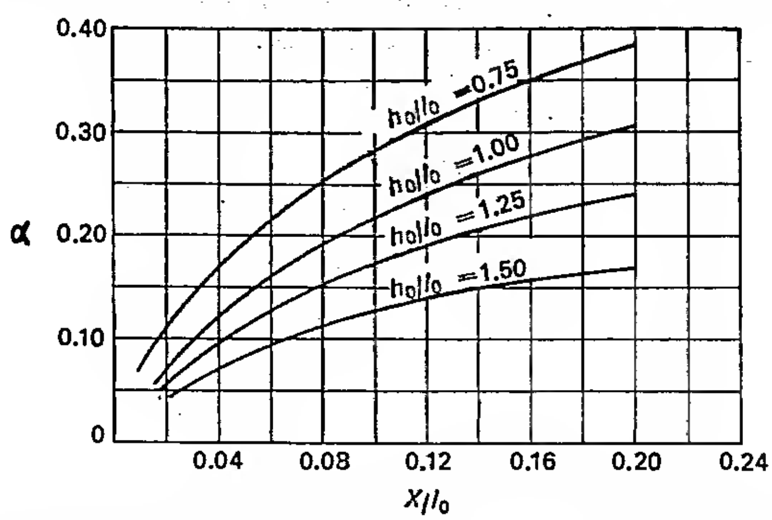


Fig. 5.9: Graph for Determination of Coefficient α for Haunched Frames (From Major, A., *Vibration Analysis and Design of Foundations for Machines and Turbines*, Akademiai Kiado, Budapest, 1962; with permission).

5.5.3 Design of Longitudinal Frames

The analysis of the longitudinal frame is carried out by any of the well-known methods for the various loading cases given below and the reinforcement is provided accordingly.

a. Static loads including self-weight of longitudinal girders and machine loads shown in loading diagram.

b. Short-circuit force as distributed load on generator support.

c. Vacuum effect of condenser. If the condenser connection with the turbine is rigid, this need not be considered.

d. Construction loads considered as distributed load on platforms projecting from longitudinal girders. This may generally be taken as 2000 kg/m².

e. Vertical dynamic forces (if any) acting on longitudinal girder.

f. Half the horizontal dynamic force which is considered to act along the axis of longitudinal girders.[C1.12]

g. Differential temperature of 20°C between the upper and lower foundation slabs.

h. Shrinkage corresponding to a temperature fall of 10°C.

i. Earthquake forces (only horizontal forces need be considered.)

For considering the worst combination of the moments due to various loads, the following points should be borne in mind.

a. The construction loads should not be considered to act with the dynamic loads. The former occur only when the machine is being assembled or at rest.

b. Vertical and horizontal dynamic loads should not be considered simultaneously.

c. Since the effects of temperature and shrinkage are mutually of opposite nature, they may not be considered together in evaluating the net moments.

d. The effects of earthquake and dynamic forces from the machinery may be considered together.

5.5.4 Design of Foundation Slab

The base area of foundation slab should be such that the maximum bearing pressure on soil is not more than the permissible bearing pressure.

Having found the soil pressures and the load transmitted by each column, the structural design of the foundation slab is done as an ordinary raft.

A practical example of the design of an under-tuned framed foundation for a 200 MW turbo-generator machine has been illustrated in Section 5.6.

5.6. Numerical Example for Design of Framed Foundation for a 200 MW Turbo-Generator

Figs. 5.10–5.12 contain the outline dimensions of the foundation in mm proposed by the

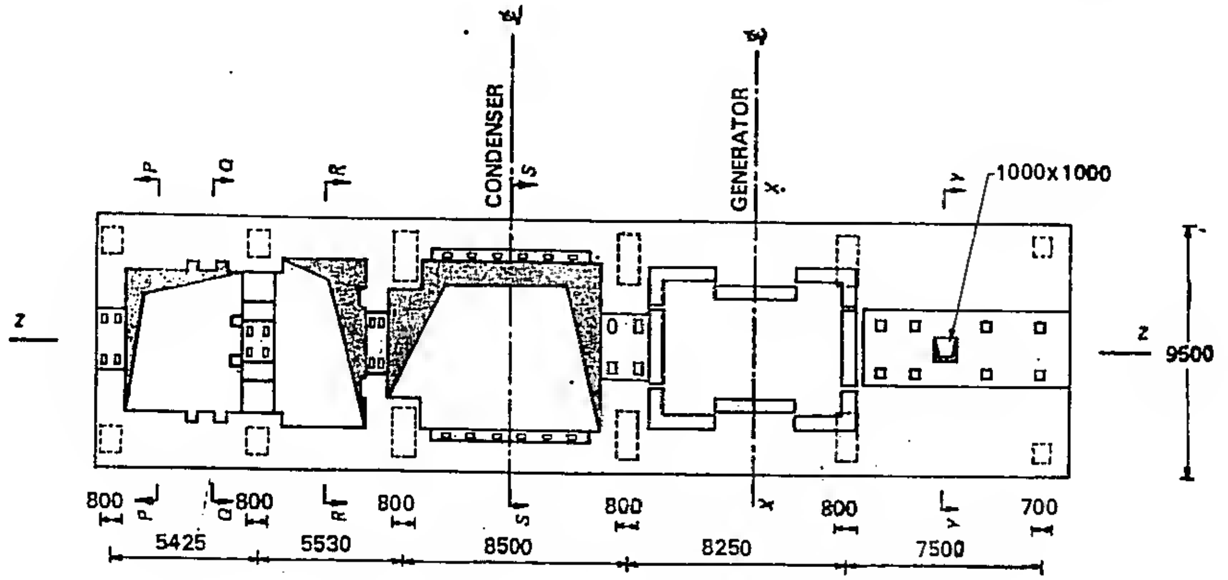


Fig. 5.10. Plan of the Upper Table.

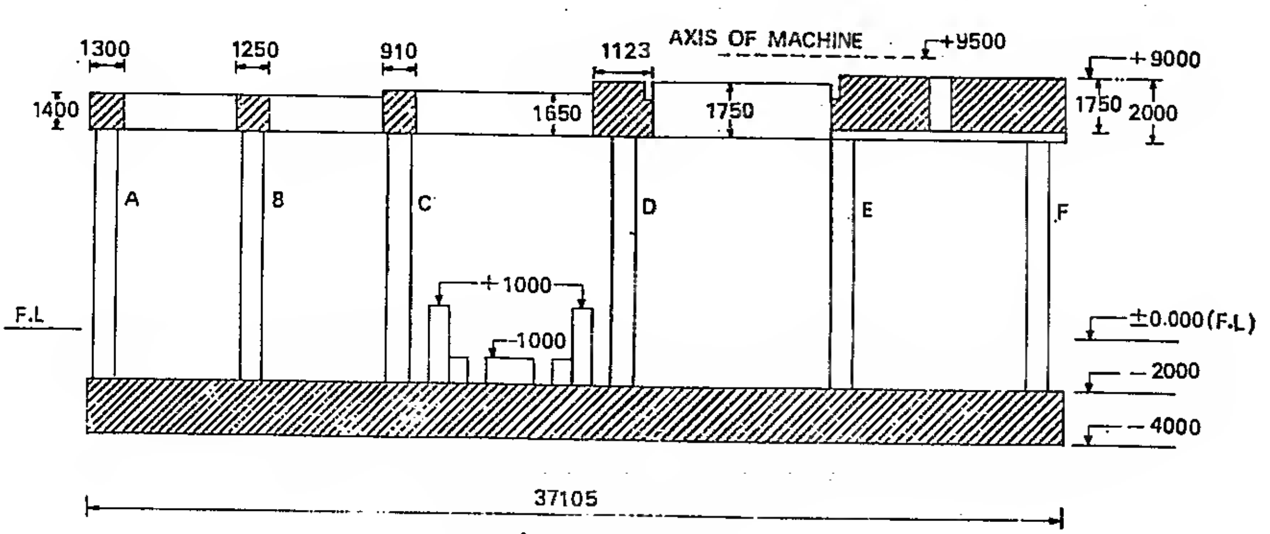


Fig. 5.11: Longitudinal Section z-z.

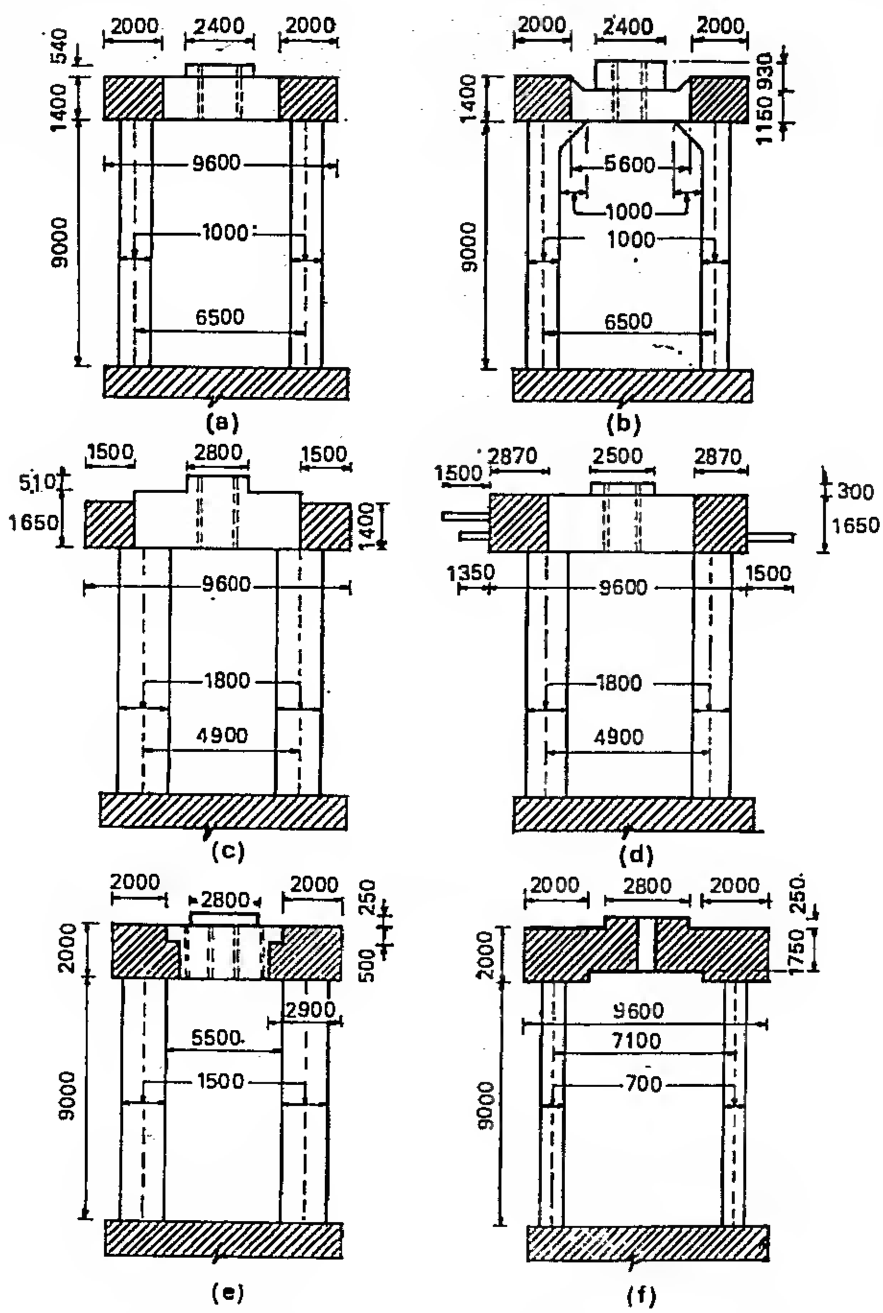


Fig. 5.12: Typical Cross-Sections—(a) Section at P-P, (b) Section at Q-Q, (c) Section at R-R, (d) Section at S-S, (e) Section at X-X, and (f) Section at Y-Y.

machine manufacturers. The loading diagram shown in Fig. 5.13 contains the total loads acting at different positions on the upper table of the foundation. The rotating loads are shown in brackets. The permissible soil stress is 2 kg/cm². Density of concrete used is 2.5 t/m³. It is required to check the dynamic stability and design the foundation.

The foundation consists of six transverse frames standing on a 2 m thick sole plate. The dynamic analysis of the foundation is carried out on the basis of the "combined method" explained in Section 5.4.3. The computations consist of the following steps:

a. Natural frequency calculations

i. Vertical frequencies of cross-frames

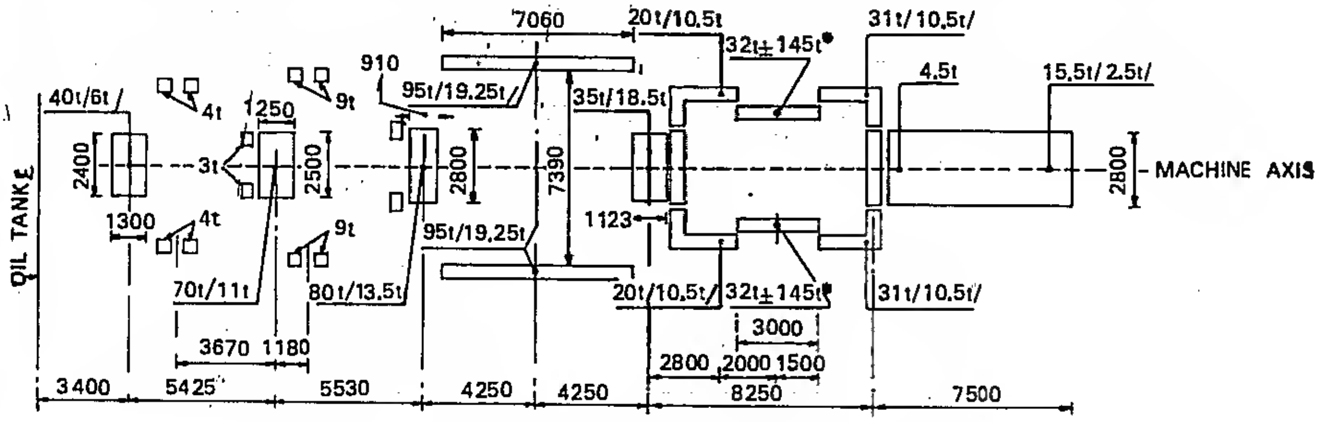


Fig. 5.13: Loading Diagram at 9,000 (* Short Circuit Force)

   ii Horizontal frequencies of the foundation

b. Amplitude calculations
   i. Vertical amplitudes
   ii. Horizontal amplitudes

c. Dynamic forces
   i. Vertical dynamic forces
   ii. Horizontal dynamic forces

d. Design of cross-frames

e. Design of longitudinal frames

f. Design of foundation slab

a. Natural Frequency Calculations

i. *Vertical Frequencies of Cross-Frames:* The geometrical data and loading on the cross-frames are given in Table 5.1.

Frame A

The moment of inertia of the frame beam

$I_b = 1/12 \times 1.3 \times 1.4^3 = 0.30 \text{m}^4$

The moment of inertia of frame column

$I_c = 1/12 \times 0.8 \times 1.0^3 = 0.067 \text{ m}^4$

Area of cross-section of frame beam

$A_b = 1.4 \times 1.3 = 1.82 \text{m}^2$

Area of cross-section of frame column

$A_c = 1.0 \times 0.8 = 0.8 \text{ m}^2$

Effective span (l) and height (h) :

Referring to Fig. 5.8

$l_0 = 7.5$ m : $h_0 = 9.7$ m

$Z = 0.7$: $X = 0.5$

$$\frac{h_0}{l_0} = \frac{9.7}{7.5} = 1.293$$

$$\frac{X}{l_0} = \frac{0.5}{7.5} = 0.067$$

Referring to Fig. 5.9, $\alpha = 0.13$

Effective span $(l) = l_0 - 2X\alpha = 7.5 - 2 \times 0.5 \times 0.13 = 7.37$ m

Table 5.1

VERTICAL NATURAL FREQUENCIES OF CROSS-FRAMES

Frame	l (m)	h (m)	I_b (m^4)	A_b (m^2)	I_c (m^4)	A_c (m^2)	K	P (t)	Q* (t)	N (t)	δ_1 (μm)	δ_2 (μm)	δ_3 (μm)	δ_4 (μm)	δ (μm)	f_v (cpm)
A	7.37	9.61	0.30	1.82	0.067	0.80	5.84	44.2	29.6	32.85	331.02	136.44	47.78	279.3	794.54	1064.2
B	6.81	9.44	0.16	1.44	0.067	0.80	3.31	77.3	32.1	54.5	760.30	192.15	88.30	429.52	1470.25	782.4
C	6.39	9.56	0.34	1.5	0.389	1.44	1.31	83.25	29.4	127.15	242.46	50.56	83.45	406.02	782.49	1072.5
D	6.41	9.69	0.86	3.78	0.389	1.44	3.34	38.6	46.3	176.7	59.04	43.11	20.94	491.57	614.66	1210.1
E	6.74	9.85	0.416	1.77	0.225	1.20	2.70	9.348	33.1	155.02	32.54	69.76	19.72	482.22	604.24	1220.4
F	7.72	9.79	0.58	2.26	0.020	0.49	36.78	20.348	86.24	44.10	107.58	284.68	43.36	648.63	1084.25	911.0

*$Q = ql$

Effective height $(h) = h_0 - \zeta a = 9.7 - 0.7 \times 0.13 = 9.61$ m
The frame constant (K) is given by

$$K = \frac{I_b}{I_c} \frac{h}{l} = \frac{0.30}{0.067} \times \frac{9.61}{7.37} = 5.84$$

Static loads on frame :

Beam weight $(Q = ql)$	=	$1.82 \times 2.5 \times 6.5 = 29.6$ t
Machine weight (P')	=	40.0 t
Grouting on beam (P'')	=	$0.54 \times 1.3 \times 2.4 \times 2.5 = 4.2$ t
Total (P)	=	44.2 t
Load on column (N)		
Load from longitudinal beam	=	$3.36 \times 2.0 \times 1.4 \times 2.5 = 23.5$ t
One third self-weight of column	=	$0.8 \times 2.5 \left(\frac{10.4}{3} - 1.4 \right) = 4.1$ t

Load due to pipe lines resting on longitudinal beam $= \frac{3.6}{5.43} \times 6.0 + \frac{1.15}{5.43} \times 60 = 5.25$ t

Total load (N) = 32.85 t

The deflection at beam centre due to concentrated load (P)

$$\delta_1 = \frac{Pl^3}{96\, EI_b} \frac{2K+1}{K+2}$$

$$= \frac{44.2 \times 7.37^3}{96 \times 3 \times 10^6 \times 0.30} \frac{2 \times 5.84 + 1}{5.84 + 2}$$

$$= 331.02 \times 10^{-6} \text{ m}$$

The deflection at beam centre due to uniformly distributed load (Q)

$$\delta_2 = \frac{Ql^3}{384\, EI_b} \frac{5K+2}{K+2}$$

$$= \frac{29.6 \times 7.37^3}{384 \times 3 \times 10^6 \times 0.30} \frac{5 \times 5.84 + 2}{5.84 + 2}$$

$$= 136.44 \times 10^{-6} \text{m}$$

The deflection at centre of beam due to shearing forces

$$\delta_3 = \frac{3}{5} \frac{l}{EA_b} \left[P + \frac{ql}{2} \right]$$

$$= \frac{3}{5} \frac{7.37}{3 \times 10^6 \times 1.82} \left[44.2 + \frac{29.6}{2} \right]$$

$$= 47.78 \times 10^{-6} \text{ m}$$

The axial compression of column due to vertical load

$$\delta_4 = \frac{h}{EA_c} \left[N + \frac{P+ql}{2} \right]$$

$$= \frac{9.61}{3 \times 10^6 \times 0.8}\left[32.85 + \frac{44.2 + 29.6}{2}\right]$$
$$= 279.3 \times 10^{-6} \text{m}$$

The total vertical displacement of the centre of beam (δ)

$$\delta = \delta_1 + \delta_2 + \delta_3 + \delta_4$$
$$= 794.54 \times 10^{-6} \text{ m}$$

The vertical natural frequency of frame A is

$$(f_v)_A = \frac{30}{\sqrt{\delta}} = \frac{30}{\sqrt{794.54 \times 10^{-6}}}$$
$$= 1064.2 \text{ cpm}$$

Table 5.1 contains the vertical deflections and vertical natural frequencies of all of the frames.

ii. *Horizontal Frequencies:* The loads carried by the frames (W_i):

$W_i = P_i + Q_i + 2N_i$
$W_A = 44.2 + 29.6 + 65.70 = 139.50$ t
$W_B = 77.3 + 32.1 + 109 = 218.4$ t
$W_C = 83.25 + 29.4 + 254.3 = 366.95$ t
$W_D = 38.6 + 46.3 + 353.4 = 438.3$ t
$W_E = 9.348 + 33.10 + 310.04 = 352.49$ t
$W_F = 20.348 + 86.24 + 88.2 = 194.79$ t

From Table 5.2, the distance of the centre of inertia from the axis of Frame A (X_G) is given by

$$X_G = \frac{30355.08}{1710.43} = 17.75 \text{ m}$$

Table 5.2

HORIZONTAL FREQUENCIES

Frame	Total load (t)	Distance of frames B to F from the axis of Frame A		Rigidity factor			
(i)	(W_i)	d_i	$W_i\,d_i$	K_{hi}	$K_{hi}\,d_i$	$x_G = (X_G - d_i)$	$x_H = (X_H - d_i)$
A	139.5	—	—	4.79×10^3	—	17.75	16.43
B	218.4	5.43	1184.82	4.78×10^3	25.93×10^3	12.32	11.00
C	366.95	10.96	4019.94	20.89×10^3	228.80×10^3	6.79	5.47
D	438.3	19.46	8527.13	23.35×10^3	454.20×10^3	− 1.71	− 3.03
E	352.49	27.71	9765.68	13.05×10^3	361.62×10^3	− 9.96	−11.28
F	194.79	35.21	6857.51	1.48×10^3	52.17×10^3	−17.46	−18.78
Sum	1710.43		30355.08	68.34×10^3	1122.72×10^3		

Displacement due to unit load acting horizontally along the axis of the frame beam (δ_h)

$$\delta_h = \frac{h^3}{12\,EI_c}\,\frac{3K+2}{6K+1} + \frac{6h}{5EA_c}\left[1+\frac{A_c h}{A_h l}\,\frac{18K^2}{(6K+1)^2}\right] + \frac{h^3}{EA_c l^2}\,\frac{18K^2}{(6K+1)^2}\ \text{m/t}$$

$$(\delta_h)_A = \frac{9.61^3}{12\times 3\times 10^6\times 0.067}\,\frac{3\times 5.84+2}{6\times 5.84+1.0} + \frac{6\times 9.61}{5\times 3\times 10^6\times 0.8}$$

$$\left[1+\frac{0.8\times 9.61}{1.82\times 7.37}\,\frac{18\times 5.84^2}{(6\times 5.84+1.0)^2}\right] + \frac{9.61^3}{3\times 10^6\times 0.8\times 7.37^2}$$

$$\frac{18\times(5.84)^2}{(6\times 5.84+1.0)^2} = 208.76\times 10^{-6}\ \text{m/t}$$

Similarly for other frames

$(\delta_h)_B = 209.21\times 10^{-6}$ m/t $(\delta_h)_D = 42.83\times 10^{-6}$ m/t $(\delta_h)_F = 675.68\times 10^{-6}$ m/t

$(\delta_h)_C = 47.87\times 10^{-6}$ m/t $(\delta_h)_E = 76.63\times 10^{-6}$ m/t

Rigidity factors $K_h = \left(\frac{1}{\delta_h}\right)$ of frames A to F are given in Table 5.2.

The distance of the resultant of forces H (centre of elasticity) from the axis of frame A

$$X_H = \frac{1122.72\times 10^3}{68.34\times 10^3} = 16.43\ \text{m}$$

The eccentricity $(e) = X_G - X_H = 17.75 - 16.43 = 1.32$ m

Table 5.2 contains the distances x_G and x_H of the axis of each individual frame from the position G and H respectively.

$$\begin{aligned} I_G = \Sigma W x_G^2 &= 139.5\times 17.75^2 + 218.4\times 12.32^2 + 366.95\times 6.79^2 + 438.3\times(-1.71)^2 \\ &\quad + 352.49\times(-9.96)^2 + 194.79\times(-17.46)^2 \\ &= 189.65\times 10^3 \end{aligned}$$

$$\begin{aligned} I_H = \Sigma K_h x_H^2 &= 4.79\times 10^3\times 16.43^2 + 4.78\times 10^3\times 11.00^2 + 20.89\times 10^3\times 5.47^2 \\ &\quad + 23.35\times 10^3\,(-3.03)^2 + 13.05\times 10^3\times(-11.28)^2 + 1.48\times 10^3 \\ &\quad \times(-18.78)^2 \\ &= 4893.28\times 10^3 \end{aligned}$$

Factor α_0 is given by Eq. (5.34)

$$\alpha_0 = \tfrac{1}{2}\left[\frac{e^2\Sigma K_{hi}}{I_G} + \frac{\Sigma K_{hi}}{\Sigma W_i} + \frac{I_H}{I_G}\right]$$

$$\begin{aligned} &= \tfrac{1}{2}\left[\frac{(1.32)^2\times 68.34\times 10^3}{189.65\times 10^3} + \frac{68.34\times 10^3}{1.710\times 10^3} + \frac{4893.28\times 10^3}{189.65\times 10^3}\right] \\ &= 33.20 \end{aligned}$$

The horizontal natural frequencies are from Eq. (5.33)

$$(f_n)_h = 30\sqrt{\alpha_0 \pm \sqrt{\alpha_0^2 - \frac{\Sigma K_{hi}}{\Sigma W_i}\,\frac{I_H}{I_G}}}$$

$$(f_n)_1 = 30\sqrt{33.20-\sqrt{33.20^2 - \frac{68.34\times 10^3\times 4893.28\times 10^3}{1710.43\times 189.65\times 10^3}}}$$
$$= 149.37 \text{ cpm}$$

and $$(f_n)_2 = 30\sqrt{33.2+\sqrt{33.2^2 - \frac{68.34\times 10^3\times 4893.28\times 10^3}{1710.43\times 189.65\times 10^3}}}$$
$$= 193.51 \text{ cpm}$$

b. Amplitude Calculations

i. *Vertical Amplitudes:* Rotating weights on cross beam (R_b)

$(R_b)_A=6.0$ t $(R_b)_B=11.00$ t $(R_b)_C=13.5$ t $(R_b)_D=18.5$ t $(R_b)_E=0$ $(R_b)_F=2.5$ t

Rotating weights on frame columns (R_c) taken together

$(R_c)_A=0$ $\qquad (R_c)_D=19.25+2\times 10.5\times \frac{5.45}{8.25}=33.10$ t

$(R_c)_B=0$ $\qquad (R_c)_E=2\times 10.5+2\times 10.5\times \frac{2.80}{8.25}=28.1$ t

$(R_c)_C=\frac{2\times 19.25}{2}=19.25$ t $\qquad (R_c)_F=0$

Centrifugal forces (C_b) on beams are given by

$$C = 0.2\,R\left(\frac{f_v}{f_m}\right)^2$$

$$(C_b)_A = 0.2\times 6\left(\frac{1064.2}{3000}\right)^2 = 0.150 \text{ t}$$

$$(C_b)_B = 0.2\times 11\left(\frac{782.4}{3000}\right)^2 = 0.150 \text{ t}$$

$$(C_b)_C = 0.2\times 13.5\left(\frac{1072.5}{3000}\right)^2 = 0.350 \text{ t}$$

$$(C_b)_D = 0.2\times 18.5\left(\frac{1210.1}{3000}\right)^2 = 0.600 \text{ t}$$

$$(C_b)_E = 0$$

$$(C_b)_F = 0.2\times 2.5\left(\frac{911.0}{3000}\right)^2 = 0.046 \text{ t}$$

Centrifugal forces on columns (C_c) taken together

$$(C_c)_A = 0$$
$$(C_c)_B = 0$$
$$(C_c)_C = 0.2\times 19.25\left(\frac{1072.5}{3000}\right)^2 \doteq 0.492 \text{ t}$$
$$(C_c)_D = 0.2\times 33.10\left(\frac{1210.1}{3000}\right)^2 = 1.077 \text{ t}$$
$$(C_c)_E = 0.2\times 28.1\left(\frac{1220.4}{3000}\right)^2 = 0.929 \text{ t}$$
$$(C_c)_F = 0$$

The vertical amplitudes $(a)_v$ are given by

$$a_v = \delta_v \times 7.85$$

where δ_v is vertical displacement at beam centre due to centrifugal forces and 7.85 is the maximum dynamic factor.

$$\delta_v = \frac{C_b l^3}{96 E I_b} \frac{2K+1}{K+2} + \frac{3}{5} \frac{l}{E A_b} C_b + \frac{h}{E A_c}\left[\frac{C_b + C_c}{2}\right]$$

$$(\delta_v)_A = \frac{0.150 \times 7.37^3}{96 \times 3 \times 10^6 \times 0.30} \frac{2 \times 5.84 + 1}{5.84 + 2} + \frac{3}{5} \frac{7.37}{3 \times 10^6 \times 1.82} \times 0.150 +$$

$$\frac{9.61}{3 \times 10^6 \times 0.8} \frac{0.15}{2} = 1.556 \times 10^{-6} \text{ m}$$

The vertical amplitude at the middle of cross-beam of Frame A is

$(a_v)_A = 1.556 \times 10^{-6} \times 7.85 = 12.22\ \mu\text{m}$

The deflections (δ_v) and the amplitudes (a_v) for all of the frames are contained in Table 5.3.

Table 5.3

VERTICAL AMPLITUDES

Frame	Vertical displacement $(\delta_v) \times 10^{-6}$ m	Vertical Amplitude (a_v) μm
A	1.556	12.22
B	1.907	14.97
C	2.225	17.47
D	3.08	23.61
E	1.272	9.98
F	0.429	3.37

ii. *Horizontal Amplitudes* (a_h): Substituting the higher of the two natural frequencies in Eq. (5.36) with $R = \Sigma(R_b + R_c)$

$$\Sigma C_h = 0.2\, \Sigma (R_b + R_c) \left(\frac{f_n}{f_m}\right)^2$$

$$= 0.2 \times 131.95 \left(\frac{193.51}{3000}\right)^2 = 0.1098 \text{ t}$$

The centrifugal force being proportional to the rotating weights, the centre of gravity of centrifugal forces is the same as the centre of gravity of rotating weights on frames.

Total rotating weight $\quad \Sigma R = \Sigma R_b + \Sigma R_c$

$= 131.95$ t

The horizontal amplitudes are computed using two alternative methods as shown below:

iia. Based on single-degree system:

Lateral deflection of upper table $(\delta_h) = \dfrac{\Sigma C_h}{\Sigma K_h}$

$$= \frac{0.1098}{68.34 \times 10^3}$$

$$= 1.606 \times 10^{-6} \text{ m}$$

Considering a maximum dynamic factor of 7.85,

Horizontal amplitude $(a_h) = 1.606 \times 10^{-6} \times 7.85 = 12.61\ \mu m$

iib. Based on combined method (Section 5.4.3):

The distance of the resultant of the rotating weights from the axis of Frame A is

$$X_R = \frac{1}{131.95}\left\{11 \times 5.43 + 32.75 \times 10.96 + 51.6 \times 19.46 + 28.1 \times 27.71 + 2.5 \times 35.21\right\}$$

$$= 17.35 \text{ m}$$

Eccentricity (e_1) of centre of elasticity with respect to centre of gravity of centrifugal forces is

$$e_1 = X_H - X_R$$
$$= 16.43 - 17.35$$
$$= -0.92 \text{ m}$$

The horizontal centrifugal force (C_h) acting on each individual frame is from Eq. (5.40).

$$C_{hi} = \Sigma C_h \frac{K_{hi}}{\Sigma K_{hi}} + e_1 \Sigma C_h \frac{K_{hi}}{I_H} x_H$$

Substituting the values for Frame A

$$(C_h)_A = 0.1098 \times \frac{4.79 \times 10^3}{68.34 \times 10^3} + (-0.92) \times \frac{0.1098 \times 4.79 \times 10^3 \times 16.43}{4893.28 \times 10^3}$$
$$= 0.00608 \text{ t}$$

Similarly for other frames

$(C_h)_B = 0.0066$ t
$(C_h)_C = 0.0312$ t
$(C_h)_D = 0.0389$ t
$(C_h)_E = 0.0240$ t
$(C_h)_F = 0.0029$ t
$\Sigma C_h = 0.1098$ t

Lateral displacement $(\delta_h)_i = \dfrac{C_{hi}}{K_{hi}}$

Substituting the values

$$(\delta_h)_A = \frac{0.00608}{4.79 \times 10^3}$$
$$= 1.269 \times 10^{-6} \text{ m}$$

Similarly $(\delta_h)_B = 1.378 \times 10^{-6}$ m
$(\delta_h)_C = 1.493 \times 10^{-6}$ m
$(\delta_h)_D = 1.668 \times 10^{-6}$ m
$(\delta_h)_E = 1.839 \times 10^{-6}$ m
$(\delta_h)_F = 1.993 \times 10^{-6}$ m

Since the higher of the two horizontal natural frequencies (193.51 cpm) is below the operating speed (3000 cpm) the maximum dynamic factor of 7.85 shall be used to obtain the horizontal amplitudes of individual frames.

Thus

$$(a_h)_A = (\delta_h)_A \times 7.85$$
$$= 1.269 \times 10^{-6} \times 7.85$$
$$= 9.96 \ \mu m$$

The amplitudes of other frames are similarly computed and are found to be ncarly the same as that of frame A.

c. Dynamic Forces

i. *Vertical Dynamic Forces:* The corrected frequencies (f'_n) are given by

$$f'_n = 1.2 \times f_n$$
$$(f'_n)_A = 1.2 \times 1064.2$$
$$= 1277.04 \text{ cpm}$$

Since the corrected natural frequency is less than the operating frequency, the vertical dynamic force is obtained from the Eq. 5.44a

$$F_b = 16\, R_b \left[\frac{f'_n}{f_m} \right]^2$$

Substituting

$$(F_b)_A = 16 \times 6 \left(\frac{1277.04}{3000} \right)^2 = 17.40 \text{ t}$$

Similary the dynamic forccs on columns are obtained by substituting R_c for R_b in Eq. 5.44a.

The minimum design vertical dynamic force ($\bar{F}$) is four times the corresponding rotating load on the cross frames.

Table 5.4 gives the vertical dynamic forces which are used in structural design.

Table 5.4

VERTICAL DYNAMIC FORCES

Frame	Modified frequency (f'_n) cpm	Rotating loads (t) on beam R_b	Rotating loads (t) on column R_c	Vertical dynamic force (t) on beam F_b	Vertical dynamic force (t) on column F_c	Design values of vertical dynamic forces (t) on cross-beam $\bar{F}_b = 4 \times R_b$	Design values of vertical dynamic forces (t) on column taken together $\bar{F}_c = 4 \times R_c$
A	1277.04	6.0	0	17.40	0	24	0
B	938.87	11.0	0	17.24	0	44	0
C	1286.95	13.5	19.25	39.75	56.68	54	77.0
D	1452.06	18.5	33.10	69.35	124.10	74	132.4
E	1464.48	0	28.10	0	107.14	0	112.4
F	1093.2	2.5	0	5.31	0	10	0

ii. *Horizontal Dynamic Forces:* The corrected horizontal frequencies are 1.2 times the calculated values

$$f'_{n1} = 149.37 \times 1.2 = 179.24 \text{ cpm}$$
$$f'_{n2} = 193.51 \times 1.2 = 232.21 \text{ cpm}$$

The total horizontal dynamic force (F_h) is given by

$$F_h = 16\,\Sigma\,[\,(R_b) + (R_c)\,]\left(\frac{f'_{n2}}{f_m}\right)^2$$

$$= 16 \times 131.95 \left(\frac{232.21}{3000}\right)^2 = 12.66 \text{ t}$$

The dynamic force $(F_h)_i$ shared by frame i is given by

$$(F_h)_i = F_h \frac{C_{hi}}{\Sigma C_h}$$

Substituting for Frame A

$$(F_h)_A = \frac{12.66 \times 0.00\,608}{0.1098}$$
$$= 0.701 \text{ t}$$

Since this is less than the specified minimum horizontal dynamic force which is equal to the rotating weight ($R_b + R_c$) on the cross-frame, the design values should be as follows:

$(F_h)_A = 6.0$ t.
$(F_h)_B = 11.0$ t.
$(F_h)_C = 13.5 + 19.25 \quad = 32.75$ t.
$(F_h)_D = 18.5 + 33.1 \quad = 51.6$ t.
$(F_h)_E = 0 + 28.1 \quad = 28.1$ t.
$(F_h)_F = 2.5$ t.

d. Design of Cross-Frames

The various loads on the cross-frames are as follows:

i. Concentrated static load (P) at the beam centre.
ii. Distributed static load on beam (ql).
iii. Vertical dynamic load (F_b) at beam centre
iv. Horizontal dynamic load (F_h)
v. Lateral force due to earthquake (F_E)
vi. Load due to differential temperature between base slab and frame beam. For design purpose a temperature difference of 20°C is assumed.
vii. Load due to shrinkage corresponding to a temperature drop of 10°C.

The worst combination of these loading cases should be considered in design.

FRAME A: The expressions for moments and shears for various loading cases are taken from Kleinlogel*.

i. Concentrated static load (P) at beam centre (Fig. 5.14a)
Load $P = 44.2$ t

* Kleinlogel (1952), *Rigid Frame Formulae*, Frederick Unger Publ. Co., N.Y.

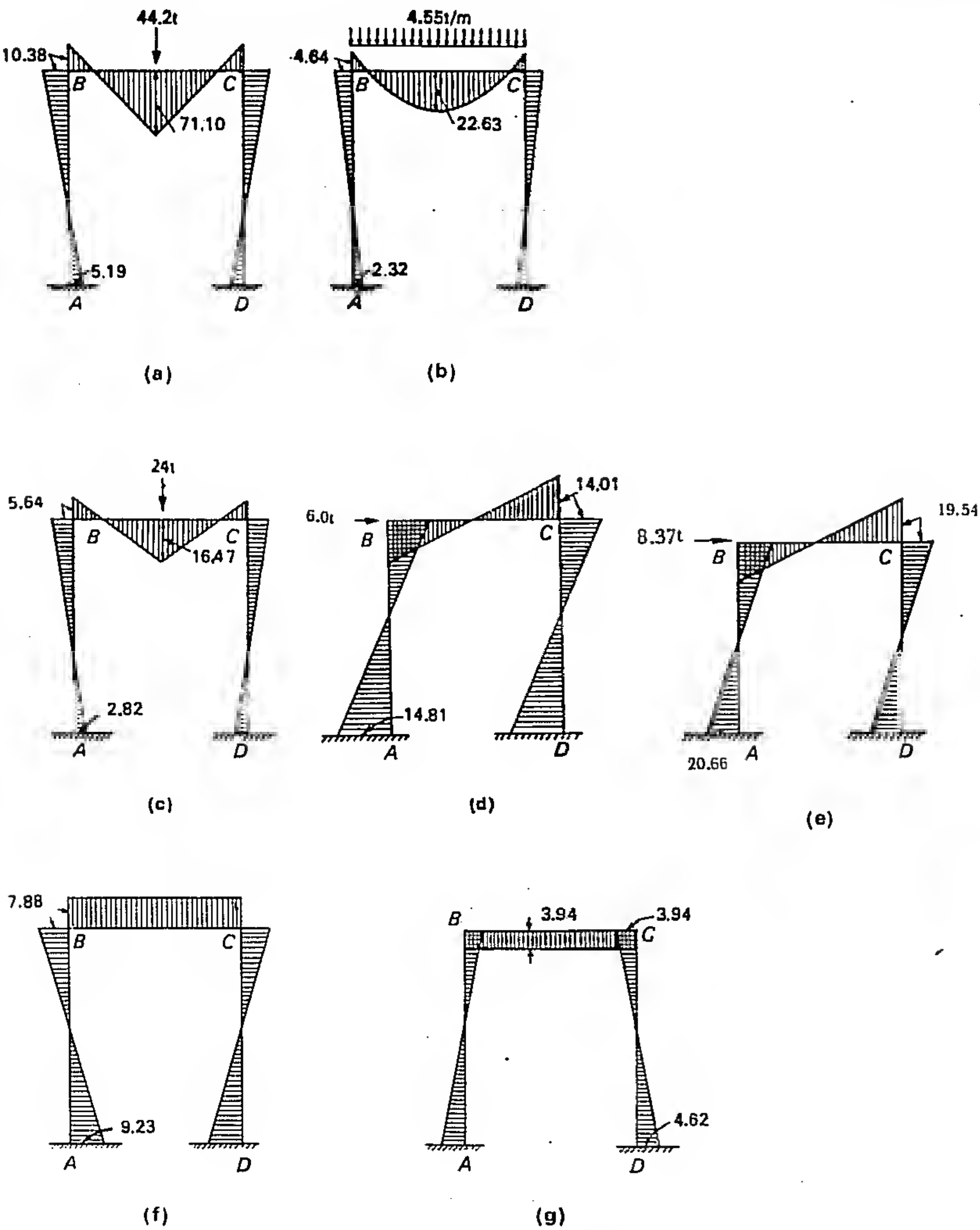


Fig. 5.14: Moment Diagram Due to **(a)** Concentrated Static Load, **(b)** Uniformly Distributed Load, **(c)** Vertical Dynamic Force, **(d)** Horizontal Dynamic Force, **(e)** Earthquake Load, **(f)** Differential Temperature (20°C), **(g)** Shrinkage (−10°C).

Frame constant $K=5.84$ (Table 5.1)

Joint Moments

$$M_A = M_D = \frac{Pl}{8(K+2)}$$
$$= \frac{44.2 \times 7.37}{8(5.84+2)}$$
$$= 5.19 \text{ t.m}$$
$$M_B = M_C = -2M_A$$
$$= -10.38 \text{ t.m.}$$

Maximum moment at beam centre (Mp):

$$M_P = \frac{Pl}{4} + M_B$$
$$= \frac{44.2 \times 7.37}{4} - 10.38 = 71.10 \text{ t.m}$$

Shears (Q):

$$Q_B = Q_C = 22.1 \text{ t}$$

ii. Distributed static load (ql), (Fig. 5.14b)

Total distributed load (ql) = 29.6 t

Joint moments:

$$M_A = M_D = \frac{ql^2}{12(K+2)}$$
$$= \frac{29.6 \times 7.37}{12(5.84+2)} = 2.32 \text{ t.m}$$
$$M_B = M_C = -2M_A = -4.64 \text{ t.m.}$$

Maximum moment (M_P)

$$M_P = \frac{ql^2}{8} + M_B$$
$$= \frac{29.6 \times 7.37}{8} - 4.64 = 22.63 \text{ t.m.}$$

Shears (Q)

$$Q_B = Q_C = 14.8 \text{ t.}$$

iii. Vertical dynamic forces (F_b), (Fig. 5.14c)

$$(F_b)_A = \pm 24 \text{ t}$$

Moments (M) and Shears (Q)

Using the same formulae for moments and shears used in (i) above

$$M_A = M_D = \frac{24 \times 7.37}{8(5.84+2)}$$
$$= \pm 2.82 \text{ t.m}$$
$$M_B = M_C = -2M_A$$
$$= \mp 5.64 \text{ t.m}$$
$$M_P = \pm\left(\frac{24 \times 7.37}{8} - 5.64\right)$$
$$= \pm 16.47 \text{ t.m}$$
$$Q_B = Q_C = \pm 12 \text{ t}$$

iv. Horizontal dynamic force (F_h) (Fig. 5.14d)

$(F_h)_A \pm 6.0$ t

Joint moments

$$M_A = \frac{(F_h)_A \times h}{2}\left(\frac{3K+1}{6K+1}\right)$$
$$= \frac{6.0 \times 9.61}{2}\left(\frac{3\times5.84+1.0}{6\times5.84+1.0}\right)$$
$$= \mp 14.81 \text{ t.m.}$$
$$M_D = -M_A = \pm 14.81 \text{ t.m}$$
$$M_B = \frac{(F_h)_A \times h}{2}\left(\frac{3K}{6K+1}\right)$$
$$= \frac{6\times9.61}{2} \quad \frac{3\times5.84}{6\times5.84+1.0} = \pm 14.01 \text{ t.m}$$
$$M_C = -M_B = \mp 14.01 \text{ t.m}$$

Shear (Q)

$$Q_B = -\frac{2M_B}{l} = \mp \frac{2\times14.01}{7.37} = \mp 3.80 \text{ t}$$
$$Q_B = -Q_C = \pm 3.80 \text{ t}$$

v. Earthquake force (F_E) (Fig. 5.14e)

Total load on Frame A = 139.5 t

Assuming the structure to be located in seismic zone III, the seismic coefficient according to IS: 1893-1970 is taken as 0.04. In view of the post-earthquake importance of turbo-generator foundations located in power stations, a 50 per cent increase in seismic coefficient should be considered (vide Clause 3.44, IS: 1893-1970). The design seismic coefficient is therefore 0.06.

The lateral force due to earthquake$=0.06\times139.5$
$=8.37$ t

Taking proportionate values given under (iv),

Joint moments are

$$M_A = \mp 20.66 \text{ t.m}$$
$$M_D = -M_A = \pm 20.66 \text{ t.m}$$
$$M_B = \pm M_C = \mp 19.54 \text{ t.m}$$

Shear (Q)

$$Q_B = \frac{-2M_B}{l} = \mp \frac{2\times19.54}{7.37} = \mp 5.30 \text{ t}$$
$$Q_C = -Q_B = \mp 5.30 \text{ t}$$

vi. Load due to differential temperature between base slab (Fig. 5.14f) and frame beam

$$M_B = M_C = -\frac{3EI_b\,\epsilon T}{h(K+2)}$$

where E = Modulus of elasticity

ϵ = Thermal expansion coefficient ($\epsilon=11\times10^{-6}/^\circ C$)

T = Change in temperature in degrees (20°C)

$$M_B = M_C = -\frac{3\times3\times10^6\times0.30\times11\times10^{-6}}{9.61\,(5.84+2)}$$
$$= -7.88 \text{ t.m}$$
$$M_A = M_D = -M_B\left(\frac{K+1}{K}\right)$$

$$= 7.88 \times \frac{5.84+1}{5.84} = 9.23 \text{ t.m}$$

vii. Load due to shrinkage equivalent to $-10°C$ (Fig. 5.14 g). Taking proportionate values from case (vi) with reversed sign

$$M_A = M_D = -\frac{10}{20} \times 9.23 = -4.62 \text{ t.m}$$

$$M_B = M_C = -\frac{10}{20}(-7.88) = +3.94 \text{ t.m}$$

The bending moments are summarized in Table 5.5.

Table 5.5

JOINT MOMENTS IN FRAME A

Moment due to	M_A (t.m)	M_B (t.m)	M_P (t.m)
1. Concentrated static load	5.19	−10.38	+71.10
2. Distributed static load	2.32	− 4.64	+22.63
3. Vertical dynamic load	± 2.82	∓ 5.64	±16.47
4. Horizontal dynamic load	∓14.81	±14.01	—
5. Earthquake	∓20.66	±19.54	—
6. Differential temperature	+ 9.23	− 7.88	− 7.88
7. Shrinkage	− 4.62	+ 3.94	+ 3.94
Maximum	+52.21	+22.47	+114.14
Minimum	−32.58	−56.45	+ 69.38

The steel requirement in the frame members can be obtained as per normal design practice.

e. Design of Longitudinal Frames

The various loads which are to be considered for design of longitudinal frames are as follows (Fig. 5.15):

i. Static loads comprising of:

a. Self-weight of longitudinal girders (W_l)

b. Loads due to machinery placed on longitudinal girders (W_m).

ii. Short-circuit force (W_s) equal to twice the value given in the loading diagram. In this case W_s is equal to 290 t. This load is considered as a uniformally distributed load (Fig. 5.15) over 3 m length.

iii. Construction loads (W_s) which are assumed as 2000 kg/m².

iv. Loads due to differential temperature (equal to 20°C) between the base slab and the upper table.

v. Load due to shrinkage equal to a temperature drop of 10°C.

vi. Vertical dynamic loads (W_d) acting at points where the rotating loads are placed on longitudinal girders (Table 5.6). As the longitudinal girders of the individual spans considered as fixed beams are essentially over-tuned, the vertical dynamic loads in these spans are computed from Eq. 5.46.

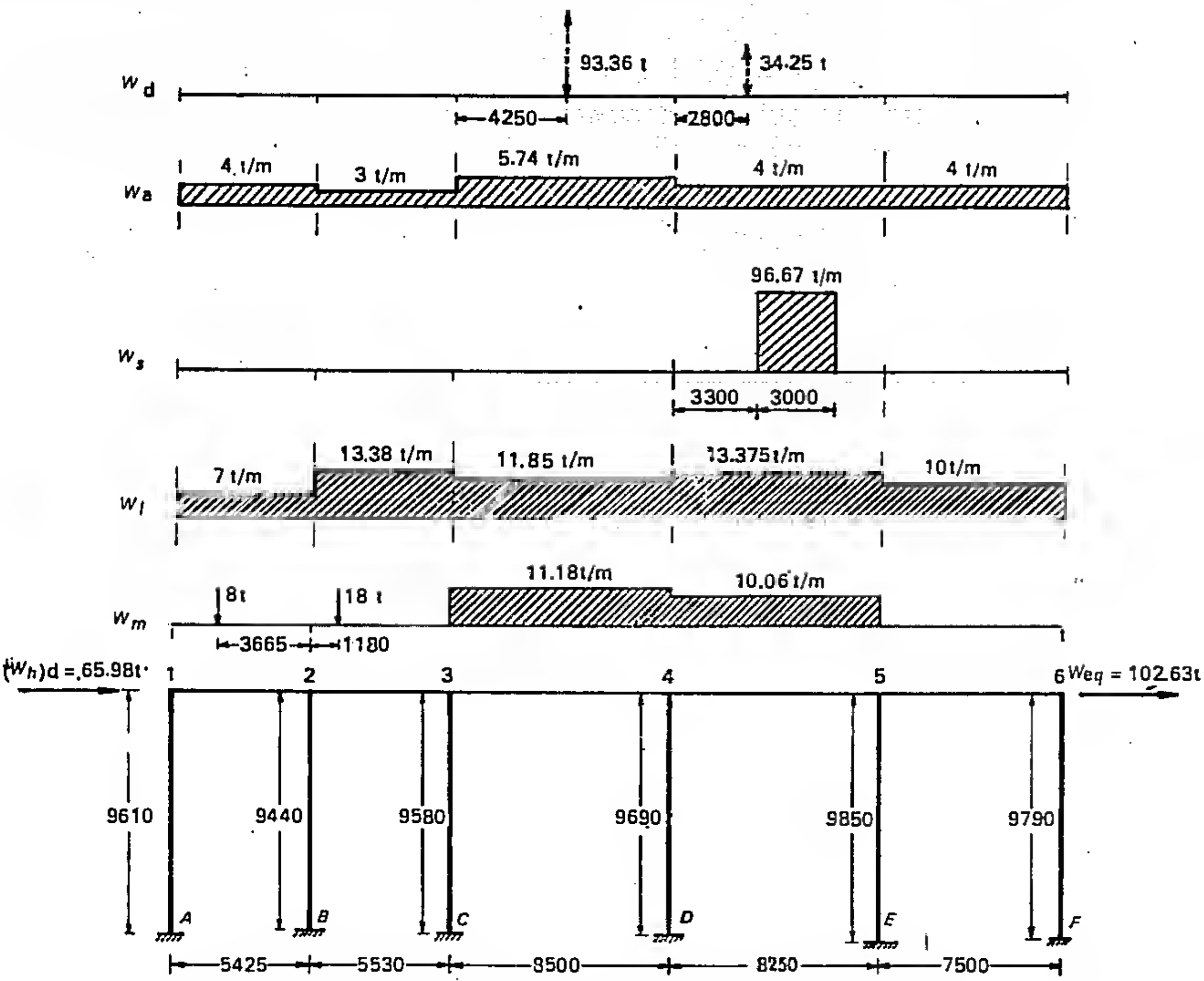


Fig. 5.15: Loads Acting on Longitudinal Frame.

Table 5.6

VERTICAL DYNAMIC LOADS ON LONGITUDINAL BEAMS

Member (Fig. 5.15)	Span (l) (m)	Moment of Inertia (I) m^4	Natural frequency (f_n) in cpm from Appendix A.6 (iv)	Rotating load on span, (t)	Vertical dynamic load (Eq. 5.46)
3–4	8.5	1.0743	4836.19	19.25	93.36
4–5	8.25	1.65	5988.72	10.50	34.25

vii. Longitudinal dynamic force which may be assumed equal to half of the total dynamic force in the lateral direction $(W_h)_d$.

viii. Earthquake loads (W_{eq}).

ix. Loads due to non-uniform heating of the upper slab for which a temperature difference of 20°C (interior warmer than exterior) is assumed. For this calculation, Rausch[01.15] recommends that reduced values of modulus of elasticity ($= 0.5E$) and moment of inertia of beam section ($= 0.3I_b$) may be considered. These values have been

Table 5.7

JOINT MOMENTS (t.m) IN LONGITUDINAL FRAME (Fig. 5.15)

Loading Case	M_{12}	M_{1A}	M_A	M_{21}	M_{23}	M_{2B}	M_{B2}	M_{32}	M_{34}	M_{30}	M_{C3}
1. (a) Self-weight	−1.33	+1.33	+0.755	+14.11	−13.56	−0.58	−0.205	+29.51	−32.41	+2.89	+1.595
(b) Superimposed loads	−0.544	+0.544	+0.364	+3.648	−3.80	+0.137	+0.163	+24.19	−27.17	+2.98	+1.657
2. Construction loads	−0.615	+0.615	+0.361	+3.97	−4.00	+0.04	+0.075	+17.12	−18.55	+1.42	+0.81
3. Short circuit load	±0.21	∓0.21	∓0.105	±6.44	∓6.89	±0.45	±0.225	∓23.1	±26.36	∓3.27	∓1.635
4. Longitudinal dynamic load	±40.97	∓40.97	∓41.98	∓26.40	∓17.50	∓43.92	±44.23	±24.70	±50.80	∓75.43	∓76.82
5. Vertical dynamic load	∓1.14	±1.14	±0.855	∓20.42	±21.20	+0.78	∓0.095	±72.24	±83.40	±11.16	±6.09
6. Earthquake load	∓64.0	±64.0	±65.57	∓41.21	∓27.39	±68.6	±69.10	∓38.54	±79.26	±117.81	±120.00
7. Differential temperature	−1.30	+1.30	+1.36	+1.70	+6.94	−8.63	−8.72	+10.58	+22.11	−32.66	−33.27
8. Shrinkage	+0.65	−0.65	−0.68	−0.85	−3.47	+4.32	+4.36	−5.29	−11.06	+16.33	+16.65
9. Maximum positive moment	103.96	108.35	110.13	93.51	45.06	116.85	117.87	198.16	151.55	218.71	218.36
10. Maximum negative moment	108.35	103.96	107.01	57.14	76.31	122.04	122.32	85.47	259.66	223.3	228.47

	M_{43}	M_{45}	M_{4D}	M_{D4}	M_{54}	M_{56}	M_{5E}	M_{E5}	M_{65}	M_{6F}	M_{F6}
1. (a)	+85.75	−85.73	+0.02	+0.16	+73.34	−73.64	−0.29	+0.265	+1.04	−1.04	−0.47
(b)	+82.33	−82.39	+0.06	+0.192	+16.45	−15.56	−0.89	−0.315	−0.27	+0.27	+0.18
2.	+34.77	−34.48	−0.31	−0.055	+23.55	−23.92	+0.37	+0.26	+0.35	−0.35	−0.15
3.	±131.35	∓139.63	±8.28	±4.14	±138.13	∓128.42	∓9.71	∓4.855	∓1.7	±1.7	±0.85
4.	±36.16	±39.10	∓75.12	∓75.66	±39.6	±20.79	∓60.53	∓60.99	±19.10	∓19.10	∓19.24
5.	±183 74	∓179.76	∓3.98	∓1.50	∓12.49	±10.86	±1.62	±1.21	∓0.27	±0.37	±0.31
6.	∓56.48	∓61.10	±117.32	±118.17	∓61.81	∓32.48	±94.54	±95.26	∓29.82	±29.82	+30.1
7.	+18.13	+24.19	−42.32	−42.43	+21.10	+12.04	−33.18	−33.48	+9.95	−9.93	−9.98
8.	−9.07	−12.10	+21.16	+21.22	−10.55	−6.02	+16.59	+16.74	−4.97	+4.97	+4.94
9.	557.78	236.56	221.96	219.54	350.43	104.53	180.19	177.80	61.34	54.82	54.84
10.	212.56	560.71	242.96	240.05	160.30	276.91	199.14	194.64	54.82	61.32	60.46

used in the example for computing moments in the upper slab due to non-uniform heating[O4.8] (Fig. 5.16).

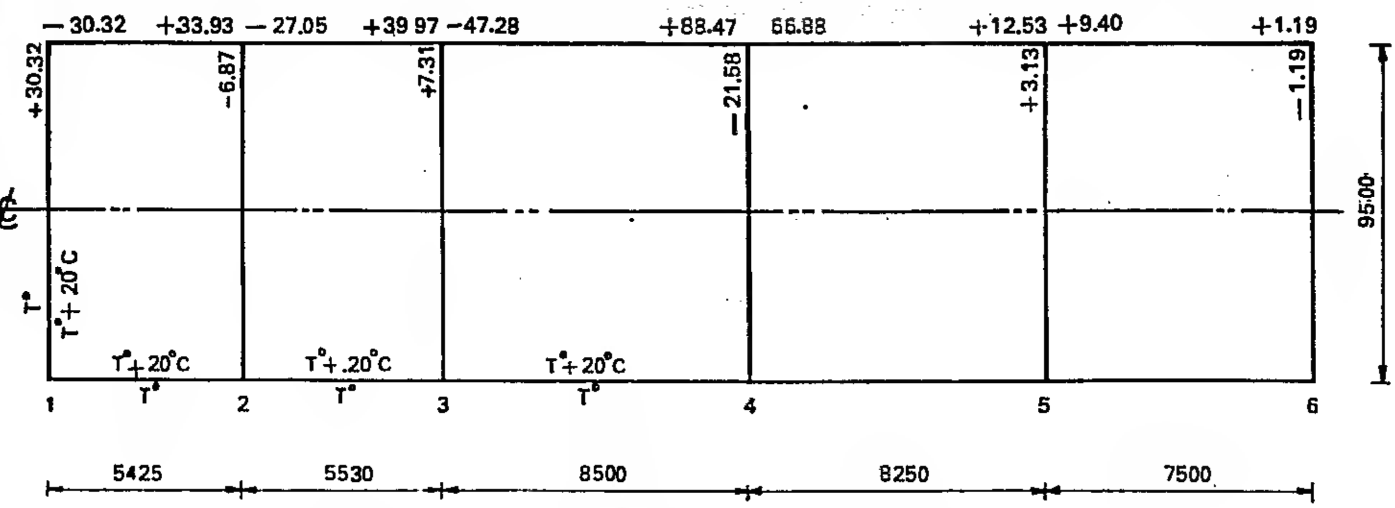


Fig. 5.16: Moments in Upper Table Due to Non-uniform Temperature.

Table 5.7 shows the moments induced in the frame members for the loading cases (i) to (viii) explained above. The net moments are evaluated in the same manner as for the cross-frame.

CHECK FOR TORSION

The torsional moments induced as a result of eccentric loading of the machine (static plus dynamic loads) on the frame members should be duly accounted for in the structural design.

f. Design of Foundation Slab

Table 5.8 shows the loads transferred to the foundation slab through the columns.

Table 5.8

LOADS TRANSMITTED TO BASE SLAB

Frame i	Concentrated load (P_i) (t)	Distributed load (Q_i) (t)	Load on each column including full weight of column (N_i) (t)	Vertical dynamic load on Beam ($\bar{F}_b$)	Vertical dynamic load on Column ($\bar{F}_c$)	Total load on each column of the frame: When the dynamic force acts vertically W_{i1} (t)	Total load on each column of the frame: When the dynamic force acts horizontally W_{i2} (t)
A	44.2	29.6	41.10	24	0	90.0	78.0
B	77.3	32.1	63.4	44	0	140.1	118.1
C	83.25	29.4	140.8	54	77	262.6	197.1
D	38.6	46.3	190.4	74	132.4	336.05	233.0
E	9.348	33.1	166.02	0	112.4	243.44	187.24
F	20.348	86.24	48.60	10	0	106.89	101.89
						1179.08	915.33

In Table 5.8

$W_{i1} = \left(\frac{P_i + Q_i}{2}\right) + \left(\frac{F_b + F_e}{2}\right) + N_i \qquad W_{i2} = \left(\frac{P_i + Q_i}{2}\right) + N_i$

CHECK FOR THE SOIL STRESSES

Area of foundation provided $= 37.105 \times 9.5$
$= 353.4 \text{ m}^2$

i. Longitudinal direction:

(a) Static loads *plus* vertical dynamic loads

The distance of centre of gravity of column loads in the x direction is

$$X_G = \frac{\Sigma W_i x_i}{\Sigma W_i}$$

$$= \frac{1}{2358.16}[(2 \times 90\,(1.0) + 2 \times 140.1\,(6.425) + 2 \times 262.6\,(11.955) + 2 \times 336.15\,(20.45) + 2 \times 243.44 \times 28.705) + 2 \times 106.894 \times 36.205)]$$

$= 18.54$ m

Eccentricity $(e) = 18.54 - 18.6025 = -0.063$

The end ordinates of soil stress $= \Sigma \frac{W_i}{A}\left[1 \pm \frac{6e}{L}\right]$

$$= \frac{2358.16}{9.5 \times 37.205}\left[1 \pm \frac{6\,(-0.063)}{37.205}\right]$$

$= 6.482\,[1 \mp 0.01]$

Maximum soil stress $= 6.739 \text{ t/m}^2 < (20 \text{ t/m}^2)$

Minimum soil stress $= 6.605 \text{ t/m}^2$

The pressure distribution diagram is as shown in Fig. 5.17.

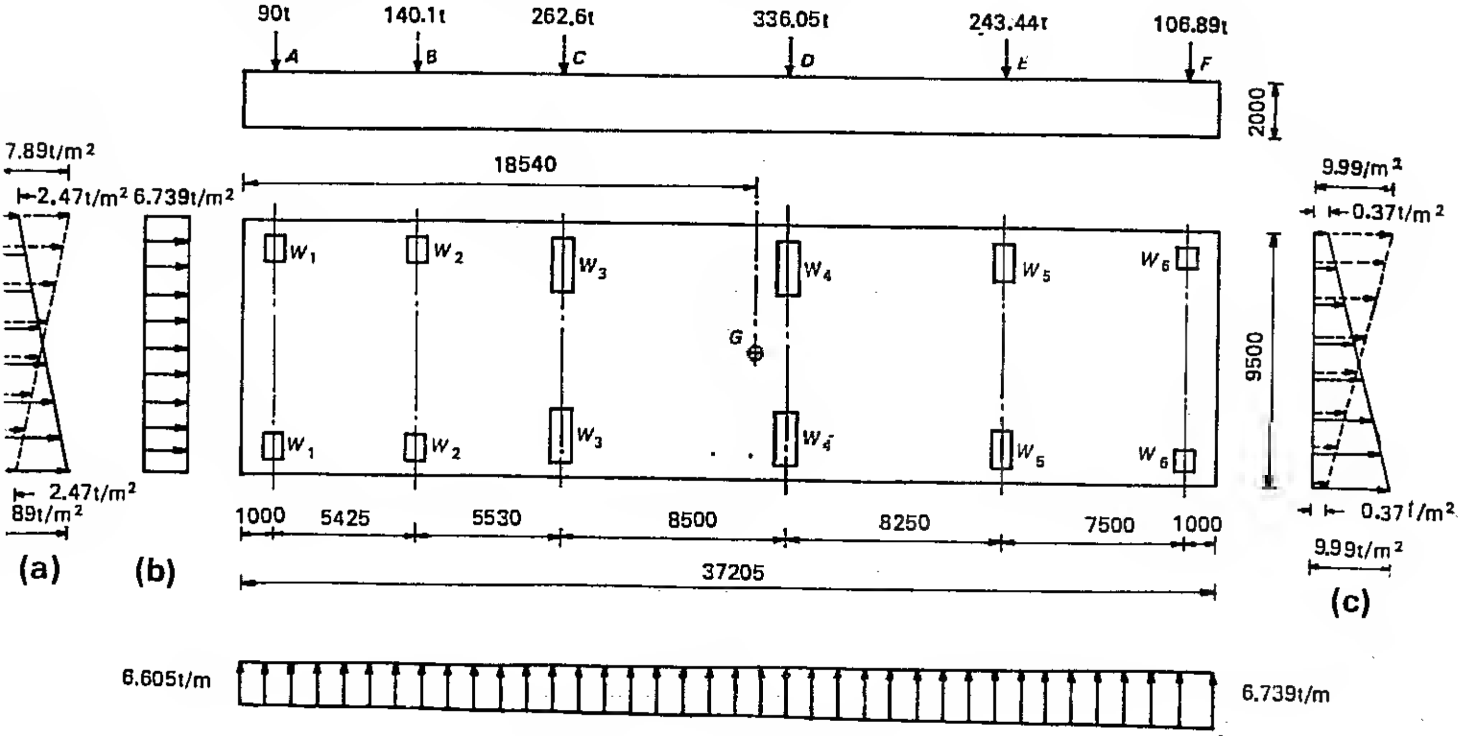


Fig. 5.17: Loading on Base Slab.

b. Static loads *plus* horizontal dynamic loads

$$X_G = \frac{1}{915.33} \begin{aligned}[t] &[78 \times 1 + 118.1 \times 6.425 + 197.1 \times 11.95 + 233 \times 20.455 + 187.24 \times \\ &\times 28.705 + 101.89 \times 36.205]\end{aligned}$$

$= 18.5951$ m

Eccentricity $(e) = 18.5951 - 18.6025 = -0.0074$

$$\text{Soil stress} = \frac{1830.66}{9.5 \times 37.205} \left[1 \pm 6 \times \frac{(-0.0074)}{9.5} \right]$$

Maximum soil stress $= 5.20$ t/m^2
Minimum soil stress $= 5.15$ t/m^2

ii. Cross-direction:

a. Static loads *plus* vertical dynamic load

When the dynamic force is acting vertically due to symmetry, the eccentricity in the cross-direction is zero. The intensity of uniform soil pressure is then 6.739 t/m^2 (Fig. 5.17b).

b. Static load *plus* horizontal dynamic loads

When, however, the dynamic force is acting in the transverse (horizontal) direction, the moment caused by these forces about the base should be taken into account.

Moment about the base $= \Sigma F_h . h$

where h is the height of the axis of the shaft above the base.

Since $F_h = 131.95$ and $h = 11.5$ m

the moment $= 131.95 \times 11.5 = 1517.43$ t.m

Total load (W) transmitted through the columns (Table 5.8)

$$= 2 \times 915.33 = 1830.66$$

$$\text{Eccentricity } (e) = \frac{1517.43}{1830.66} = 0.83 \text{ m}$$

$$\text{Soil stress} = \frac{P}{A}\left(1 \pm \frac{6\,e}{B}\right)$$

$$= \frac{1830.66}{353.4}\left[1 \pm \frac{6 \times 0.83}{9.5}\right]$$

$$= 7.89 \text{ t/m}^2;\ 2.47 \text{ t/m}^2$$

The variation of the soil pressure is shown in Fig. 5.17a.

c. Static load *plus* horizontal dynamic load *plus* earthquake load

The equivalent earthquake load F_E is taken as 0.06 times the total load on the top table (Table 5.2).

$$F_E = 0.06 \times 1710.43$$
$$= 102.63 \text{ t}$$

$$\text{Moment about the base} = (131.95 + 102.63)\ 11.5$$
$$= 2697.67 \text{ t.m}$$

$$\text{Eccentricity } (e) = \frac{2697.67}{1830.66} = 1.47 \text{ m}$$

Resultant stress on soil due to combined effect of dynamic load and earthquake force

$$= \frac{1830.66}{353.4}\left[1 \pm \frac{6\times 1.47}{9.5}\right]$$

$$= 9.99 \text{ t/m}^2;\ 0.37 \text{ t/m}^2$$

The maximum soil pressure is within the permissible limit. The variation of soil pressures in cross-direction is shown in Fig. 5.17c.

DESIGN

The foundation slab may be designed as a raft subjected to the soil pressures and loads shown in Fig. 5.17.

CHAPTER SIX

Foundations for Miscellaneous Machines

6.1 Rotary-Type Machines with Low Frequency

The machines under this category include rolling mills, crushing mills, pumps, motor generators and rotary compressors having operating speeds less than 1500 rmp.

6.1.1 Design Data

For the type of machines listed above the following information should be furnished by the machine manufacturers to the foundation designer.

a. Rolling Mills

i. Weight of rolling mill.
ii. Weight of motor driving the rolling mill. The weight of rotor and stator should be separately specified.
iii. The maximum torque on the shaft.
iv. Loads expected during erection and assembly of machine.

b. Crushing Mills

i. Weight of crusher and its various parts.
ii. Weight of motor.
iii. Speed of main shaft.
iv. Generating forces or data required for their evaluation.

c. Pumps

i. Weight of the pump.
ii. Speed of the pump.
iii. Frequency of pressure fluctuations in the discharge through the pump.
iv. Number of impeller vanes.

d. Grinding Mills

i. Characteristics of driving motor.
ii. Distance of the axis of drum from the upper plane of the foundation.
iii. Loading data consisting of the weight of mill casing, the ball charge and the weight of material to be ground.

e. Motor Generators

i. Weights of driving motor and generator.
ii. Weights of rotors—separately for motor and generator.
iii. Weight of fly-wheel.
iv. Operating speed.
v. Short-circuit force or moment.

Apart from this, for every machine a detailed lay-out diagram showing the position of anchor bolts, channels and other embedments in the foundation should be furnished along with the loading diagram showing the magnitude and points of application of all of the loads (static and rotating) to be considered in the design of foundation. As in the case of other types of foundation, the soil data which include the soil profile (bore log) upto a depth equal to three times the width of the foundation below the base (or upto hard strata, if hard soil is met with within that depth), dynamic shear modulus of soil and the level of ground water table in various seasons should be obtained from site investigations.

6.1.2 Design Criteria

Foundations for machines having low operating frequency should be so designed as to have a higher natural frequency. For the purpose of foundation design, the permissible limit of the amplitude may be taken from Fig. 1.4. IS 2974 (Part IV)-1968 recommends a value of 0.3 mm as a general guidance to the designer.[C4.4]

It is recommended that all bearings of the main shaft be located on one common foundation. This avoids unequal settlement which may cause distortion of the machine shaft. When several machines are to be accommodated on one long and continuous mat (which may be necessitated by soil conditions and close spacing between the machines themselves), the dynamic analysis of the foundation may be carried out independently as explained in Section 4.4 for reciprocating engines. The permissible limit of amplitude may then be increased by about 30 per cent.

To avoid transmission of vibrations to adjacent parts, it is desirable to provide suitable isolation between the machine foundation and adjacent structures. Where it becomes unavoidable to support minor structural elements of adjacent structures on the machine foundation, suitable measures should be taken to make the connection resilient by introducing gaskets made of rubber, cork, etc.

Foundations of low-frequency rotary-type machines maybe of either block-type or framed-type, depending on the operating floor level, positioning of auxiliary equipment, etc. For preliminary dimensioning, the weight of foundation may be taken as 2.5 times the weight of the machine.[C4.4]

6.1.3 Principles of Design

a. Rolling Mill

Rolling mills generally have the following three main components:

i. Driving motor, ii. Motor generator set, and iii. Roller stand and gear box.

i. *Driving motor:* This is a DC motor which controls the speed of rollers. It usually has a block foundation, the analysis for which may be carried out as explained in Section 4.3.

As the bloom enters or leaves the rollers, the dynamic effect gives rise to a torque on the shaft of the rotor. This causes the foundation to rotate. If θ is the amplitude of rotation caused by this dynamic torque, the maximum stress on the soil will be given by[C4.4]

$$\sigma_{max} = \frac{W}{A} + C_\theta b\theta \tag{6.1}$$

where W is total weight of machine and foundation, b is half the length of the foundation in the plane of torque and A is base area.

As a conservative measure, the dynamic torque may be taken in the design as an equivalent static moment equal to twice the normal torque suggested by the manufacturers.

ii. *Motor generator set:* The motor generator set supplies direct current to the motor of the rolling mill. The following loads should be considered in design:

a. Constructional loads during assembly.
b. Weight of machine multiplied by a dynamic factor of two.
c. A torque (T) given by[C4.4]

$$T = \frac{2\pi\varphi}{60}\frac{dV}{dt} \tag{6.2}$$

where φ is mass moment of inertia of all rotating masses of the set and V is the speed; $\frac{dV}{dt}$ is the rate of change of speed which varies in practice from 2.8 to 10.4 and is generally specified by manufacturers. This torque shall be used with a dynamic factor of two.

If the foundation is of the framed type, the design may be carried out on the lines explained in Section 5.4.3.

iii. *Roller stand and gear box:* The purpose of the roller stand is to support the bearings of rollers, and the forces arising during rolling are transmitted by it to the foundation. The gear box contains the gears which drive the rollers.

The gear box is subjected to a torque which is of the same order of magnitude as that of the shaft of the driving motor (Equation 6.2). The foundation for the gear box, if provided separately, should be designed in the same way as for the driving motor.

If the roller stand, gear box and driving motor are supported on a common mat, only the load of the machinery and the weight of the foundation need be considered.

The following loads should be considered in design of foundation for rolling mills:

i. Weight of rolling mill equipment multiplied by two.
ii. Maximum moment on the motor shaft.
iii. Weight of driving motor multiplied by two.
iv. Horizontal force transmitted to the foundation during its working.
v. Constructional loads.

The analysis of foundation consists of the following steps:

i. Stress analysis of individual units of foundation such as those weakened by openings, cantilevers, etc.

ii. Computation of pressures transmitted to the soil.

In the above calculations, the weight of the rolling mill equipment and the weight of driving motor should be multiplied by a dynamic coefficient of two.[C4.4]

b. Crushing Mills

Two types of crushing mill commonly encountered are: (1) jaw crushers, and (2) cone crushers.

In both types, forced vibrations are caused by the unbalanced inertial forces. Where framed-type foundations are adopted, the analysis may be carried out as explained in Section 5.4.3.

The unbalanced inertial forces in vertical $(F_m)_z$ and horizontal $(F_m)_x$ directions for a jaw crusher are given in Table 6.1 for various schematic arrangements.

The following notation is used in the expressions given in Table 6.1

m_b Mass of moving jaw

m_c Mass of the connecting rod

m_0 Mass of the eccentric or half of the mass of the crank shaft

m_d Mass of the counter weights

r Distance between the centre line of the shaft and the pivot of the arm

r_1 Distance between axis of rotation and centre of gravity of counter weights

ω_m Angular velocity of machine

The unbalanced inertial force for a cone crusher is given by[C4.4]

$$F_m = (m_1 r_1 - m_2 r_2)\,\omega^2 \tag{6.3}$$

where

m_1 is total mass of main shaft and crushing cone attached to it

m_2 is mass of cam shaft and units such as gears, counter weights, etc. rigidly connected with it

r_1 is distance between shaft of the crusher and centre of gravity of the main shaft

r_2 is distance between the shaft of the crusher and centre of gravity of cam shaft

ω is angular frequency of the cam shaft

This force acts in all directions in the plane of rotation.

The loads to be considered in the design of foundation are:

i. The weight of machine multiplied by five[C4.4]

ii. Weight of foundation

iii. Constructional loads, if any

iv. The unbalanced inertial force (Eq. 6.3)

The analysis may be carried out as explained in Sec. 4.3.

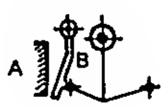
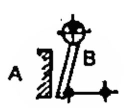
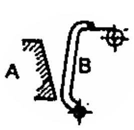
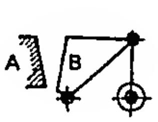
c. Pumps

Vibration is not a major problem in the design of foundations for pumping installations. Pumps are generally operated at low speeds. However, sometimes due to insufficient clearance between the impeller and casing, the pressure surges increase and the ensuring wave propagates through the water to the casing and the foundation. The frequency of such induced vibration in cps is given by.[C4.4]

Table 6.1

UNBALANCED INERTIAL FORCES IN JAW CRUSHERS

(After, Major, A., *Vibration Analysis and Design of Foundations for Machines and Turbines,* Akademiai Kiado, Budapest, Collet's Holdings, London, 1962; with permission).

Type	Schematic Arrangements of Jaw Crushers		Approximate Values of Mass Forces
1		I	$(F_m)_z = (m_0+m_c)\, r\omega^2 \sin \omega t$ $(F_m)_x = (m_0+0.8\, m_c)\, r\omega^2 \cos \omega t$
		II	$(F_m)_z = [(m_0+m_c)\, r - m_d r_1]\, \omega^2 \sin \omega t$ $(F_m)_x = 0.25\, m_b r\omega^2 \sin \omega t$
2		I	$(F_m)_z = (m_0+m_b)\, r\omega^2 \sin \omega t$ $(F_m)_x = (m_0+0.5\, m_c)\, r\omega^2 \cos \omega t$
		II	$(F_m)_z = [(m_0+m_b)\, r - m_d r_1]\, \omega^2 \sin \omega t$ $(F_m)_x = [(m_0+0.5\, m_b)\, r - m_d r_1]\, \omega^2 \cos \omega t$
3		I	$(F_m)_z = (m_0+0.7\, m_c)\, r\omega^2 \sin \omega t$ $(F_m)_x = (m_0+m_c+0.5\, m_b)\, r\omega^2 \cos \omega t$
		II	$(F_m)_z = 0$ $(F_m)_x = [(m_0+m_c+0.5\, m_b)\, r - m_d r_1]\, \omega^2 \cos \omega t$
4		I	$(F_m)_z = (m_0+m_c)\, r\omega^2 \sin \omega t$ $(F_m)_x = (m_0+0.8\, m_c)\, r\omega^2 \cos \omega t$
		II	$(F_m)_z = [(m_0+m_c)\, r - m_d r_1]\, \omega^2 \sin \omega t$ $(F_m)_x = 0.25\, m_b\, \omega^2 \sin \omega t$
5		I	$(F_m)_z = (m_0+0.7\, m_c)\, r\omega^2 \sin \omega t$ $(F_m)_x = (m_0+m_c+0.5\, m_b)\, r\omega^2 \cos \omega t$
		II	$(F_m)_z = 0$ $(F_m)_x = [(m_0+m_c+0.5\, m_b)\, r - m_d r_1]\, \omega^2 \cos \omega t$
6		I	$(F_m)_z = (m_0+m_c+0.5\, m_b)\, r\omega^2 \sin \omega t$ $(F_m)_x = (m_0+0.7\, m_c+0.5\, m_b)\, r\omega^2 \cos \omega t$
		II	$(F_m)_z = [(m_0+m_c+0.5\, m_b)\, r - m_d r_1]\, \omega^2 \sin \omega t$ $(F_m)_x = [(m_0+0.7\, m_c+0.5\, m_b)\, r - m_d r_1]\, \omega^2 \cos \omega t$
7		I	$(F_m)_z = (m_0+m_c)\, r\omega^2 \sin \omega t$ $(F_m)_x = (m_0+0.8\, m_c)\, r\omega^2 \cos \omega t$
		II	$(F_m)_z = [(m_0+m_c)\, r - m_d r_1]\, \omega^2 \sin \omega t$ $(F_m)_x = 0.25\, m_b r\omega^2 \sin \omega t$

A—Fixed jaw
B—Movable jaw
I—Without counter weights
II—With counter weights

$$f = \frac{Nn}{60} \tag{6.4}$$

where N is speed of rotation of pump in rpm and n is the number of impeller vanes.

The foundation should be so designed as to avoid resonance with this operating frequency.

d. Grinding Mills

Tube mills and drum mills are examples of this type. For the design of mill foundations, dynamic analysis is generally not necessary. However, it is necessary to check the soil stresses underneath the foundation. For checking the soil stress, both the direct load due to the weight of machine and the weight of foundation and the moment caused by the horizontal component of the centrifugal force acting at a certain height above the base of the foundation should be taken into account. The horizontal component of centrifugal force acting in the direction of motion is taken as $0.1W$ and $0.2W$ respectively for short drum type and tube-type grinding mills, where W is the weight of the mill (excluding the ball charge) and the material to be ground.[C4.4]

In case of poor soils ($\sigma_p < 1.5$ kg/cm^2) it is recommended that the entire mill be placed on a common foundation. If soil is favourable, separate foundations may be provided for both the intake and discharge ends of the mill. The motor and gear box should, however, be placed on a common foundation.

The design of a foundation for a tube mill has been illustrated in Section 6.8.

e. Motor Generators

Block or framed-type of foundations are adopted for these machines whose principles of analysis are already given in the preceding chapters. The loads to be considered in design are given under rolling mills (item ii, Section 6.1.3a).

6.2 Machine Tools

For foundations of machine tools, the dynamic analysis is usually not needed. Consideration of static loads with due margin of safety is generally sufficient. Examples of this type are lathes and milling, drilling and boring machines. For some of the precision machines, it may be necessary to protect the machine foundation against external vibrations such as those caused by hammers and compressors located nearby. The foundation in such cases should be mounted on suitable isolating layers in order to limit the amplitude of motion caused by the vibration of floor itself. The theory of passive isolation applicable in such cases is given in Chapter 7.

6.2.1 Design Data

a. Magnitude and position of loads
b. Weight of material to be processed on the machine tool
c. Layout diagram showing the holes for foundation bolts, recesses, channels, etc. if any, needed in the foundation body
d. Soil data including bearing capacity
e. Any special requirements in connection with foundation.

6.2.2 Design Criteria

Medium type machine tools such as lathes, milling machines, drilling and boring machines

may be rigidly bolted to the concrete floor, which should be at least 20 cm thick. If separate foundations are provided, the depth of the foundation block may be taken as $0.5\sqrt{L}$, where L is length of foundation in metres.[01.12]

6.3 Impact-Type Machines other than Hammers

Examples are forging and stamping presses and pig breakers. The general guidelines for design of foundations of this group of machines are given below:

6.3.1 Forging and Stamping Presses

a. Data Required for Design

- i. Layout drawing of the installation showing position of anchor bolts, embedded pieces, etc.
- ii. Thrust exerted by the press.
- iii. Stroke of the press.
- iv. Weight of the press equipment.
- v. Maximum weight of material to be forged.
- vi. Load-time relationship of the pulse or dynamic overloading due to forging or stamping.
- vii. Dynamic torque in horizontal plane (in case of friction presses).
- viii. Soil data.

b. Principles of Analysis and Design

Three types of stamping press are commonly adopted:

i. Hydraulic-type presses, ii. Eccentric-type presses, and iii. Friction-type presses.

The dynamic effect produced by hydraulic presses is very little. Dynamic analysis may, therefore, be omitted for these machines. IS 2974-Pt. V recommends a dynamic overload factor of 2 and that twice the weight of the material to be forged should be considered in design. In stamping presses, the dynamic overload is caused by the drop of the upper ram on the forge piece. In case of friction-type presses, the dynamic torque in the

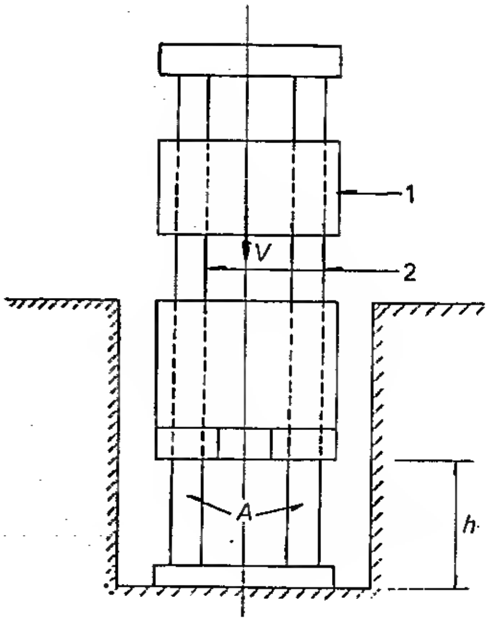


Fig. 6.1: Hydraulic Press—(*1*) Cross Head, (*2*) Anchor Columns.

horizontal plane of the foundation should also be considered.

In the case of hydraulic presses of large capacities, the dynamic overloading can be considered as follows.[C1.15] If h and A denote the height and cross-sectional area of anchor columns (Fig. 6.1), the elastic compression δ of these columns is given by

$$\delta = Wh/EA \tag{6.5}$$

where W is the weight of the machine minus weight of the moving cross-head.

The dynamic factor η is given by[C1.15]

$$\eta = \frac{v}{\sqrt{g\delta}} \tag{6.6}$$

where v is velocity of impact.

The dynamic force P is given by

$$P = \xi\eta W \tag{6.7}$$

where ξ is the fatigue factor, which may be taken as 1.5 in this case.

In the case of large eccentric presses (Fig. 6.2), other forms of dynamic load also occur. As seen from the Fig. 6.2, the operation of the press causes an impact moment M accompanied by a centrifugal force P. The latter acting at a height causes an additional moment on the foundation.

If the period of impact is not given, Rausch[C1.15] suggests that five times the normal torque M and five times the normal centrifugal force P be considered as an equivalent statically applied moment and load respectively.

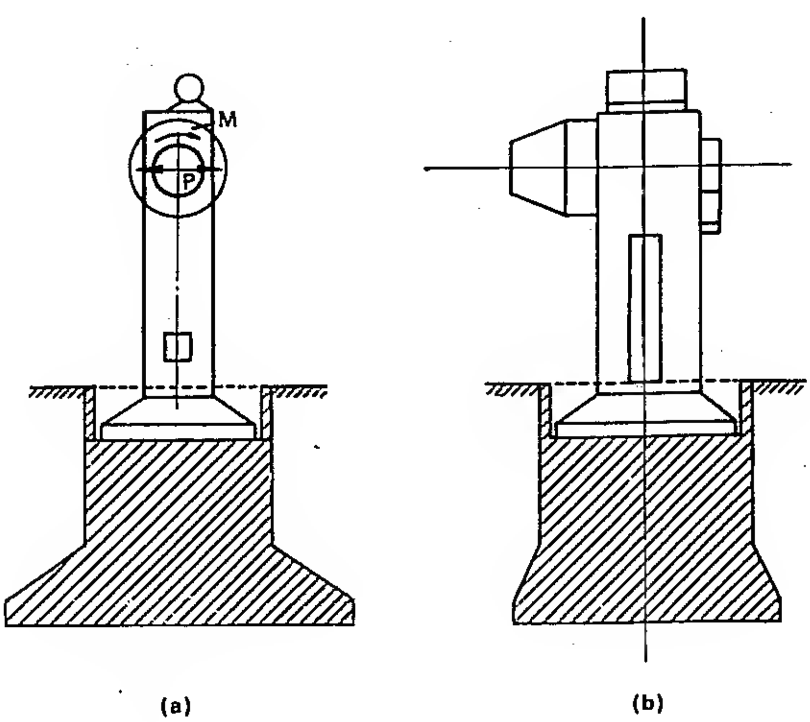


Fig. 6.2: Eccentric Press—(a) Front View, (b) Side View.

6.3.2 Pig Breakers or Drop-Weight Crushers

These are characterized by the very high impact energy caused by the fall of ram. To avoid damage to the environment due to vibration, these machines should be located as far away as possible from structures susceptible to vibration and sensitive equipment. The minimum distances suggested are given in Table 6.2.

Table 6.2

SAFE DISTANCES FROM DROP-WEIGHT CRUSHERS

[After IS: 2974 (Pt. V)-1970]

Type of soil	Safe distance in metres for a ram weight (in tonnes)		
	<3 t	3-7 t	>7 t
1. Rocky soil	20	30	50
2. Dry sandy soil, clay, sandy clay	30	40	60
3. Water-logged soil	50	80	100

a. Data Required for Design

i. Layout and outline drawing of whole installation showing also the position of anchor bolts and other embedded parts.
ii. Weight of scrap to be crushed.
iii. Weight of ram and its height of fall.
iv. Construction loads.

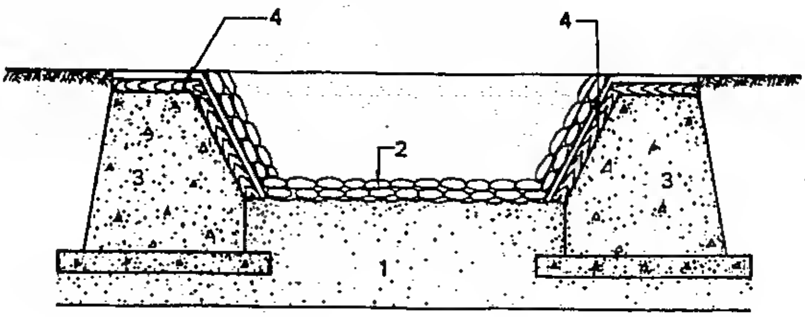
b. Principles of Design

Fig. 6.3a shows one type of foundation for a crusher platform. To avoid damage due to impact, the top surface may be covered with blast furnace slag. To increase the crushing efficiency, a layer of steel ingots with interstices filled with fine scrap may be placed between the sand and slag layers. The anvil of the crusher may also be placed on an RCC block in a rectangular or circular well as shown in Fig. 6.3b.

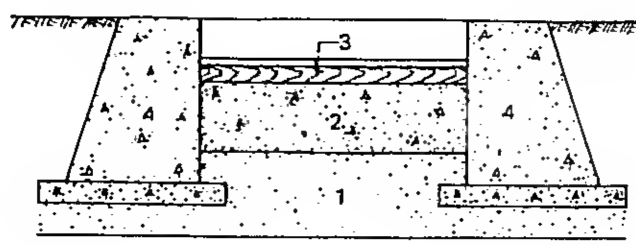
The design criteria for such a foundation are similar to those for a hammer foundation given in Section 4.5.3. The permissible amplitude of foundation may be taken as 1.2 mm. It is recommended that suitable arrangement be made for intercepting the flying chips from crusher platform. For this purpose, timber battens with rubber lining inside may be suspended on hinges from a metallic ring installed above the well. The battens may be tied at intervals by a rope.

6.4 Fans and Blowers

A fan consists of a rotor with several blades so arranged as to cause an axial flow of air, whereas in a blower the air flows in axial direction and then radiates out in all directions. Machines of this type operate in wide range of speeds depending on size and service loads. In general, the operating speed decreases as the size of the fan or blower increases. In this case the predominant sources of vibration are rotor imbalance and forces in connecting belts as well as the motor impulse. The machine manufacturers furnish the data necessary for the calculation of exciting forces.



(a)



(b)

Fig. 6.3: Types of Foundation for a Drop Weight Crusher—**(a)**(*1*) Compact Sand, (*2*) Slag, (*3*) Side Retaining Walls, (*4*) Timber Lining; **(b)** (*1*) Compact Sand, (*2*) Concrete Block, (*3*) Timber Layer, (*4*) Retaining Walls.

Generally block foundations are provided for such machines and for the analysis, the method recommended in Section 4.3 may be employed.

6.5 Looms

The principal components of a cloth-weaving loom are the lay and the shuttle. The lay, which is a relatively heavy member undergoing horizontal reciprocating motion, is driven by a pair of cranks and connecting rods. A "shuttle" travels alternately in opposite directions across the lay from one shuttle box to another. The two principal sources of vibration and shock resulting from the operation of a loom are:

a. The inertial force created by the reciprocating motion of the lay. This is a harmonic force acting in the horizontal direction and the reaction upon the frame of the loom is at the level of the crank shaft.

b. The force that propels the shuttle in the form of an impact. The complexity of mechanism employed for this purpose makes the exact nature and direction of this impact uncertain.

When looms are installed on the upper floors of mills, the entire building may sway at the frequency of loom motion. The floors supporting the looms are prone to sway and bending under the influence of the moment resulting from the lay force acting on the loom frame at the height of the crank shaft. The natural frequencies of the floors should be well

away from the operating speed of the loom and the building should be designed adequately for the calculated dynamic (or equivalent static) forces.

6.6 Testing Machine with Pulsator

6.6.1 Design Procedure

A testing machine connected with pulsator (which is generally used for fatigue tests) operates in a range of frequencies (usually 300–1500 rpm) and not at one particular frequency like other machines. The supporting structure for this machine should be designed, therefore, to avoid resonance in the entire range of operating frequencies. The natural frequency of the foundation should be at least 20 per cent below the lower limit or 20 per cent above the upper limit of the operating range. To obtain low natural frequencies it may be necessary to mount the foundation on springs or on rubber discs. The sum of the unbalanced forces due to the working of the pulsator and the testing machine should be considered in the design of the foundation. The unbalanced force P is given by

$$P = \frac{W}{g} S\omega_m^2 \tag{6.8}$$

where W is the moving weight, S is the stroke and ω_m^2 is the operating frequency (sec^{-1}).

The maximum dynamic factor (μ) is given by

$$\mu = \frac{f_n^2}{f_n^2 - f_m^2} \tag{6.9}$$

where f_m is the operating speed nearest to the natural frequency (f_n). The foundation is analysed as explained in Section 4.4.

An example has been worked out in Section 6.8 illustrating the design of a block foundation for a testing machine with pulsator.

6.7 Machines Installed on Building Floors

Machines such as small electric motors, textile machines and machine tools which do not induce excessive vibration may be set directly on the building floors. However, during the installation, the following points should be considered:

i. Suitable vibration isolating layers should be provided under the base plate of the machine in order to reduce the transmission of dynamic forces, even if they are negligible. One such arrangement is shown in Fig. 6.4. Use of rubber washers is also recommended between the machine stand and the foundation bolts. Fig. 6.5 shows an arrangement for fixing the machine leg to the floor.

ii. The natural frequency of the building floor should be well away from the lowest operating frequency of the machine. Accurate determination of natural frequency of building is difficult due to many influencing factors involved. Therefore, for the determination of natural frequencies, experimental data are most reliable in existing cases.

iii. Where the environment is sensitive to vibrations the machine should be placed on soft springs made of steel or rubber. The machine may either be placed directly on these resilient supports (if the machine is rigid enough) or mounted on another reinforced concrete block which rests on springs placed underneath it.

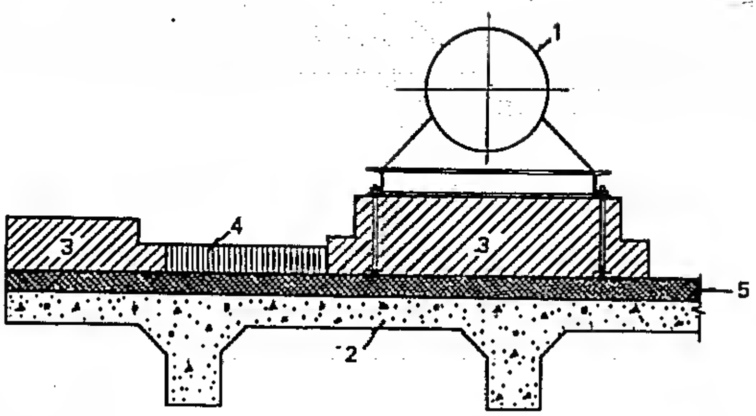


Fig. 6.4: Setting Machine on Building Floor (After, Major, A., *Vibration Analysis and Design of Foundations for Machines and Turbines*, Akademiai Kiado, Budapest, 1962; with permission)—(*1*) Machine, (*2*) Floor Slab, (*3*) Concrete Block, (*4*) Lean Concrete, (*5*) Vibration Absorbing Layer.

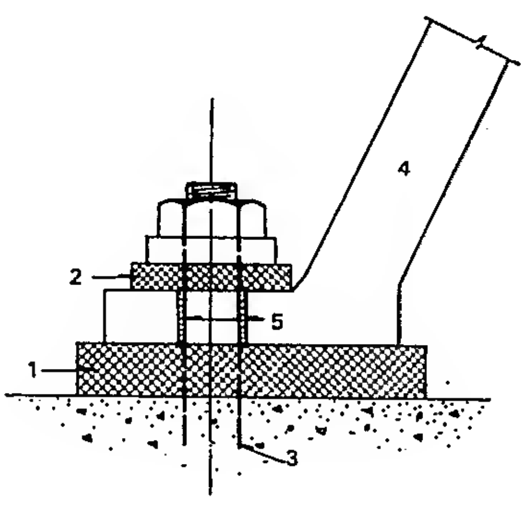


Fig. 6.5: Fixing Machine Leg on Floor—(*1*) Rubber Block, (*2*) Rubber Washer, (*3*) Anchor Bolt, (*4*) Machine Leg, (*5*) Rubber Sleeve.

iv. Reinforced concrete floors should be protected from chemical attack occurring due to leakage of machine oil, etc. Vitreous glazed tiles may be used to cover the top surface of the machine hall. Oil traps or ducts should also be provided to avoid accumulation of oil on the floor.

6.8 Numerical Examples

1. Design of Foundation for a Tube Mill

A. DATA

The schematic arrangement of the foundation for a tube mill is shown in Fig. 6.6. The site investigation had given the value of permissible stress of soil as 1.5 kg/cm^2. The data of the machine are as follows:

i.	Weight of the cylindrical tube without charge (W_t)	80 t
ii.	Weight of steel balls (W_b)	40 t
iii.	Maximum weight of material to be pulverized (W_p)	8 t
		128 t

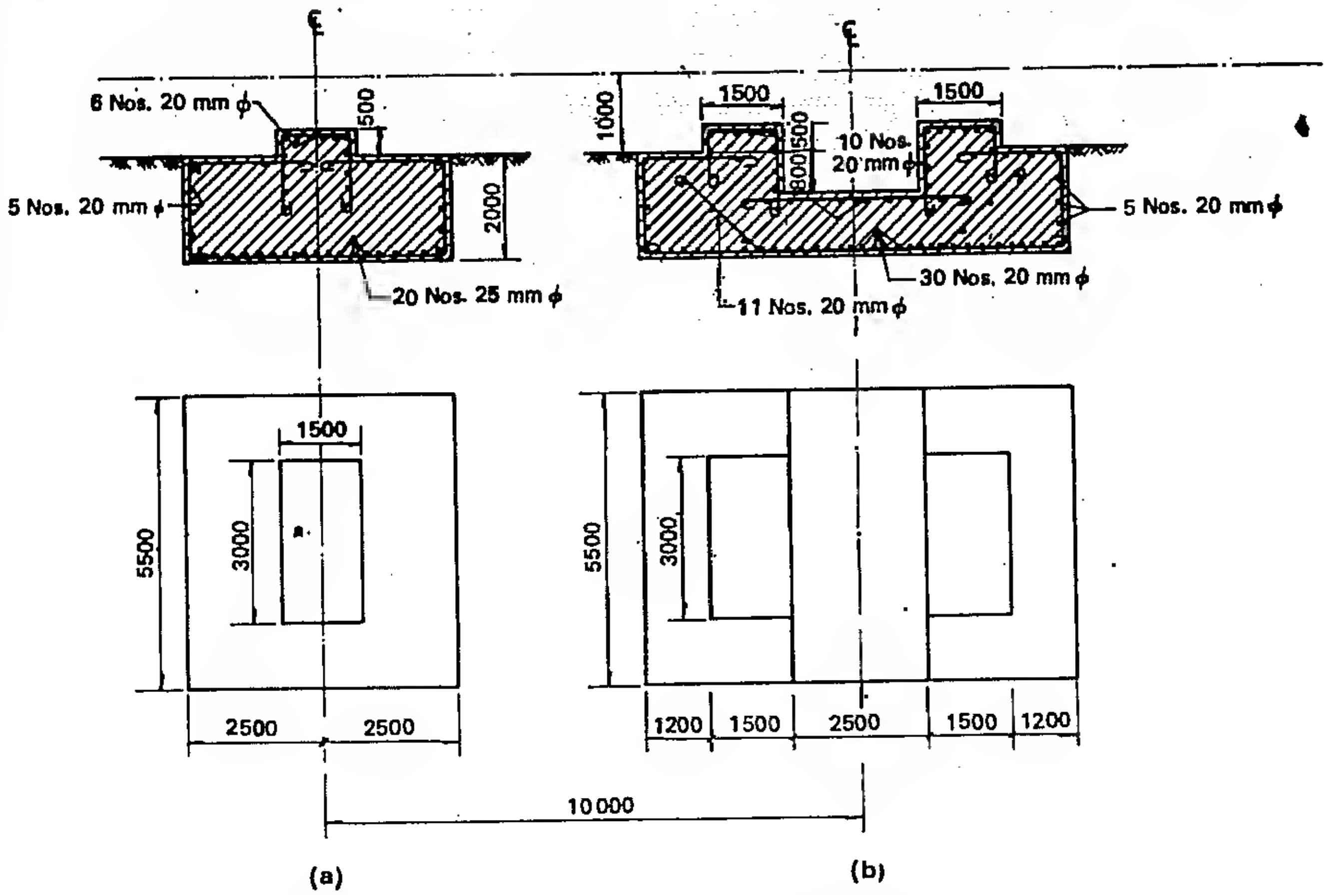


Fig. 6.6: Foundation for a Tube Mill—**(a)** Inlet End, **(b)** Discharge End.

Weight of the machinery at the discharge end, including motor and gear-box assembly (W_m) 30 t

The inlet and discharge ends of the mill are provided on separate foundations as shown in Fig. 6.6.

B. ANALYSIS

i. Analysis of foundation at the inlet end:

a. Static load

Weight of machine $= \frac{1}{2} \times 128 \text{ t} = 64 \text{ t}$

Weight of foundation block (Fig. 6.6) $= 5.0 \times 5.5 \times 2.0 \times 2.4 + 3.0 \times 1.5 \times 0.5 \times 2.4$

$= 137.4$ t

Total static weight $= 201.4$ t

Base area provided $= 5 \times 5.5 = 27.5 \text{ m}^2$

Stress due to static load on the soil (σ_{st}) $= 201.4/27.5 \text{ t/m}^2 = 0.7323 \text{ kg/cm}^2$

b. Dynamic load

The horizontal centrifugal force $= 0.2\ W_t$

$$P_x = 0.2 \times \frac{80}{2} = 8 \text{ t}$$

The dynamic moment about the base (M_d) $= 8 \times 3 = 24$ t.m

The stress induced at the base due to dynamic moment (σ_d) $= \dfrac{24}{\frac{1}{6} \times 5.0 \times (5.5)^2}$ t/m² $= 0.0952$ kg/cm²

Stresses on soil $= \sigma_{st} + \sigma_d$

$= 0.7323 \pm 0.0952 = 0.8275$ kg/cm² and 0.6371 kg/cm²

The maximum stress on soil is thus within the permissible limit.

ii. Foundation at the discharge end

a. Static loads

Weight of the machinery at the discharge end $= 64 + 30$

$= 94$ t

Self-weight of foundation (Fig. 6.6) $= 7.9 \times 5.5 \times 2.0 \times 2.4 + 2(1.5 \times 3.0 \times 0.5 \times 2.4) - (2.5 \times 5.5 \times 0.8 \times 2.4)$

$= 208.56 + 10.8 - 26.4 = 193$ t

Base area $= 7.9 \times 5.5 = 43.45$ m²

$$\sigma_{st} = \frac{94 + 193}{43.45}$$

$$= \frac{287}{43.35} = 6.605 \text{ t/m}^2$$

i.e., 0.66 kg/cm²

b. Dynamic load

The moment due to dynamic load at the base $= 24$ t.m

$$\sigma_d = \frac{24}{1/6 \times 7.9 \times (5.5)^2}$$

$= 0.06$ kg/cm²

The stresses on soil $= \sigma_{st} \pm \sigma_d$

$= 0.66 \pm 0.6$ kg/cm²

$= 0.72$ or 0.6 kg/cm²

< 1.5 kg/cm² (safe)

c. Design

Having obtained the variation of reactive pressures from soil, the structural design of the foundation may be carried out as in a normal raft. Although the structural requirements do not necessitate the provision of reinforcing steel in the foundation, nominal reinforcement consisting of 16–20 mm diameter bars at 25 cm centres are provided on all faces of the foundation block.

Fig. 6.6 shows the disposition of reinforcement in the foundation.

2. Design of a Foundation for a Testing Machine with Pulsator

A. DATA

i. Weight of machine complex 8.3 t (total)

ii. Permissible bearing capacity of soil (broadly classified as sandy)	1.5 kg/cm^2
iii. Data for unbalanced forces in machine	
Moving weight of pulsator (W_p)	45 kg
Stroke length (S_p)	$\pm$3.5 cm
Moving weight of testing machine (W_t)	700 kg
Stroke length (S_t)	$\pm$0.5 cm
Operating frequency	From 300–750 cps
iv. Permissible amplitude	=0.5 mm

B. DYNAMIC ANALYSIS

This foundaton is analysed primarily for vertical vibrations.

Base area (A_f) provided, as suggested by the machine manufacturers $=2.7 \text{ m} \times 2.84 \text{ m} = 7.668 \text{ m}^2$

Height of the block $=1.2$ m (assumed)

C_z corresponding to an area of 10 m^2 and $\sigma_p = 1.5$ kg/cm^2 (Table 3.3) $=3$ kg/cm^3

C_z corresponding to actual area

$$= \frac{3 \times \sqrt{10}}{\sqrt{2.7 \times 2.84}} = 3.426 \text{ kg/cm}^3$$

Effective spring constant (K_z)

$$=C_z A_f$$
$$=3.426 \times 2.7 \times 2.84 \times 10^4 \text{ kg/cm}$$
$$=26.271 \times 10^4 \text{ kg/cm}$$

Weight of foundation $=2.7 \times 2.84 \times 1.2 \times 2.4 = 22.083$ t

Weight of machine $=8.30$ t

Total weight $=30.383$ t

Corresponding mass

$$= \frac{30.383 \times 10^3}{981} = 30.97 \text{ kg.sec}^2/\text{cm}$$

Vertical natural frequency in cpm

$$= \frac{1}{2\pi} \times 10^2 \sqrt{\frac{26.271}{30.97}} \times 60$$
$$=880 \text{ cpm}$$

Max. operating frequency of machine $=750$ cpm

Frequency margin $=130 \text{ cpm} \approx 20\%$

Unbalanced forces (P) in

i. Testing machine

$$\frac{W_t}{g} \times S_t \times \omega_m^2$$
$$= \frac{700}{981} \times 0.5 \left(\frac{2\pi \times 750}{60}\right)^2 = 2.2 \text{ t}$$

ii. Pulsator

$$\frac{W_p}{g} \times S_p \times \omega_m^2$$
$$= \frac{45}{981} \times 3.5 \left(\frac{2\pi \times 750}{60}\right)^2 = 0.99 \text{ t}$$

Total unbalanced force (P) $=2.20+0.99 \text{ t} = 3.19$ t

Dynamic factors (μ):

CASE 1: $f_n = 880$ cpm; $f_m = 300$ cpm

$$\mu_1 = \frac{f_n^2}{f_n^2 - f_m^2}$$

$$= \frac{880^2}{880^2 - 300^2} = 1.131$$

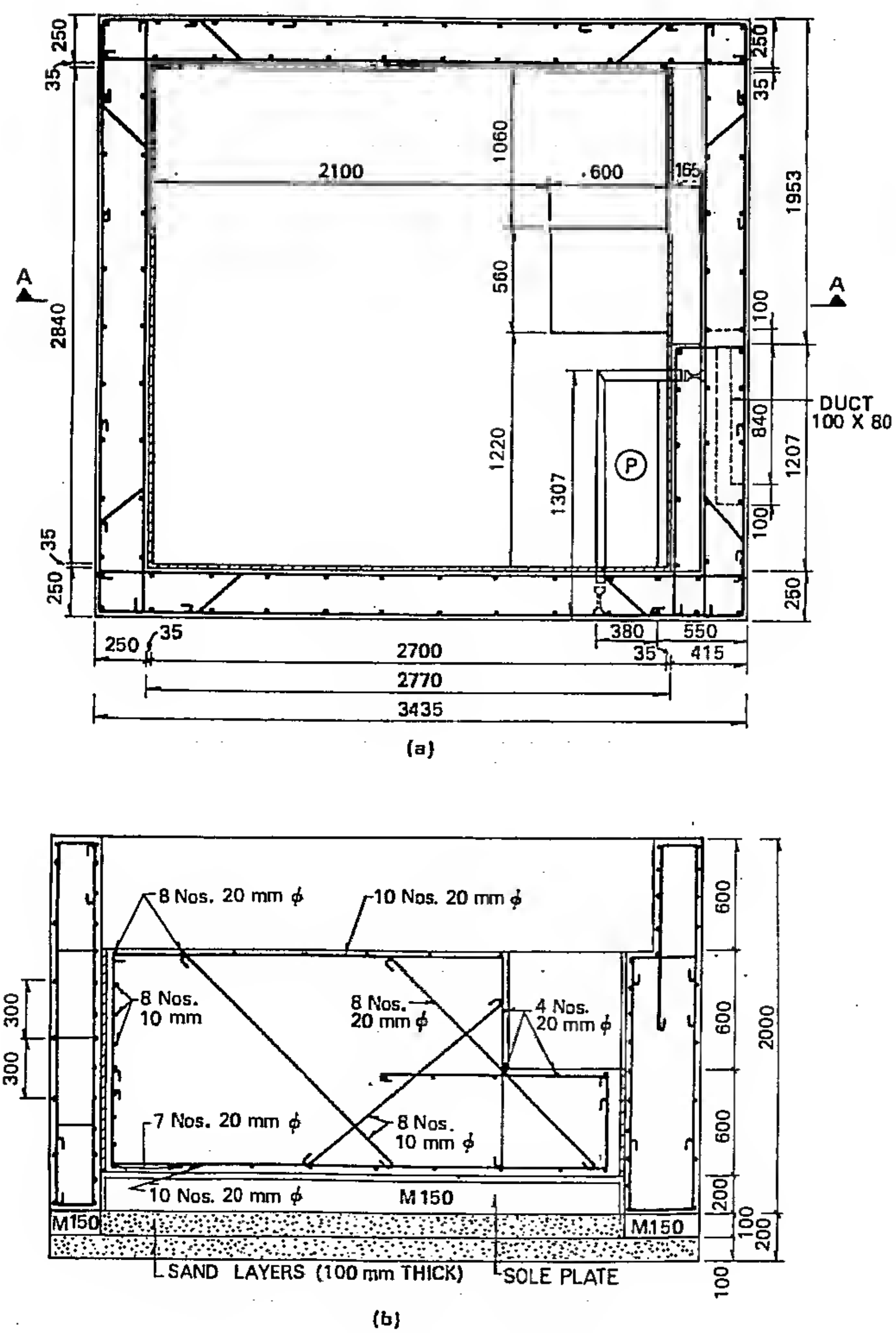


Fig. 6.7: Pulsator Foundation—(a) Plan, (b) Section at *A-A*.

CASE 2: $f_n=880$ cpm; $f_m=750$ cpm

$$\mu_2=\frac{(880)^2}{880^2-750^2}=3.655$$

The latter governs the design:

Static equivalent force $(F)=\xi\,\mu\,P$

where ξ is fatigue factor which may be assumed as three

Substituting $F=3\times 3.655\times 3.19$

$=34.98$ t

Total load on soil $=$Static$+$dynamic

$=30.383+34.98=65.37$ t

Stress on soil $=\dfrac{65.37\times 10^3}{270\times 284}=0.852$ kg/cm^2 (safe)

Amplitude $y=\left(\dfrac{P}{K}\right)\mu=\dfrac{3.19\times 10^3\times 3.655}{3.426\times 7.688\times 10^4}=0.04$ cm

This is less than the permissible value of 0.5 mm.

C. STRUCTURAL DETAIL

Fig. 6.7 shows the structural details of the foundation. The portion marked (P) in the figure shows the detail of a bracket support necessary for the control panel of the pulsator.

The foundation is separated from the surrounding retaining walls by a layer of insulation boards 35 mm thick (shown hatched in Fig. 6.7).

CHAPTER SEVEN

Vibration Isolation

IF A MACHINE is rigidly bolted to the floor, the vibratory movement of the machine itself may be reduced, but the vibration transmitted to the floor will be large. This may produce harmful effects even at large distances. On the other hand, if a flexible support is provided under the machine or its foundation, the vibration transmitted to the floor will be considerably reduced, but this may cause significant motion to the machine itself during normal operation or during the starting and stopping stages. Some compromise has, therefore, to be reached between the two requirements. This is achieved in design practice by selecting a suitable natural frequency for the machine foundation.

For machines running at a steady speed the degree of isolation is determined by the ratio η (defined as the ratio of the operating frequency of the machine f_m to the natural frequency of foundation f_n). By choosing a suitable natural frequency, therefore, it is possible to obtain the required degree of isolation which obviously depends on the environmental conditions at site.

To avoid excessive vibration due to the working of a machine the following points should be considered in the planning stage.

a. *Selection of site:* Vibration causing machinery (forge presses, hammers, compressors, etc.) should be located far away from the region which is meant for precision work.

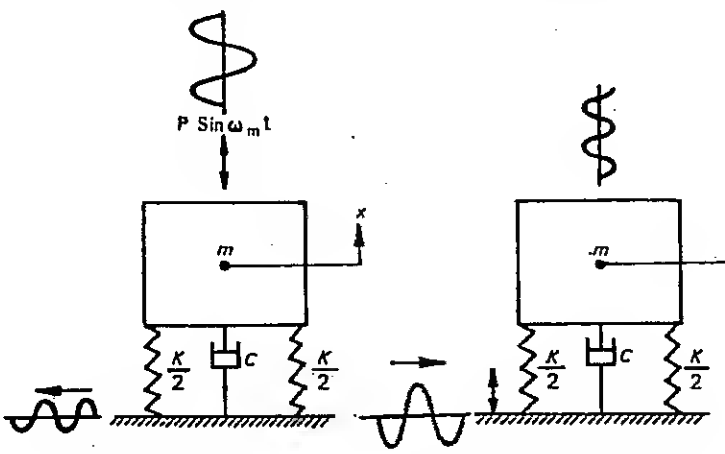
b. *Balancing of dynamic loads:* The machine should be dynamically balanced to limit the unbalanced forces generated by its operation.

c. *Adopting suitable foundations:* The foundation for the machine should be designed using accepted criteria and not by rule of thumb. The necessary design parameters such as soil constants should be evaluated at the site where the machine foundation is to be located. This is especially necessary in cases where vibration-causing machinery such as hammers, compressors are to be installed.

d. *Providing isolation:* Machine foundations should be completely separated from adjoining floors and building components by providing suitable isolating layers in between.

7.1 Active and Passive Types of Isolation—Transmissibility

From the point of view of isolation, two types of vibration problems are encountered in industrial practice: (a) active isolation, and (b) passive isolation. In the active type, the isolation is required against vibration caused by the machine itself. The foundation for such a machine should be so designed as to reduce the transmitted vibration (to the environment) to the permissible level prescribed. In the passive type of vibration isolation, the foundation for a delicate machinery is designed in such a way that the amplitude of its motion due to floor vibration (caused by a disturbing source in the vicinity) is reduced to an acceptable limit. These two cases are illustrated in Fig. 7.1.



(a) (b)

Fig. 7.1: Vibration Isolation—(**a**) Active Type, (**b**) Passive Type.

The term "transmissibility" is defined in the case of active isolation (Fig. 7.1a) as the ratio of force transmitted to the foundation to the vibratory force developed by the machine itself. In the case of passive type of isolation (Fig. 7.1b), the term is defined as the ratio of the amplitude of the sensitive instrument to the amplitude of the base. From the theory of vibration, a common expression for transmissibility can be derived for both these cases as

$$T=\sqrt{\frac{1+4\eta^2\zeta^2}{(1-\eta^2)^2+4\eta^2\zeta^2}} \tag{7.1}$$

where η is frequency ratio and ζ is the damping factor.

The variation of transmissibility with damping and frequency ratio is shown in Fig. 7.2. If the degree of damping is so small that it may be neglected, then a simplified expression for transmissibility can be used, i.e.,

$$T=\left|\frac{1}{1-\eta^2}\right|=\left|\frac{f_n^2}{f_m^2-f_n^2}\right| \tag{7.2}$$

It is obvious from the above relation that with greater values of η ($\eta>\sqrt{2}$) the transmissibility

will be less. Hence, for effective isolation, the value of η should be as high as possible. This means that the natural frequency of the isolated system should be made as low as possible, in relation to the forcing frequency. It may also be seen from Fig. 7.2 that for values of

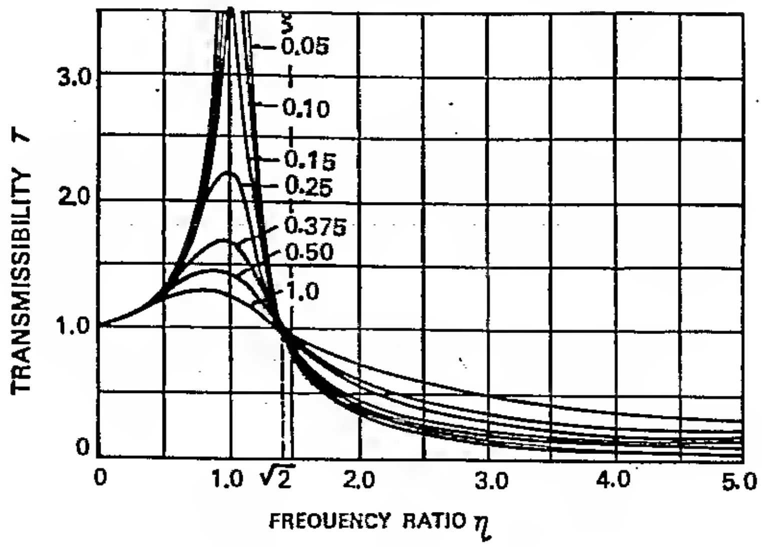


Fig. 7.2: Transmissibility (T) versus Frequency Ratio (η).

$\eta < \sqrt{2}$ the transmissibility will be more than unity, which is not desirable. It is recommended that the frequency ratio be at least equal to two in all cases of vibration isolation.

Fig. 7.3 gives data useful in problems of isolation. This data is obtained from Eq. 7.2

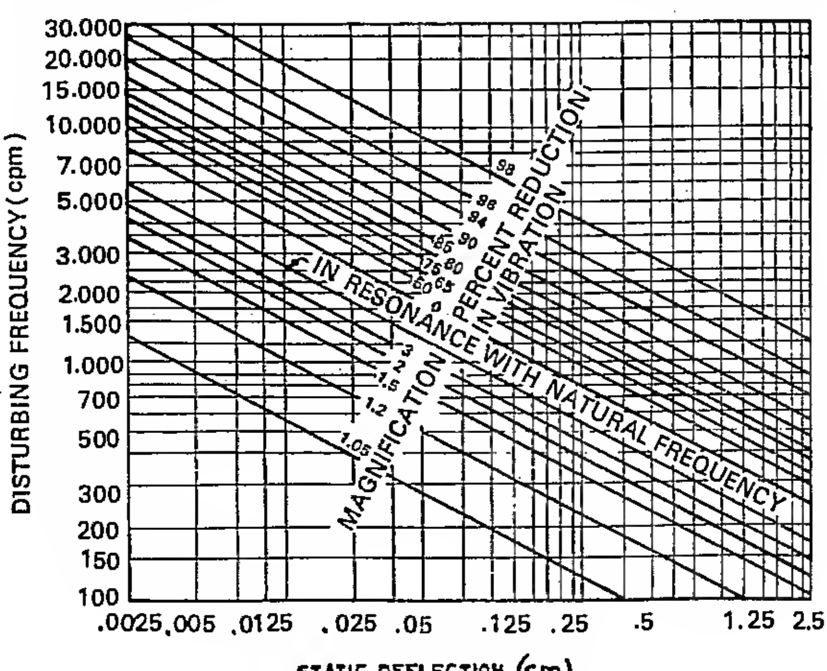


Fig. 7.3: Isolation Efficiency for Resiliently Mounted Systems.

by replacing $f_n = \frac{1}{2\pi}\sqrt{g/\delta}$ where δ is the static deflection of the system. The transmissibility is then expressed as

$$T = \frac{1}{\left(2\pi f_m\right)^2 \frac{\delta}{g} - 1} \tag{7.3}$$

Solving for f_m in cpm, the following equation may be obtained.

$$f_m = 300\sqrt{\frac{1}{\delta}\left(\frac{1}{T}+1\right)} \qquad (\delta \text{ in cm}) \tag{7.4}$$

which may also be written as

$$f_m = 300\sqrt{\frac{1}{\delta}\left(\frac{2-R}{1-R}\right)} \tag{7.5}$$

where $R = (1 - T)$ represents the relative reduction of the transmitted vibration.

So far, the discussion has been limited to bodies with motion in one direction only. In general, a rigid body mounted on a spring has six degrees of freedom and hence six modes of vibration. The design should ensure adequate isolation in all possible modes of vibration. Eq. 7.1 applies to translatory as well as rotatory modes of vibration.

7.2 Methods of Isolation in Machine Foundations

It was the conventional belief that a heavy foundation block would provide adequate isolation against vibrations produced from the operating machinery mounted on it. This concept had led to suggesting many empirical formulae for the weight of the foundation block in relation to the capacity or the weight of the machine itself. Subsequently, it was felt desirable to place the machine on a foundation block which is placed in a reinforced concrete trough lined with isolating material. It is now realized that to provide effective isolation, the machine or its foundation should be mounted on a suitable isolating medium properly designed on the basis of the theory of transmissibility explained in the preceding section. Different forms of isolating media are available in commercial practice. Rubber carpet mountings of different patterns are available. Steel helical springs are widely used in practice. Two forms of spring coil assemblies used for supporting machine foundations are shown in Fig. 7.4. In some of the advanced countries rubber-in-shear mountings commercially available in many forms are commonly used. Such mountings are placed directly between the base of the machine and the floor, thus avoiding expensive foundations. A comparatively recent system of anti-vibration mounting makes use of air springs, such as those used in vehicle suspension. Grootenhuis[C3.17] suggested an air-bellow mounting system for isolation of low-frequency vibrations.

7.3 Isolation in Existing Machine Foundations

The occurrence of resonance and the consequent effect on increase of vibration amplitudes is one of the most common sources of trouble in machine foundations. This is evidently due to faulty design based on improper estimation of design parameters such as stiffness of supporting media and unbalanced forces in the machine. The high ground water table is sometimes responsible for excessive propagation of vibration. If the level of

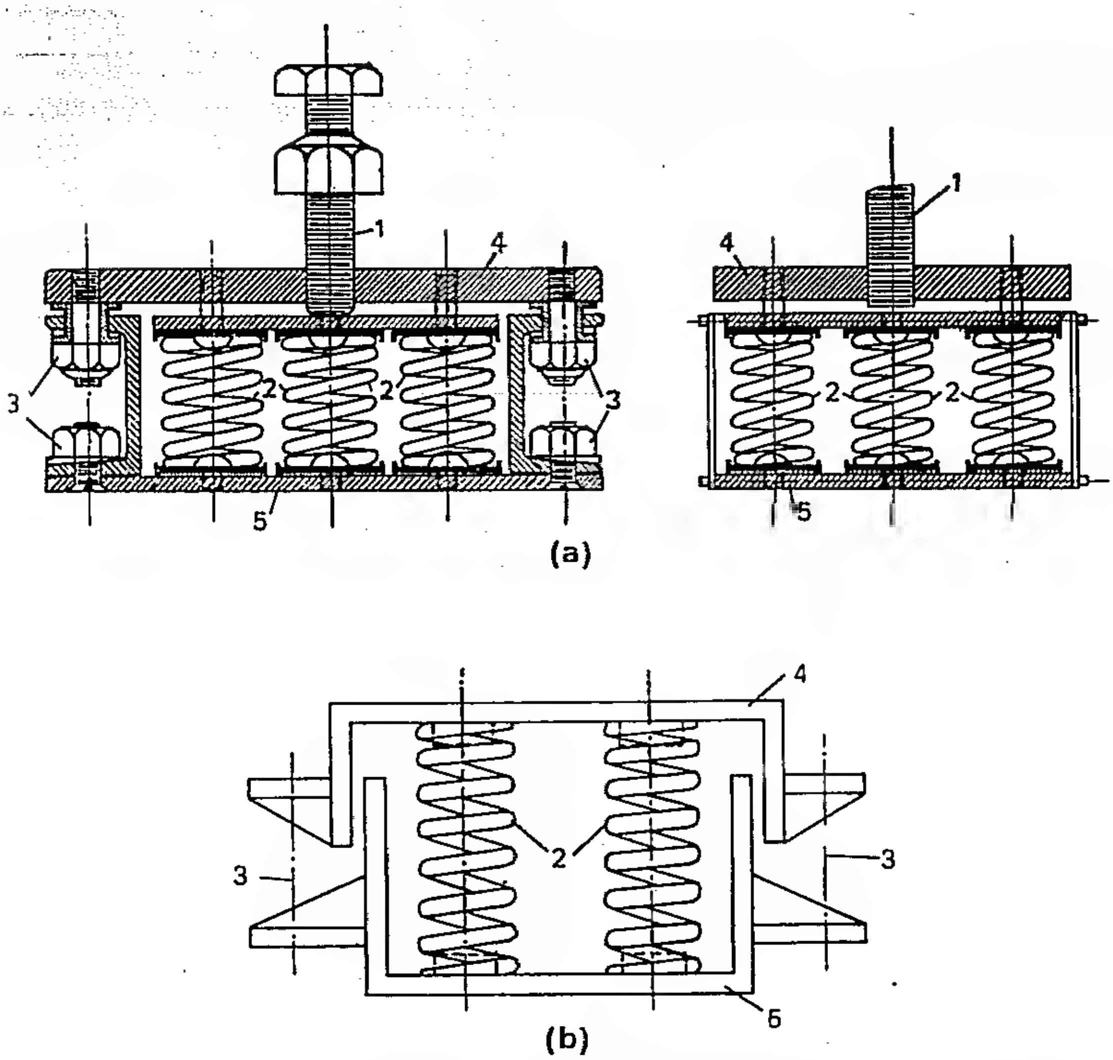


Fig. 7.4: Two Forms of Spring Coil Assembly (After Major, A., *Vibration Analysis and Design of Foundations for Machines and Turbines*, Akademiai Kiado, Budapest, 1962; with permission)—(*1*) Restraining Anchor Bolt, (*2*) Springs, (*3*) Assembly Bolts, (*4*) Upper Casing, (*5*) Lower Casing.

water table rises above the base level of the foundation, the vibration is felt up to large distances from the source. The possible methods of vibration isolation in machine foundations arc discussed below.

a. Counter-Balancing the Exciting Loads

The best way of reducing the vibration is to treat the source itself. In the case of rotating-type machinery, it is possible to counter-balance completely the exciting forces in the direction perpcndicular to the motion of piston and partly in the direction of motion of the piston. The efficiency of a certain mcthod of counter-balancing depcnds on the type of engine and the nature of vibration. For example, in the case of horizontal reciprocating engines, the dangerous vibrations are those occurring in the direction of sliding (in the direction of axis of piston) accompanied by rocking in the vertical plane containing the axis of shaft. In this case, the method of counter-balancing should be such that the first harmonic of the exciting forces in the direction of piston are

balanced, although this slightly increases the component in the perpendicular direction.

The method of counter-balancing does not require long interruption in the operation of the machine. The interruption is only for the time necessary for attaching the counter-weights. This operation is generally carried out by the mechanical. engineers.

b. Stabilization of Soils

Stabilization of soil increases the rigidity of the base and, therefore, increases the natural frequencies of the foundation resting directly on soil. This is, however, possible only in the case of sandy soils for which chemical or cement stabilization is generally adopted. Further, this method does not involve prolonged interruption of the working of the machine.

The nature of vibration determines the limits of stabilized zones of soil. For foundations subjected to rocking vibrations, only a portion of the soil near the edges of the foundation need be stabilized to a depth generally not less than about 2 m.

c. Use of Structural Measures

Suitable structural measures may also be adopted to change the natural frequencies of a foundation and to ensure the required margin of safety from the operating frequency of the machine. The choice of structural measures depends on the nature of vibration and the ratio of natural frequency to the operating frequency. Following are the possible structural measures that can be adopted in appropriate cases.

i. *Increasing Base Area or Mass of Foundation*

If the operating frequency of the machine is less than the natural frequency (i.e., for over-tuned type foundation), the structural measures are directed to further increase the natural frequencies of the foundation. This purpose is achieved by enlarging the base area of the foundation.

For under-tuned foundations whose natural frequency is lower than the operating frequency, the desired purpose is achieved by increasing the foundation mass without appreciably increasing the area of contact with soil.

If a vibrating foundation lies close to another foundation, it may be helpful to connect the two foundations so as to increase the rigidity as a whole.

In doubtful cases, it is recommended to leave projecting reinforcement from the foundation block to facilitate increasing the base area or mass (as the situation may necessitate) at a later stage if excessive vibration is noticed.

ii. *Use of Slabs Attached to Foundation*

Fig. 7.5 shows a foundation with an attached slab. The dimensions of this slab are so chosen that the amplitude of vibration in rocking mode of the foundation-slab system is reduced to the required limit.

Referring to Fig. 7.5, the differential equation of motion for forced vibration in rocking mode is given by

$$(\varphi_0 + m_1 h^2)\,\ddot{\theta} + (C_\theta I - WS + h^2 C_\tau A_1)\theta = P_0 H \sin \omega_m t \tag{7.6}$$

where m_1 is mass of the attached slab and A_1 is its base area. Other symbols have been defined in Chapter 4. The amplitude and frequency of rocking vibrations are given by

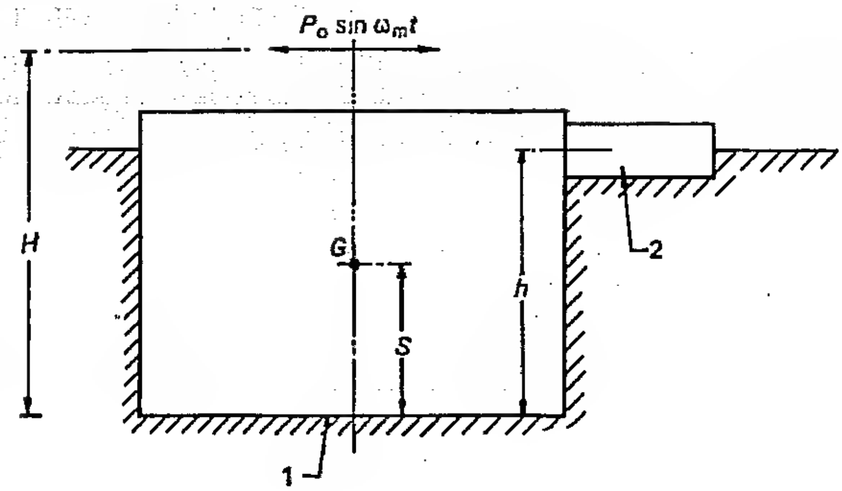


Fig. 7.5: Use of Attached Slab—(*1*) Foundation, (*2*) Attached Slab.

$$a_\theta = \frac{P_0 H}{(\varphi_0 + m_1 h^2)(\omega_\theta^2 - \omega_m^2)} \tag{7.7a}$$

and

$$\omega_\theta^2 = (C_\theta I - WS + h^2 C_\tau A_1)/(\varphi_0 + m_1 h^2) \tag{7.7b}$$

In order that the slab may be effective in changing the natural frequencies and reducing the amplitudes of the original system, the following inequalities should apply:

$$\omega_x \neq \omega_\theta^0 \tag{7.8a}$$

$$a_\theta < a_\theta^0 \tag{7.8b}$$

The superscript "o" suggests that the values correspond to the situation when the slab does not exist as part of the system.

Values a_θ^0 and w_θ^0 are obtained from Eq. 7.7, omitting the terms pertaining to the slab.

Eqs. 7.8a and 7.8b yield the following relations after substitution:

$$\frac{C_\tau A_1}{m_1} \neq \frac{C_\theta I}{\varphi_0} \tag{7.9a}$$

and

$$\frac{a_\theta}{a_\theta^0} = \frac{1}{1 + \dfrac{h^2 C_\tau A}{C_\theta I}} < 1 \tag{7.9b}$$

Eq. (7.9a) suggests that the dimensions of the slab should be so selected that its natural frequency of horizontal translation is not equal to the frequency of rocking vibrations of the foundation.

To satisfy Eq. (7.9b), h and $C_\tau A_1$ should be made as large as possible. In other words, the slab should be located as close as possible to the ground level and the natural frequency of horizontal vibration of the slab should be made as large as possible (by installing it on piles or otherwise). Short frictional piles made of reinforced concrete may be used for the purpose.

iii. *Use of Auxiliary Spring-mass Systems*

a. Vibration neutralizers: One of the methods of reducing excessive vibration of a foundation is to attach to it a suitably designed auxiliary mass m_2 by means of a spring having appropriate stiffness K_2 in the desired mode (Fig. 7.6a). Assuming that the primary system—comprising of the foundation mass m_1 resting on soil (or any other elastic layers) having stiffness K_1—behaves essentially as a single-degree freedom system, and is in resonance with the speed of the machine it supports, i.e., $\omega_m = \sqrt{\frac{K_1}{m_1}}$

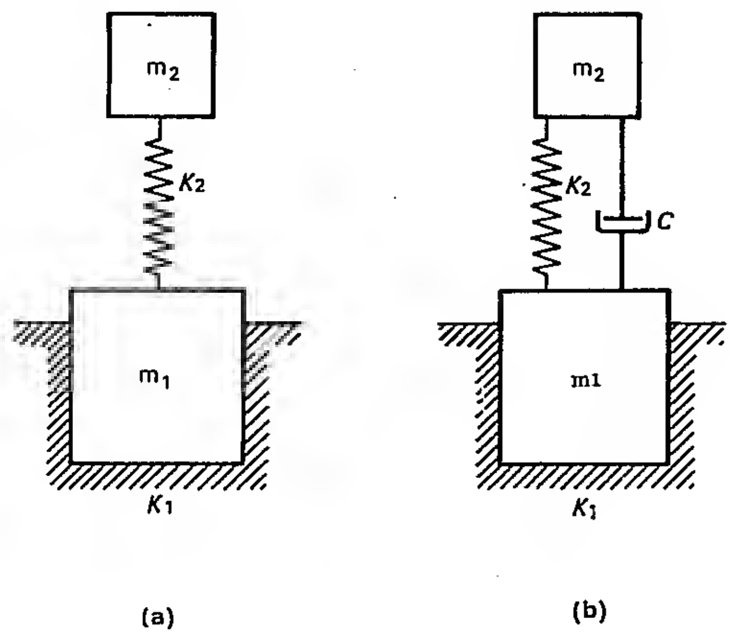


Fig. 7.6: Use of Auxiliary Mass System for Vibration Isolation—**(a)** Without Damping, **(b)** With Damping.

the parameters of the auxiliary system, viz., mass m_2 and stiffness K_2, may be so chosen that the vibration of the parent system is completely eliminated. This remedy is, however, possible "only if" the exciting frequency (ω_m) is constant.

When the auxiliary system is appended to the primary system, the resulting system possesses two degrees of freedom, the theoretical treatment for which has already been given in Section 2.3.

It was shown in Sec. 2.3 (see Fig. 2.5a) that the displacement of the primary system is zero when

$$\bar{\omega}_{n2} = \sqrt{\frac{K_2}{m_2}} = \omega_m \tag{7.10}$$

Also from Eq. 2.35a, the amplitude of mass m_2 is given by

$$a_2 = \frac{P_0}{m_2\omega_m^2} \tag{7.11}$$

From Eq. 7.10 again,

$$K_2 = m_2\,\omega_m^2 \tag{7.12}$$

For the design of auxiliary system, the following steps apply:

1. Choose m_2 such that a_2 is within reasonable limits considering the available space for its movement (use Eq. 7.11).
2. Determine K_2 from Eq. 7.12.
3. Knowing K_2, m_2 and ω_m, design a suitable auxiliary spring-mass system.

The above procedure applies for any uncoupled mode of vibration (e.g., translation along or rotation about vertical axis).

Since the need for the mounting of an auxiliary mass system arises only when the primary system is in resonance, i.e., when $\frac{K_1}{m_1} = \omega_m$, and from the above theoretical considerations the auxiliary system should satisfy Eq. 7.10, it may be deduced that the particular case, viz. $\frac{K_1}{m_1} = \frac{K_2}{m_2}$, considered in Sec. 2.3 has a practical significance in problems of vibration isolation. The data compiled in Tables 2.2 and 2.3 will, therefore, be useful to designers.

If the operating frequency of the machine is not constant or it varies in wide limits, it is not possible to design an "auxiliary mass vibration neutralizer" unless it is possible to design such a system that the natural frequency ($\bar{\omega}_{n2}$) changes correspondingly with the operating frequency of the machine (ω_m) and Eq. 7.12 is always satisfied. Systems of this category have not yet been perfected and will not be considered further here.

b. Vibration dampers : It has been stated earlier that if the exciting frequency is not constant, it is not possible to reduce undesirable vibrations by using the principle of vibration neutralizer. However, by introducing appropriate damping (c) in the auxiliary system, it is possible to keep the movement of the primary system within tolerable limits (Fig. 7.6b). The theoretical treatment for this case has also been described in Section 2.3.

i. The frequency of the auxiliary system ($\bar{\omega}_{n2}$) is given by

$$\bar{\omega}_{n2} = \sqrt{\frac{K_2}{m_2}} = \frac{1}{1+\alpha}\sqrt{\frac{K_1}{m_1}} \tag{7.13}$$

ii. The optimum damping to be provided in the auxiliary system is given by

$$\zeta = \frac{C}{C_c} = \sqrt{\frac{3\alpha}{8(1+\alpha)^3}} \tag{7.14}$$

Where $\alpha = m_2/m_1$ and $C_c = 2\sqrt{K_2 m_2}$

iii. The maximum displacement a_{max} of the primary system is given by

$$a_{max} = a_{st}\sqrt{\frac{1+2}{\alpha}} \tag{7.15}$$

Where $a_{st} = P_0/K_1$

The procedure for the design of the damper is as follows.

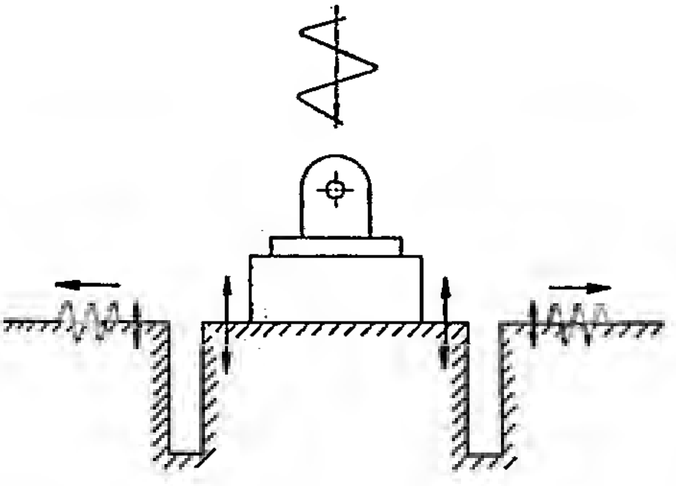
i. Select the auxiliary mass m_2 from Eq. 7.15 such that the maximum displacement

of foundation (a_{max}) is within tolerable limits. Eqs. 7.13 and 7.14 then give the values of spring stiffness K_2 and damping ζ.

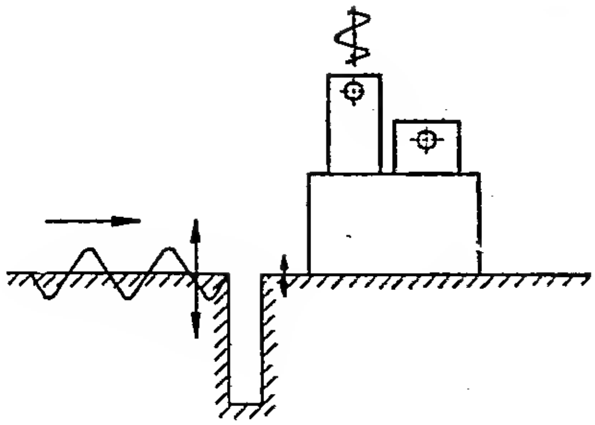
ii. The damping is provided by a viscous dash-pot suitably designed to yield the desired viscous coefficient C obtained from Eq. (7.14).

d. Isolation by Trench Barriers

It has been suggested that the presence of a trench in the path of a wave reduces the onward transmission of vibration (Fig. 7.7). Experience has shown that trenches are not suitable for general application and are less effective, particularly for isolating low-frequency vibrations. According to Barkan,[C1.1] for effective isolation the depth of the trench should be at least one-third of the wavelength of vibration. Thus, if the velocity of vibration in a particular soil is 200 m/sec and the frequency of vibration is 10 cps, the wavelength would be 20 m and the trench would have to be at least 7 m.



(a)



(b)

Fig. 7.7: Vibration Isolation by Trenches—(**a**) Active, (**b**) Passive.

Trenches filled with bentonite slurry are reported to have shown better isolation characteristics.

e. Isolation in Buildings

Vertical separation between parts of a building would help to prevent vibrations due to machinery located in one part of the building from causing trouble elsewhere. Additional rigidity imparted to the floors may sometimes help to reduce local vibration. This should, however, be applied cautiously and may be tried only when the natural frequency of the floor is above the operating frequency of machinery placed on it.

7.4 Case Histories

In this section, are described a few selected case histories concerning structural vibrations arising out of defective planning, design or construction of machine foundations in an industrial environment.

a. Resonance as the Main Cause of Vibration

i. Barkan[c1.1] reports the problem of a foundation for a steam engine (650 HP) that caused vibrations in an adjacent 45 m high structure. The speed of the engine was 150 rpm. Vibration measurements had shown that the resonance curve of the structure had a very sharp peak (0.37 mm) corresponding to the speed of the engine (150 rpm). The amplitude at resonance was so sensitive that a small change of frequency of even 5 per cent of the machine speed either way, had shown marked effect in reducing the amplitudes. The resonant range was thus very narrow and the measurements confirmed that this range was in close proximity to the first natural frequency of the structure itself.

ii. Yet another interesting example was given by the same author.[c1.1] A framed structure 20 m high was experiencing annoying vibrations caused by compressors working at a speed of 100 rpm. Due to variations in the operating speed, typical of reciprocating engines, and the occurrence of "resonance" at a particular speed of machine within this range of variation, the vibration record showed amplitudes of the order of 2.23 mm. Further, the waveform gradually reduced to a near zero amplitude condition and again increased as the speed of the machine approached the resonant stage.

Obviously in both the examples given above, the natural frequency of the structure was in close proximity to the frequency of the wave propagating from the foundation. In other words, resonance was the main cause of vibration of these structures.

Situations are often encountered where a structure located close to a machine foundation may not experience excessive vibration while another located farther away may undergo perceptible or even dangerous vibration. This can be explained by the fact that the frequency of the farther structure is close to the frequency of the propagating waves, while that of the nearer one is at a safer margin from the frequency of the waves.

The obvious remedy in such cases is to change the speed of the machine if such a proposal is feasible from the operational point of view. Alternatively, the natural frequency of the structure itself may be altered by any of the methods discussed earlier.

b. Change in Soil Conditions and High Level of Ground Water Table as Causes of Vibration

Vibration in machine foundations are influenced to a large extent by the seasonal changes in the supporting soil medium. The changes in soil moisture and fluctuations in ground

water level, especially in areas close to sea or river beds, cause a change in the resonant frequency of the system and correspondingly the amplitudes of motion. An increase in soil moisture causes a decrease in the effective stiffness. This in turn causes a decrease in the natural frequency of structures founded on such soils. Other conditions remaining same, this situation results in an increased transmission of wave energy to the surrounding structure. The foregoing points will be illustrated by practical examples explained below.

i. A heavy duty double acting hammer installed in a factory was reported to be causing excessive annoyance to personnel working in the vicinity when it was working. It was also alleged that the vibration resulting from its working was affecting the functioning of certain sensitive machines located elsewhere in the same factory. The site inspection revealed that the foundation bolts had become loose and the grout had failed. The concrete beneath the foundation had also failed causing the anvil to tilt. A few interim measures were suggested to restore the working of the hammer without causing loss of production to the factory. These included resetting of the base plate on a rich concrete bed and proper installation of foundation bolts in a non-shrink grout (embeco, epoxy or equivalent). An open trench was provided around the foundation to reduce the onward transmission of vibration from the sides of the foundation. The performance of hammer appeared satisfactory for some period after its reinstallation. But the problem reappeared in the monsoon season, though to a relatively less extent.

Detailed calculations based on soil data supplied by the factory had shown that the amplitude of vibration was (1.36 mm) only marginally in excess of the usually acceptable limit of 1.2 mm. The amplitudes measured at the time when the water table was very close to ground level were, however, still higher—of the order of 1.8 mm. This was attributed to the high water table resulting in a low stiffness of soil in saturated condition. The amplitudes evidently increase under these conditions. Besides, ground water is known to be a good conductor of vibration waves. The problem was temporarily solved in monsoon by lowering the water level by artificial means. A permanent solution would be to re-erect the machine on a new foundation suitably designed and constructed taking into account the range of soil characteristics in different seasons.

ii. Crockett[C2.6] has described the problem of a large forge hammer foundation, the falling weight of the hammer being about 30 t. The soil consisted of hard marl containing water logged lenses of rock. The anvil rested on a timber pad giving a natural frequency of 13 cps. After some period, it was noticed that the frequency of vibration was decreasing, finally reaching about 7 cps. Exploratory investigation of the soil revealed that the vibration was stirring up water in the lenses of rock into marl producing a soft slurry of practically no strength. The slurry was further being pumped into the voids of the foundation due to impact and this led to a total break down of the system.

The machine had to be re-erected on a new foundation designed as a three dimensionally prestressed block to improve its fatigue resistance. The block is further mounted on very soft rubber springs to reduce the live load transmitted to the soil below. The entire foundation block was placed in a prestressed concrete trough to avoid a direct contact of the main foundation body with the surrounding soil.

The above two illustrations show how changes in soil conditions give rise to vibration problems in machine foundations.

c. Fatigue as the Cause of Failure

Crockett[C2.6] refers to yet another example of a forging hammer foundation in prestressed concrete. Dynamic calculations of the hammer foundation showed that the oscillatory live load acting on the foundation block (which supported the anvil) was about six times greater than the dead load. Experience had shown that reinforced concrete cannot withstand a ratio of live to dead load of this magnitude and would have probably failed in a short period of time by concrete itself fatiguing and consequent other effects following it. The foundation was therefore decided to be built in prestressed concrete, the stressing of the block having been done in three directions. It was claimed that after the construction and commissioning of the hammer, the foundation performed better and had withstood more than three hundred million stress reversals.

As illustrated in the two examples quoted above from the experiences of Crockett, prestressed concrete foundations possess increased fatigue resistance as compared to normal reinforced concrete ones. Partial prestressing (class III) may be used where adequate damping capacity is needed to cope up with extreme resonant vibrations, in the case of steady state machinery. Although the number of prestressed machine foundations actually built and reported about are few, yet there is potential scope for its application to machine foundations in future, considering the present trend of development in machine industry.

d. Use of Air bellow Mountings for Vibration Isolation

Grootenhuis[C3.17] reports the application of air suspension for supporting a hydraulic fatigue machine which caused a low frequency continuous disturbing force and a shock load (caused by a sudden brittle type fracture of the test specimen) as well. The machine was fixed to a 40 tons inertia block which is mounted on two long air bellows. The stability in the lateral direction which was lacking otherwise was provided by cantilever springs while the longitudinal stability was good. The installation had proved satisfactory both for steady state vibration isolation as well as for shock isolation.

Yet another example showing application of air bellows is narrated by the same author[C3.17] for a strong floor in a structural laboratory supporting fatigue testing machines running at low frequency. It is claimed that with air suspension springs, one could realise as low a frequency as 1 cps. Air bellow mountings are, therefore, ideal for low frequency vibration isolation and have a promising future in foundation installations supporting low speed machines.

e. Use of Dynamic Absorbers

Russel[C2.6] illustrates an interesting application of a dynamic vibration absorber (earlier referred to as auxiliary mass vibration neutralizer) to a compressor foundation to eliminate excessive vibration. The compressor was running at 600 rpm and the natural frequency of foundation was also found to be of the same order. The compressor produced a vertical dynamic force of 7t which acted eccentrically on the foundation. While all other possible methods of solving the problem were found to be not feasible the adoption of vibration absorber (described in Sec. 7.2) was finally decided. Calculations showed that 10 dynamic absorbers were required and these were positioned on the sides of the foundation in pairs so that they produced an equal but opposite force to that created by the compressor. The foundation was thus made "out of tune" with the operating speed of the compressor mounted on it.

f. Use of Trenches

A jig boring machine was wrongly located in the stamping shop of a factory. The expected tolerance of this machine was 1 μm. Vibration measurements conducted at site have shown that the peak amplitude recorded on this foundation when the largest press (500 t) was working at a distance of about 50 m was of the order of 5 μm peak to peak. Besides, a few other sensitive machines were also planned to be located very near to the jig boring machine. It was, therefore, felt appropriate to structurally isolate the region of the heavy duty presses from the location of the sensitive machines. A trench 2 m deep and 30 cm wide was suggested to be dug across the shop floor with retaining walls on either side which are covered on top by precast boards placed on flexible pads on the bearing area. The depth of trench was arrived at as little more than one-third of the wavelength of vibration which was measured at site. The trench was suggested to be kept unfilled (air gap) to ensure better isolation. The isolation thus provided was found to be good enough for the desired purpose.

In the foregoing paras, only a few selected and typical examples of case studies were narrated. It may be realised that the solutions offered in each of the cases explained above may not be unique. However, the narration of a case study generates new ideas and alternativc solutions possible for each problem. For fuller details of each of the case study examples briefly explained above, the readers may rcfer to the relevant sources catalogued at the end of the text.

7.5 Properties of Isolating Materials

a. Cork

Cork is an effective isolating medium against vibration, shock and sound. It has low density, high compressibility and high impermeability. It is used generally in the form of slabs which are made by pressing cork particles under high pressure and subsequently baking them with steam. Cork slabs are placed either directly under the base of machine or under the concrete foundation. The stiffness of cork is relatively large and the surface area of cork required in most applications is very small. Consequently, the cork is applied in the form of spaced pads. Cork has a relatively small range of stiffnesses and is available only in slab form capable of carrying loads only in compression. In contrast to this, rubber can be moulded in many complex shapes and can be loaded in compression, shear or flexure.

Cork has low density varying from 2 to 4 g/cm^3. The maximum recommended loading is equivalent to a pressure of 2 kg/cm^2 for low density and 4 kg/cm^2 for high-density cork. The natural frequencies for several densities of cork as function of the intensity of load will be supplied by the cork manufacturers for various thicknesses.

The logarithmic decrement Δ of cork in compression is approximately 0.4, and this corresponds to a damping ratio (ζ) of six per cent. Cork looses its efficiency if allowed to expand on all sides. Cork sheets need therefore to be enclosed in a steel frame to prevent their lateral expansion. The resilient properties of cork deteriorate when it comes into contact with water or oil. It is recommended that these sheets are trcated with a preservative before use.

b. Felt

Felt is a fabric built up by the interlocking of fibres by some mechanical process or chemical

action. It may consist of wool or other synthetic fibres. Felt is used in the form of small pads cut to the required area and placing them under the machine to be supported. It is generally glued to the mounted machine and to the floor. The natural frequencies as a function of intensity of load for various densities of felt are usually supplied by the felt manufacturers. The force-deflection curve of a felt pad in compression is fairly linear upto a deflection as great as 25 per cent of its thickness, but thereafter the stiffness increases rapidly. The compressive strength of felt is around 80 kg/cm^2 and its elastic modulus is about 800 kg/cm^2. When used over long periods and under conditions of alternating wet and dry conditions, felt looses its elastic properties.

c. Rubber

Rubber springs have the advantage of enduring compression as well as shear. The characteristics of rubber in compression depend on the ratio of the load carrying area to the lateral expansion area. This ratio is defined as the "area ratio" (A_r). For a rectangular block of dimensions l, b, and h, it is given by

$$A_r = \frac{lb}{2h(l+b)} \tag{7.16}$$

Pads whose area ratios are equal deflect the same percentage of their thickness when supporting loads causing the same intensity of pressure (i.e., load per unit area). Rubber may be deflected under a compressive force only if it is permitted to expand laterally. In contrast to cork, therefore, rubber should not be confined on all sides. Two forms of rubber pads used in compression are shown in Fig. 7.8. In Fig. 7.8b the area ratio is reduced since the lateral expansion area is increased by the central hole. Consequently, the pad shown in Fig. 7.8b is less stiff than that shown in Fig. 7.8a. The maximum deformation (δ) in relation to the height of the rubber block (h) may be taken as 0.2 in compression and 0.4 in shear respectively. The allowable stress may be taken as about 8 kg/cm^2 in compression and 3 kg/cm^2 in shear for rubber having a shore hardness of 40°. These

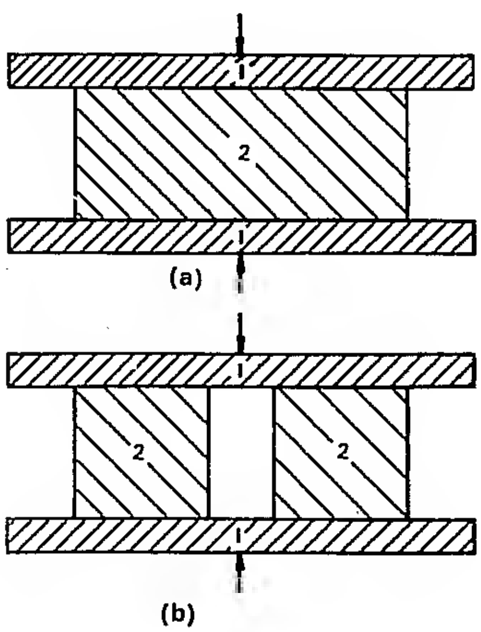


Fig. 7.8: Bonded Rubber Pads in Compression—(*1*) Metal Plate, (*2*) Rubber Prism.

values increase as the hardness increases to about 16 kg/cm² in compression, and 5 kg/cm² in shear corresponding to a shore hardness of 70°.

Table 7.1 contains the properties of natural rubber compounds.[C3.30] A property known as "shore hardness" decides the quality of rubber and hence its design characteristics. The shear stiffness (K_S) of a rubber block of thickness h and cross-sectional area A can be determined from the shear modulus G using the relation

$$K_S = \frac{GA}{h} \tag{7.17}$$

The above formula assumes that the height to width ratio is small enough to ignore deformation due to bending.

Table 7.1

PROPERTIES OF NATURAL RUBBER COMPOUNDS[C3.30]

Shore hardness $(S)^\circ$	Shear modulus (G) kg/cm²	Young's modulus (E) kg/cm²	Bulk modulus (B) kg/cm²	α
40	4.59	15.29	10193.68	0.85
45	5.50	18.35	10193.68	0.80
50	6.52	22.43	10499.49	0.73
55	8.26	33.13	11111.11	0.64
60	10.81	45.36	11722.73	0.57
65	13.97	59.63	12334.35	0.54
70	17.64	74.92	12945.97	0.53

The stiffness of a rubber pad in compression (K_c) is given by

$$\frac{1}{K_c} = \frac{h}{A}\left[\frac{1}{E(1+2\alpha A_f^2)} + \frac{1}{B}\right] \tag{7.18}$$

Where E, B and α are given in Table 7.1 and other terms defined earlier.

The application of rubber springs in compression and shear is illustrated in Example 3 given in Sec. 7.6.

d. Steel Springs

Steel springs have the advantage that their properties are known more precisely than other materials described earlier. This enables a more accurate design of spring isolators and hence they are preferred to other materials in normal practice. Springs are often used in groups (See Fig. 7.4). The properties of helical spring are given in Sec. 3.5.

7.6 Numerical Examples

1. A machine having an operating frequency of 1000 rpm is mounted on a resilient pad which has undergone a static deflection of 0.8 cm under the weight of machine. Determine the percentage reduction of the transmitted vibration.

Solution: Natural frequency (f_n) $= 300/\sqrt{0.8}$

$= 335.5$ cpm

Frequency ratio (η) $= f_m/f_n$
$= 1000/335.5$
$= 2.982$
Transmissibiliy (T) $= 1/(\eta^2 - 1)$
$= 0.127 \simeq 12.7\%$
Percentage reduction (R) $= 1 - T$
$= 87.3\%$

2. A machine weighing 200 kg is supported on a group of springs having a total stiffness of 4000 kg/cm. The machine has an unbalanced disturbing force of 50 kg at a speed of 3000 rpm. Assuming a damping factor of 0.2, determine (a) the amplitude of motion of the machine foundation due to unbalance, (b) transmissibility, and (c) transmitted force.

Solution: Static deflection of the system $= 200/4000$
$= 0.05$ cm

Natural frequency in cpm $= 300/\sqrt{0.05}$
$= 1341.6$ cpm

Frequency ratio (η) $= 3000/1341.6$
$= 2.236$

i. Amplitude of motion from Eq. (2·12a) $= \dfrac{P_0}{K\sqrt{(1-\eta^2)^2+(2\eta\zeta)^2}}$

Substituting the value $= 0.00305$ cm

ii. Transmissibility, from Eq. (7.1) $= \sqrt{\dfrac{1+4(2\cdot236\times0\cdot2)^2}{(1-2\cdot236)^2+(2\times2\cdot236\times0\cdot2)^2}}$
$= 0.3273$

iii. The transmitted force (F_T) is the disturbing force multiplied by the transmissibility (T)

$$F_T = 50 \times T$$
$$= 16.365 \text{ kg}$$

3. A rotating machine has an operating speed of 1500 rpm. Design a suitable rubber spring for one of the machine legs transferring a load of 200 kg if the degree of isolation expected is 87.5%.

A. *Rubber Block Used as a Compression Spring*

Operating frequency (ω_m) $= 1500/60 = 25$ cps
Percentage reduction in vibration $= 87.5\%$
i.e., $R = 0.875$
or $T = 1 - R = 0.125$

Neglecting damping and substituting in Eq. (7.2), the frequency ratio (η) works out to 3.
Natural frequency (f_n) $= 1500/3 = 500$ cpm
The compression (δ) of the rubber spring is given by the relation

$$\frac{300}{\sqrt{\delta}} = 500$$

This gives $\delta = 0.36$ cm
Stiffness (K) $= 200/0.36 = 555.6$ kg/cm

From Table 7.1, selecting natural rubber with a shore hardness 55° and choosing the

dimensions of the rubber block as $b = 5$ cm, $l = 10$ cm and $h = 4$ cm, the stiffness can be calculated as follows:

Area ratio $$(A_r) = \frac{10 \times 5}{2 \times 4(10+5)} = 0.4166 \qquad (7.16)$$

From Eq. (7.18)

$$\frac{1}{K_c} = \frac{4}{10 \times 5}\left[\left(\frac{1}{33.129(1+2) \times 0.64 \times 0.4166}\right) + \frac{1}{11111 \cdot 11}\right]$$

$$K_c = 504.54 \text{ kg/cm}$$

i. Deflection (δ) $= \dfrac{200}{504.54}$

≈ 0.4 cm

$<$ 0.2 times the height of the block.

ii. Stress on the Rubber block $= \dfrac{200}{5 \times 10}$

$= 4$ kg/cm^2

The deflection and compressive stress are within the permissible values.

B. Rubber Block Used as a Shear Spring

Using the same data as in case A above, and adopting a double shear sandwich (Fig. 7.9) comprising of two rubber blocks of the same dimensions (as in the previous example)

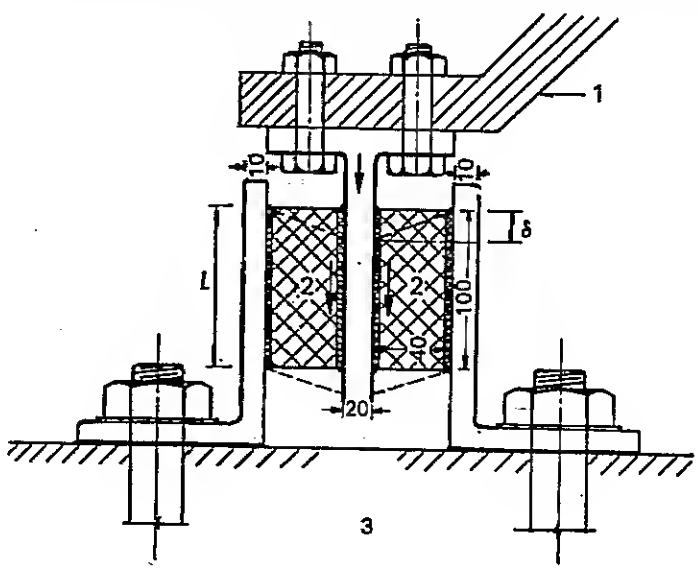


Fig. 7.9: Bonded Rubber Pad in Double Shear.

the shear stiffness K_s is found from Eq. 7.17 as

$$K_s = \frac{8.257 \times (2 \times 50)}{4}$$

$$= 206.426 \text{ Kg/cm}$$

Note that twice the area of one block is used in this case as the block is in double shear.

i. Shear deflection $= \dfrac{200}{206.426}$

$= 0.97$ cm

< 0.4 times the thickness

ii. Shear stress $= \dfrac{200}{2\,(5 \times 10)}$

$= 2 \text{ kg/cm}^2$

The shear deformation and the shear stress are thus within the allowable limits. Figure 7.9 shows one form of application of a rubber spring used as a double shear sandwich.

CHAPTER EIGHT

Constructional Details of Machine Foundations

IN THIS CHAPTER are discussed the main aspects of structural detailing of machine foundations which are of interest to the designer as well as to the engineer at site. Apart from the normal requirements of reinforced concrete construction as given in relevant codes of practice[C4.11] the additional points specially applicable to the construction of machine foundations are discussed below.

8.1 Concreting

At least M 150 grade concrete should be used for block foundations and M 200 for the superstructure of framed foundations. The foundation should be concreted in horizontal lifts. The concreting of the superstructure should preferably be done in a single operation and care taken to avoid cold joints in the body of the foundation. In the case of framed foundations, however, the base slab may be concreted first and the concreting of the super structure may be delayed. The location of construction joints should be judiciously chosen by the designer and every care taken by the site engineer to ensure monolithicity of the structure at the joint. This is achieved by providing a suitable number of dowels through the joint, providing shear keys and ensuring good quality control and supervision during concreting. To establish better bond between old and new concrete, the upper surface should be honey-combed and wired with a wire brush and a thin layer of cement mortor applied to the old surface before pouring the new concrete thereon. Cement grout with non-shrinkable additive should be used under the machine bed-plate and for the pockets of anchor bolts.

8.2 Reinforcement

Reinforcement should be used on all surfaces, around openings, cavities, etc. which are to be provided in the body of foundation for mechanical requirements. In block-type foundations (Fig. 4.33) and in the base-slab of framed foundations (Fig. 8.1) the reinforce-

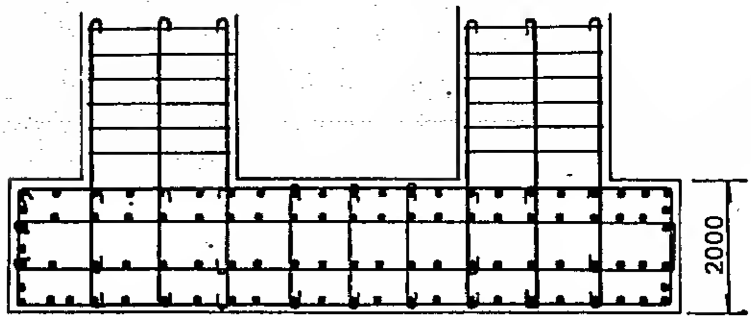
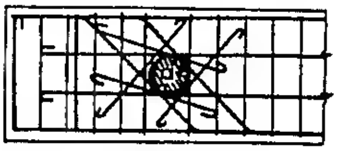


Fig. 8.1: Typical Reinforcement in Base Slab of a Framed Foundation.

ment should be used in three directions. The minimum reinforcement in block foundations should be 25 kg/m^3 of concrete. The reinforcement usually consists of 16–25 mm bars kept at 20–30 cm spacing in both directions and also on the lateral faces of the foundation. The concrete cover for protection of reinforcement should be a minimum of 75 mm at bottom and 50 mm on sides and at top. The minimum amount of steel in the base slab of framed foundations (Fig. 8.1) is generally kept around 50 kg/m^3. For circular openings, the reinforcement should overlap for a length equal to 50 times the diameter or should be extended beyond their point of intersection to 40 times the diameter. Around all openings, pits, etc., steel reinforcement equal to 0.5–0.75 per cent of cross-sectional area of the opening should be provided. This must be provided in the form of a cage (Fig. 8.2).

Fig. 8.2: Typical Reinforcement Around an Opening *a*.



Figs. 8.3–8.5 show typical details of reinforcement in framed foundations. Fig. 8.6 shows the detail at a typical beam–column junction.

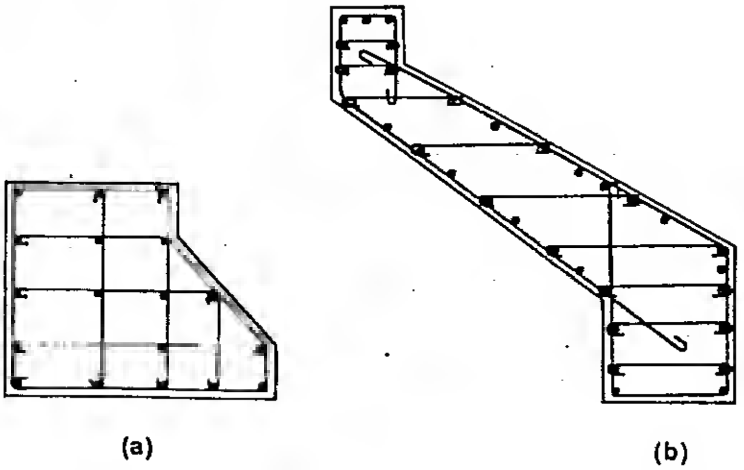


Fig. 8.3: Reinforcement in a Typical Cross-Beam of a Framed Foundation.

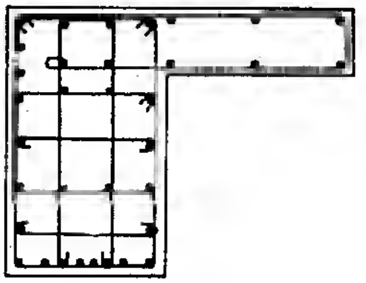


Fig. 8.4: Typical Reinforcement in Longitudinal Beams with Cantilever Projection.

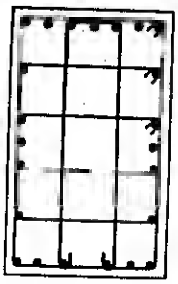


Fig. 8.5: Typical Reinforcement in Column.

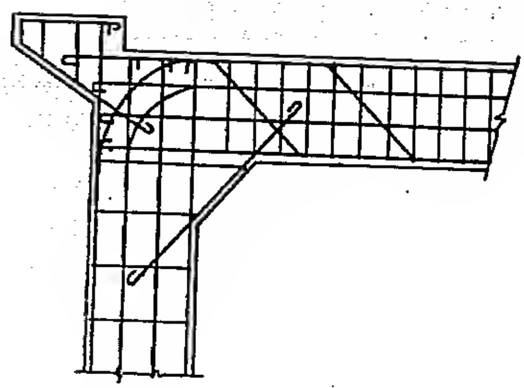


Fig. 8.6: Detail at a Beam-Column Junction.

8.3 Expansion Joints

Machine foundations should invariably be separated from adjoining structural elements to prevent transmission of vibration. The joints so provided should be kept clear of debris which may block them. Where direct contact with adjoining structural parts is unavoidable, two layers of felt or other resilient packing may be used at the interface. Foundations of adjoining structures built on different soil strata should invariably be separated by such joints to avoid failures due to differential settlement.

8.4 Connecting Elements

Machines are generally fixed to the foundation through base plates and anchor bolts. For this purpose, the concreting of the foundation should be stopped at the level of the base plate. This gap will be filled by mortar after levelling. For base plates of 20–30 cm width, the underfilling is usually 2–3 cm thick. With wider plates, the thickness may be

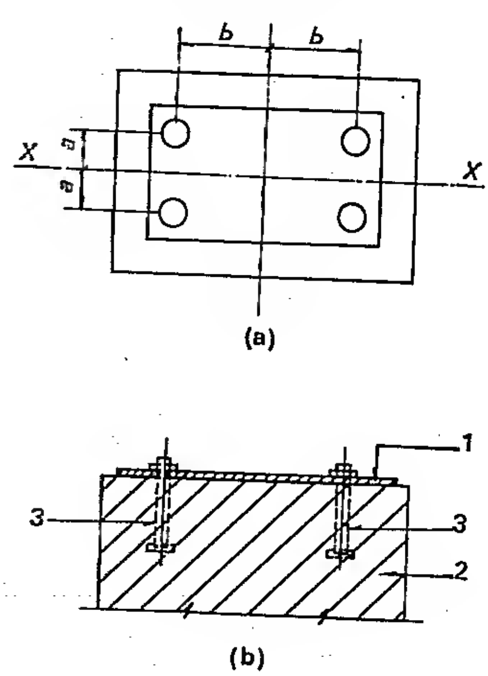


Fig. 8.7: Positioning of Anchor Bolts—(a) Plan; (b) Section x-x, (1) Template, (2) Foundation, (3) Anchor Bolts.

larger upto 5 cm. The base plate can be levelled by wedges or by screw-jacks, enabling the machine to be levelled accurately.

Base plates are fixed to the foundation block by anchor bolts, which must be correctly positioned in the foundation corresponding to the holes in the base plate. It is advisable to position the bolt holes by means of template (Fig. 8.7). All bolts must be fixed to the template in position by nuts which may be removed after the hardening of concrete. It is also possible to leave holes in the form-work to form pockets in concrete for the anchor bolts. The holes are filled with mortar after the base plate is placed and the bolts aligned. The bolt holes should be open at the bottom into a horizontal duct leading to the outer surface of foundation block (Fig. 8.8), or they should be extended throughout the thickness. This will facilitate the cleaning of holes before concreting and also to have access during fixing. The bolt holes should not be too large. A 15×15 cm hole is generally sufficient. A minimum clearance of 8 cm should be provided from the edge of the bolt hole to the

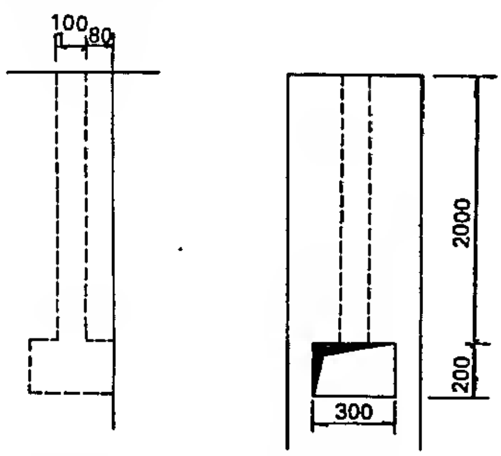


Fig. 8.8: Typical Detail of a Bolt Hole Close to Edge. (Dimensions in mm.)

nearest edge of the foundation. The length of bolt to be concreted is generally 30–40 times the diameter. If the thickness of the foundation member does not permit this length, a washer and a nut may be provided at the end for securing the bolt to the foundation. A typical form of fastening the anchor bolt is shown in Fig. 8.9. Bolt holes should invariably be filled with concrete. The locations of the bolt holes should always be made with reference to the machine axes which are firmly marked and not from edges of foundation members such as beams and columns, since the latter might be displaced during construction. To avoid excessive transmission of vibration to the foundation through the anchor bolt, the base plate of the machine may be fixed on a vibration-absorbing medium. The bolts should be placed and the bolt holes concreted only after the shrinkage of concrete is completed. Concreting the spaces under the machines should be done with extreme care using a mortar mix of 1 : 2. The sand should be well graded and optimum cement content used in the mortar so as to reduce shrinkage and increase strength. Concreting under the base plate should be done evenly and without interruption to achieve a dense concrete. Grouting is recommended wherever possible. Machines should not be operated for at least 15 days after under-filling, since vibrations from the machine have harmful effect on fresh mortar.

The edges of the foundation should be protected by providing a border of steel angles (Fig. 8.10). Holes are generally left in the body of the concrete for the lugs of the angle iron which are subsequently concreted along with the floor finish. The steel angles are

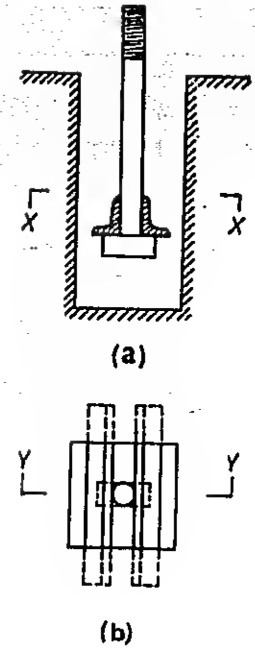


Fig. 8.9: Typical Detail of Fixing Anchor Bolt—**(a)** Section at *y-y*, **(b)** Plan at *x-x*.

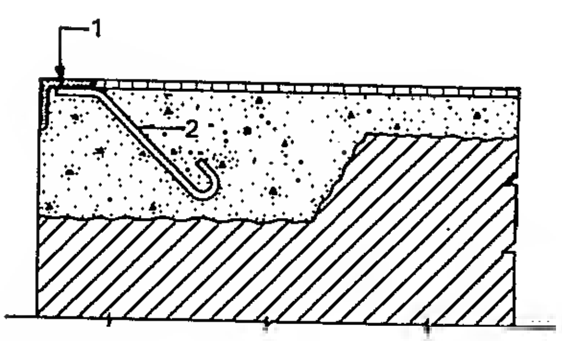


Fig. 8.10: Typical Detail at the Edge—(*1*) Angle Iron Border, (*2*) Lug.

generally 75 mm × 75 mm × 8 mm and the lugs are 12 mm diameter bars spaced at 50 cm.

In the case of generator foundation, reinforcement on either side of bus openings should be insulated to a length of at least 15 cm from the intersection point to avoid any stray currents. Haunches should be provided at beam–column junctions (in case of framed foundations) to ensure the rigidity of the joint.

8.5 Methods of Laying Spring Absorbers

It has been stated earlier that spring absorbers are commonly used for providing isolation in machine foundations. Two types of spring mounting system are adopted in machinery installations: (a) supported system, and (b) suspended system.

The principle is, however, same in both these types. While in the former, the springs are placed directly under the machine or the foundation (Fig. 8.11) in the latter type the foundation is suspended from springs which are located at or close to the floor level (see Fig. 4.32). The latter type permits easy access to the springs for future maintenance or replacement. A brief description as to when (under what circumstances) and how the two types of spring mounting systems are adopted in practice is given below.

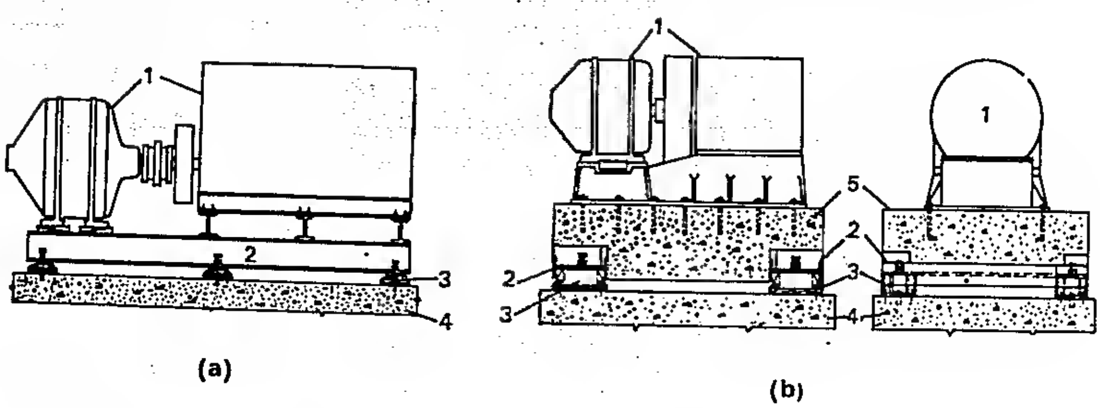


Fig. 8.11: Two Forms [(**a**) and (**b**)] of Using Spring Absorbers in Supported-Type Construction (From Barkan, D. D., *Dynamics of Bases and Foundations*, McGraw-Hill, New York, 1962; with permission)—(*1*) Machine, (*2*) Rigid Frame, (*3*) Spring Casing, (*4*) Base Slab, (*5*) Upper Foundation Block.

a. Supported System

For machines which are well balanced and for which the exciting forces associated with the higher harmonics of the operating speed are negligible, the mounting system does not necessitate a heavy mass above the springs. Such machines can be directly mounted on a rigid metal frame (made of rolled steel sections) supported by the spring casings which are placed at appropriate locations (Fig. 8.11a).

However, for machines possessing large unbalanced forces corresponding to higher harmonics of operating speed, the mass above the springs may be increased by providing an additional concrete block above the springs, as shown in Fig. 8.11b. In either case the machine or the foundation is directly supported on the springs and hence the name "supported system."

The principal stages in the construction of a spring-supported type of foundation in the general case are as follows.

i. The base slab of appropriate thickness (0.2–1 m, depending on the type and size of engine and soil properties) is laid on a levelled soil surface. The side walls forming the passage around the foundation are also constructed.

ii. After the concrete of the base slab has hardened, the top surface is covered with rubberoid, tar paper or ply wood sheet to prevent direct contact between the base slab and the upper slab to be cast over it.

iii. The lower plates of the spring absorbers are then placed at proper locations.

iv. Above these plates, a prefabricated metal frame of rolled steel beams is installed.

v. The form-work is then laid over it and the upper foundation block is cast (Fig. 8.11b). The rolled steel beams are embedded in the lower part of this block. Cavities should, however, be left in the upper foundation to facilitate access to the springs.

vi. After the concrete of the upper foundation has hardened, the springs are placed on the lower plates of the absorbers. The springs are covered on top by another plate which is rigidly bolted to the girders.

vii. Finally, the restraining anchor bolt is installed to permit lifting of the mass above the springs. The lifting should be done evenly and regulated carefully by a level.

Fig. 8.11b shows this arrangement. In cases where the upper foundation block can be avoided, stages (ii) and (v) listed above may be omitted.

Where the springs of the form shown in Fig. 7.4b are used, the upper foundation block may be cast over those spring casings which are kept in position and are precompressed by means of the side assembly bolts. After the concrete is hardened the bolts are evenly loosened, thus transferring the entire weight of the foundation on to the springs. In supported-type construction a passage should be provided all around to give access to the spring casings for periodical checking.

b. Suspended System

As already stated, suspended absorbers are preferred in cases where easy access is to be provided to the spring casings. A detail of this system is shown in Fig. 4.32. As can be noticed, this type of construction differs from the supported type only by the length of the restraining anchor bolt which passes through the absorber assembly. The lower end of the restraining anchor bolt is connected to a girder which is cantilevered out from the upper foundation block. The spring absorbers are located at the upper edge of the foundation close to the floor level.

The constructional procedure for laying the suspended type of absorber is similar to that of supported type described earlier.

8.6 Provision for Tuning

Where the design data are uncertain or the necessary margin of safety cannot be realized in design stage to avoid the resonant range, it is desirable to give due provision in the construction for tuning the foundations at a later stage. By "tuning" is meant changing the natural frequency of the foundation system as may be found necessary at a later stage. Increasing the foundation base area would result in increasing the stiffness provided the consequent addition of mass is negligible. This would increase the natural frequency. To reduce the natural frequency, addition of mass for the same base area will be useful. Due provision shall therefore be made during construction to enable these subsequent alterations. By suitably altering the natural frequency of the foundation, it would be possible to avoid resonant range $(0.7\, f_m > f_n > 1.3\, f_m)$ and consequently the build-up of amplitudes. To facilitate the subsequent enlargement of foundation, dowels should be provided in the foundation to ensure a good bond between the old and new concrete. Tschebotarioff[C3.49] has suggested the use of voids or hollows in the foundation block which may subsequently be concreted, if so needed, to increase the mass of the foundation with the same base area.

In the case of framed structures, tuning is possible by altering the rigidity of the frame. The effective length of columns can be reduced by connecting the columns at intermediate level. Dowels or projecting reinforcement should be left in the columns at appropriate levels for this purpose.

APPENDIX A

Useful Data for Ready Reference

A.1 Characteristics of Simple Spring Mass Systems under Free and Forced Vibrations

a. *Single-Degree Freedom System* (*Fig.* 2.1)

Characteristic quantity	Nature of vibration			
	Undamped case		Damped case	
	Free	Forced	Free	Forced
Circular frequency of vibration (ω_n)	$\sqrt{K/m}$	ω_m	$\sqrt{K/m}\sqrt{1-\zeta^2}$	—
Natural frequency (f_n) cps	$\frac{1}{2\pi}\sqrt{K/m}$	$\frac{\omega_m}{2\pi}$	$\frac{1}{2\pi}\sqrt{\frac{K}{m}}\sqrt{1-\zeta^2}$	—
Natural frequency (f_n) cpm	$\frac{60}{2\pi}\sqrt{K/m}$	$\frac{60\omega_m}{2\pi}$	$\frac{60}{2\pi}\sqrt{\frac{K}{m}}\sqrt{1-\zeta^2}$	
Amplitude (a)	Constant	$\frac{P_0}{K}\frac{1}{1-\eta^2}$		$\frac{P_0}{K}\sqrt{\frac{1}{(1-\eta^2)^2+(2\eta\zeta)^2}}$
Dynamic factor (μ)	—	$\frac{1}{1-\eta^2}$		$\sqrt{\frac{1}{(1-\eta^2)^2+(2\eta\zeta)^2}}$

where $\eta = \frac{\omega_m}{\omega_n}$, $\zeta = C/C_c$ and $C_c = 2\sqrt{Km}$ and P_0 is the amplitude of exciting force.

b. *Two-Degree Spring-Mass System (Fig. 2.3)*
Free vibrations
i. Circular natural frequencies

$$\omega_{n1,\,n2} = \frac{1}{2}\left[\frac{K_1}{m_1} + \frac{K_2}{m_2}\left(1 + \frac{m_2}{m_1}\right) \pm \sqrt{\frac{K_1}{m_1} + \frac{K_2}{m_2}\left(1 + \frac{m_2}{m_1}\right) - \frac{4K_1K_2}{m_1m_2}}\right]$$

Forced vibrations
ii. Amplitudes when mass m_2 is under the influence of an oscillating force $P_0 \sin \omega_m t$

$$a_1 = \frac{\bar{\omega}_{n2}^2}{m_1 f(\omega_m^2)} P_0$$

$$a_2 = \frac{(1+\alpha)\bar{\omega}_{n1}^2 + \alpha\bar{\omega}_{n2}^2 - \omega_m^2}{m_2 f(\omega_m^2)}$$

where

$$\bar{\omega}_{n2}^2 = \frac{K_2}{m_2}$$

$$\bar{\omega}_{n1}^2 = \frac{K_1}{m_1 + m_2}$$

$$\alpha = \frac{m_2}{m_1}$$

and

$$f(\omega_m^2) = \omega_m^4 - (\bar{\omega}_{n1}^2 + \bar{\omega}_{n2}^2)(1+\alpha)\omega_m^2 + (1+\alpha)\bar{\omega}_{n1}^2\bar{\omega}_{n2}^2$$

iii. Amplitudes when mass m_1 is under the influence of an oscillating force $P_0 \sin \omega_m t$

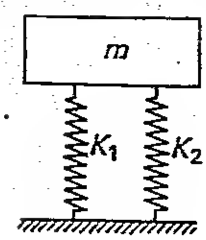
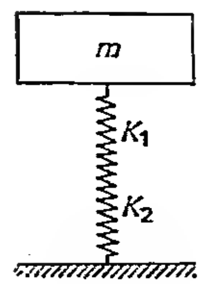
$$a_1 = \frac{P_0}{m_1 f(\omega_m^2)}\,[\omega_{n2}^2 - \omega_m^2]$$

and

$$a_2 = \frac{P_0}{m_1 f(\omega_m^2)}\,[\bar{\omega}_{n2}^2]$$

where $f(\omega_m^2)$ is as defined above.

A.2 Resultant Stiffness (K) of Springs Used in Combination

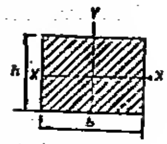
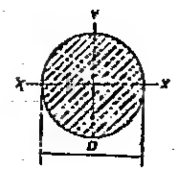
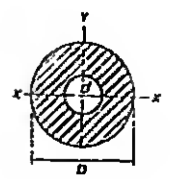
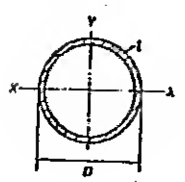
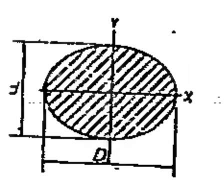
Type of Connection	Figure	Expression
(a) Parallel	 (a)	$K = K_1 + K_2$
(b) Series	 (b)	$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$

A.3 Stiffness of Helical Springs

Direction	*Expression*	
Axial	$K_s = \frac{Gd^4}{8nD^3}$	(for one spring)
	$K_z = NK_s$	(for N springs)
Lateral	$K_x = K_s\left[\frac{1}{0.385\alpha\left\{1+\frac{0.77h^2}{D^2}\right\}}\right]$	(for one spring)
	$= N\cdot K_x$ for N springs	
Bending	$K_\theta = I'_x K_s$ or $I'_y K_s$	
Torsional	$K_\psi = \frac{I'_z K_s}{0.385\alpha\left[1+\frac{0.77h^2}{D^2}\right]} = I'_z K_x$	

where d is diameter of wire
D is diameter of coil
n is number of windings
G is shear modulus
h is height of spring coil
α is a factor to be taken from Fig. 3.7
I'_x, I'_y and I'_z are to be computed from Eq. 3.3.

A.4 Moment of Inertia (I) of Common Sections

Shape	Figure	Expression I_x	I_y
a. Rectangle	 (a)	$\frac{bh^3}{12}$	$\frac{hb^3}{12}$
b. Circle	 (b)	$\frac{\pi D^4}{64}$	$\frac{\pi D^4}{64}$
c. Hollow circle	 (c)	$\frac{\pi}{64}(D^4 - d^4)$	$\frac{\pi}{64}(D^4 - d^4)$
d. Thin-walled tube	 (d)	$\frac{\pi D^3 t}{8}$	$\frac{\pi D^3 t}{8}$
e. Ellipse	 (e)	$\frac{\pi D d^3}{64}$	$\frac{\pi D^3 d}{64}$

A.5 Mass Moments of Inertia (φ) of Rectangular Prisms and Solid Cylinders

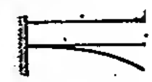
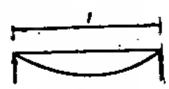
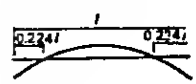
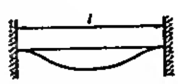
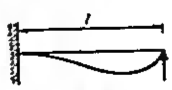
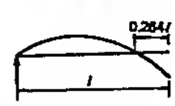
Shape	Expression for φ_x	φ_y	φ_z
i. Rectangular prism (Fig. 3.2a)	$\frac{m}{12}(l_y^2 + l_z^2)$	$\frac{m}{12}(l_x^2 + l_z^2)$	$\frac{m}{12}(l_x^2 + l_y^2)$
ii. Solid cylinder (Fig. 3.2b)	$\frac{m}{12}\left(\frac{3}{4}D^2 + l^2\right)$	$\frac{m}{8}D^2$	$\frac{m}{12}\left(\frac{3}{4}D^2 + l^2\right)$

m=mass of element

A.6 Fundamental Natural Frequency of Transverse Vibration of Beams with Various End-conditions

$$\omega_n = 9.55\,\alpha \sqrt{\frac{EI}{\mu l^4}}\ \text{cpm}$$

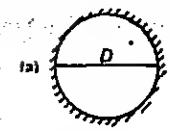
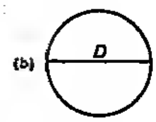
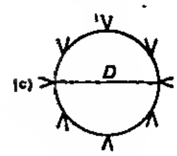
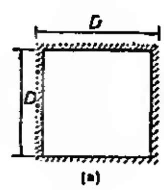
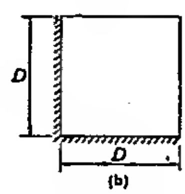
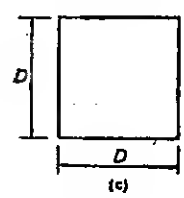
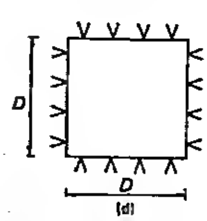
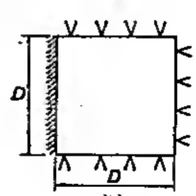
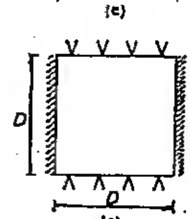
where l is effective span
I is moment of inertia of section
μ is mass per unit length of beam
E is modulus of elasticity
and α is a factor given below for various cases

Type	Figure	α
a. Cantilever	 (a)	3.52
b. Both ends simply supported	 (b)	9.87
c. Both ends free	 (c)	22.4
d. Fixed beam	 (d)	22.4
e. Fixed at one end and supported at other	 (e)	1 .4
f. One end hinged and other end free	 (f)	15.4

A.7 Fundamental Natural Frequencies of Thin Plates of Circular and Square Shapes Having Uniform Thickness

$$\omega_n = \alpha \sqrt{\frac{Et^2}{\rho D^4(1-\nu^2)}}$$

where E is Young's modulus (kg/cm^2)
t is thickness of plate (cm)
ρ is mass density (kg.sec^2/cm^4)
D is diameter of plate or side of a square plate
ν is Poisson ratio
α is a factor given below for various boundary conditions

Shape	Boundary condition	Figure	α
(1) Circular	a. Clamped		11.84
	b. Free		6.09
	c. Simply supported		4.35
(2) Square	a. Clamped		10.4
	b. Two adjacent edges clamped and other two free		2.01
	c. All edges free		4.07
	d. All edges simply supported		5.7
	e. One edge fixed, and others simply supported		6.8
	f. Two opposite edges fixed and other two simply supported		8.37

A.8 Permissible Stresses in Concrete

All values in kg/cm²

Concrete mix	Compression		Shear	Bond			
				Mild steel		High-strength steel	
	Bending	Direct		Average	Local	Average	Local
M-150	50	40	5	6	10	8.04	14.0
M-200	70	50	7	8	13	11.2	18.2
M-250	85	60	8	9	15	12.6	21.0
M-300	100	80	9	10	17	14.0	23.8
M-350	115	90	10	11	18	15.4	25.2
M-400	130	100	11	12	19	16.8	26.6

A.9 Reinforcing Characteristics

Size (mm)	Area (cm²)	Weight (kg/m)	Perimeter (cm)	Length per tonne (m)
6	0.283	0.222	1.89	4510
8	0.503	0.395	2.51	2532
10	0.785	0.617	3.14	1621
12	1.131	0.888	3.77	1125
14	1.539	1.208	4.40	829
16	2.011	1.578	5.03	633
18	2.545	2.000	5.65	500
20	3.142	2.466	6.28	405
22	3.801	2.980	6.91	336
25	4.909	3.854	7.85	260
28	6.157	4.830	8.80	207
32	8.042	6.313	10.05	159
36	10.179	7.990	11.31	125
40	12.566	9.864	12.57	101
45	15.904	12.490	14.14	80
50	19.635	15.410	15.71	65

A.10 Conversion Factors for British and Metric Units

Metric	British		Reciprocal
Length			
1 cm	0.394	in	2.54
	0.033	ft	30.50
1 m	3.28	ft	0.305
	1.094	yd	0.914
Area			
1 cm²	0.155	in²	6.452
	1.08×10^{-3}	ft²	929.0
1 m²	10.76	ft²	0.093
	1.196	yd²	0.838

A.10 Conversion Factors for British and Metric Units *(Contd.)*

Metric	British		Reciprocal
Volume			
1 cm^3	0.0619	in^3	16.387
1 m^3	35.31	ft^3	0.0283
	1.308	yd^3	0.765
Mass force			
1 g	0.0353	oz	28.35
1 kg	2.205	lb	0.454
1 tonne	2205.0	lb	0.00045
Density			
1 kg/cm^3	36.1	lb/in^3	0.0277
1 kg/m^3	0.0624	lb/ft^3	16.02
Second moment of area			
1 cm^4	0.024	in^4	41.623
1 m^4	115.9	in^4	0.00863
Pressure, stress			
1 kg/cm	5.6	lb/in	0.178
1 kg/m	0.672	lb/ft	1.488
1 kg/mm^2	1422.0	lb/in^2	0.0007
	1.422	kip/in^2	0.703
1 kg/cm^2	14.22	lb/in^2	0.0703
1 kg/m^2	0.205	lb/ft^2	4.88
1 $tonne/m^2$	205.0	lb/ft^2	0.0049
Moment, Torque			
1 kg. cm	0.868	in. lb	1.152
	0.0723	ft. lb	13.825
1 kg. m	7.231	ft. lb	0.138
1 tonne-cm	868.0	in. lb	0.00115
	72.3	ft. lb	0.0138

Terminology

FOR THE SAKE of convenience, definitions of terms which are used most frequently in the field of machine foundations are assembled here.*

Acceleration. A quantity that specifies the rate of change of velocity with time.

Amplitude. Maximum value of a quantity that varies with time according to sine or cosine rule.

Angular frequency (also called **circular frequency**). A periodic quantity denoting the angular frequency (in units of rad/sec),

Balancing. Balancing is a method of adjusting the mass distribution of a rotor so that the amplitude at bearings are reduced or controlled.

Circular frequency. See "angular frequency."

Coupled modes. Modes of vibration that are interdependent and which influence each other because of energy transfer from one mode to another.

Critical damping. It is the minimum viscous damping that allows a displaced system to return to its initial position without oscillation.

Critical speed. Speed of a rotating system that corresponds to a resonant frequency of the system.

Cycle. The full sequence of a periodic quantity occurring during a period.

Damped natural frequency. Frequency of free vibration of a damped linear system.

Damping. Dissipation of energy.

Degrees of freedom. Number of degrees of freedom of a mechanical system is equal to the number of independent coordinates required to completely define the position of the system at any instant of time.

Displacement. Change of position of a body from the position of equilibrium.

Auxiliary mass vibration neutralizer. An anxiliary mass-spring system which tends to neutralize the vibration of a

*Some of the definitions listed here have been reproduced from Harris, C. M. and Crede., *Shock and Vibration Handbook*, Vol. I, McGraw-Hill Book Co. Inc., New York; with permission.

foundation to which it is attached. The basic principle of its operation is inducing vibration out of phase with the vibration of the foundation itself, thus applying a counter-acting force.

Auxiliary mass vibration damper. As above, but with damping introduced in auxiliary system to control amplitudes for a variable-frequency machine.

Excitation. External force applied to a system that causes the system to respond.

Forced vibration. Vibration in which the response is imposed by an external force (under excitation).

Foundation. A structure that supports the machinery on top.

Free vibration. Vibration that occurs in the absence of an external force acting on it.

Frequency. A periodic function in time and is inverse of period. Unit is cps [also called Hertz (Hz)].

Fundamental frequency. The lowest natural frequency of an oscillating system.

Fundamental mode. The mode of vibration associated with the lowest frequency.

Impact. A single collision of one mass in motion with a second mass which may be either at rest or in motion.

Impulse. Product of force and the time during which the force acts.

Isolation. Reduction in capacity of a system to respond to an excitation usually provided by an elastic support.

Lissajous figure. A stationary pattern seen on an oscilloscope when the frequencies of two periodic signals given to the vertical and horizontal amplifiers of the oscilloscope are related to each other as a ratio of two whole integers.

Logarithmic decrement. The natural logarithm of the ratio of any two successive amplitudes of same sign in the decay curve obtained in a free-vibration test.

Mode of vibration. A characteristic pattern assumed by a system in which the motion of every particle is simple harmonic with same frequency.

Natural frequency. Frequency of free vibration of a system.

Oscillation. Another term to mean vibration.

Over-tuned foundation. A machine foundation whose fundamental natural frequency is greater than the operating frequency of the mounted machine.

Period of vibration. Reciprocal of natural frequency in cps.

Phase of a periodic quantity. Fractional part of a period through which the variable has advanced and measured from an arbitrary reference.

Resonance. A term that occurs in forced vibrations. Resonance is the stage at which a slight change in the frequency of excitation causes a decrease in the response. In simple terms, it is the stage when the frequency of excitation becomes equal to the natural frequency of the system.

Resonant frequency. Frequency at which resonance occurs.

Response. The motion resulting from an excitation.

Simple Harmonic Motion (also called **"harmonic motion"**). The motion in which the displacement is a sinusoidal function of time.

Shock. Transient excitation—characterized by severity and suddenness of application.

Single degree of freedom system. A system in which only one coordinate is required to define completely the configuration of the system at any instant.

Steady-state vibration. Vibration in which the peak amplitude remains constant.

Stiffness. Ratio of change of force (or moment) to the corresponding change in translational (or rotational) deflection of an elastic element.

Transmissibility. Non-dimensional ratio of the response amplitude (force, displacement, etc.) of a system under steady-state forced vibration to the excitation amplitude.

Uncoupled mode. An uncoupled mode of vibration is a mode that can exist simultaneously with, but independently of, other modes.

Undamped natural frequency. Frequency of free vibration resulting from a system having no damping or energy-dissipating device.

Under-tuned foundation. A foundation whose fundamental natural frequency is below the operating speed of the machine it supports.

Vibration isolator. A resilient support that tends to isolate a system from steady-state excitation.

Viscous damping. Dissipation of energy such that every particle is resisted by a force proportional to the velocity of the particle and acting in an opposite direction.

Wave. A measure of disturbance that is propagated in a medium at any instant of time.

Wavelength. The distance between two points in a wave having a difference in phase equal to one full period.

APPENDIX C

Selected Bibliography

THE FOLLOWINO abbreviations are used in the references listed in different sections.

1.	ACI	American Concrete Institute
2.	ASCE	American Society of Civil Engineers
3.	ASME	American Society of Mechanical Engineers
4.	ASTM	American Society of Testing Materials
5.	BRS	Building Research Station
6.	ESL	Engineering Societies Library
7.	IAEE	International Association of Earthquake Engineering
8.	ICE	Institution of Civil Engineers
9.	IIS	Indian Institute of Science
10.	ISI	Indian Standards Institution
11.	INSMFE	International Conference on Soil Mechanics and Foundation Engineering
12.	ISET	Indian Society of Earthquake Technology
13.	NSSMFE	National Society of Soil Mechanics and Foundation Engineering
14.	Proc.	Proceedings
15.	RILEM	Reunion Internationale des Laboratoire d'Essais et de Recherches Sur les Matériaux et les constructions
16.	SRTEE	School of Research and Training on Earthquake Engineering
17.	Trans.	Transactions.
18.	VDI	Verein Deutscher Ingenieure (Berlin)
19.	WCEE	World Conference on Earthquake Engineering
20.	WES	US Army Engineer Waterways Experiment Station

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INDEX

Amplifier 35
Amplitude factor 53
Amplitude metbod 138, 142
Apparent mass of soil 62
Aspect ratio 40
Auxiliary spring mass system 199

Block foundation 2
Blowers 183

Carrier amplifier 35
Case histories 202
Centre of gravity 31
Coefficient of
    damping 49
    elastic uniform compression 31, 40, 67
    elastic uniform sbear 31, 40 43, 67
    elastic non-uniform compression 31, 42, 67
    elastic non-uniform sbear 31, 43, 67
Combined method 138, 139, 146
Compressor
    rotary 1
    displacement 74
Concreting 211
Cone crusber 178
Connecting elements 213
Constant force excitation 10
Convolution integral 27
Cork 205
Counter-balancing 196
Counter-blow hammer 103, 115
Crank mechanism 4
Critical damping 9
Crushing mill 175, 178

Damping coefficient 49
Damping ratio 61
Degree of freedom
    single 8, 9
    two 13, 16, 18, 19
    multiple 21, 24, 26, 28
Degree of absorption 84
Depression in condensers 150

Design parameters—evaluation of 30
Diesel engine 2, 74, 94
Drop weight crushers 183
Dynamic analysis 51
Dynamic magnification factor 11, 12, 17, 20, 21, 22
Dynamic test 33
Dynamic absorbers 204
Duhamel's integral 27

Earthquake effect 151, 152, 169
Eccentric press 181, 182
Effective mass of soil 56
Eigen value 25
Elastic supports 31
Elastic pads 2, 43
Expansion joints 3, 213
Excitation
    constant force type 10
    rotating mass type 11

Fans 183
Felt 205
Forging press 187
Forced vibrations 9, 16, 19, 26
Framed foundation 135
Free vibrations 2, 8, 13, 18, 24
Frequency factor 53

Geometrical properties 31
Geopbone 35, 38
Grinding mills 176, 180

Hammer foundation 103
High speed machines 135
Hydraulic press 181, 182

Impulsive load 150
Inertial force 81
Isolation
    active and passive 193
    methods 195, 205
    in building 202

by air bellow mountings 204
by trench barriers 201

Jaw crushers 178, 179

Lissajous figure 38
Loading diagram 4, 136
Looms 184
Lumped parameter 59

Machine foundation
types 2
general requirements of 2
Magnification factor 58
Machine tools 180
Mass ratio 53, 55, 60
modified 56
Model block 37, 40
Mode shape 25
Modes of vibration 51
Moment of inertia
of base area 32
mass 32
Motor generators 176, 177, 180

Natural frequency
definition 2
reduced 52

Oscillator 33, 37
mechanical 33, 37
Oscilloscope 38

Pen recorder 35
Permissible
amplitudes 6, 138
bearing pressures 7
Period ratio 28
Phase difference 39
Piles
bearing 46
friction 47
Pig breakers 183
Plastic clay 52
Pressure distribution 53
Pressure bulb 66, 67
Pulsator 185, 188

Rayleigh's principle 56
Reciprocating machinery 74
Rectangular pulse 28
Reinforcement 211
Resonance, definition 8
Resonance method 138
extended 139
Rolling mills 175, 176
Rotary type machines 175
Rubber 206

Settlement 3
Shear modulus 35, 37, 39, 53
Soil constant 42
Sole plate 83, 136
Spring
absorbers 83, 99, 113, 215
coefficients 43
Stabilization of soil 197
Stamping press, 181
Steam engines 74
Steel springs 44, 207
Stiffness
vertical 44
horizontal 45
rotary 45, 48
torsional 46, 49
Stiffness matrix 24
Structural dynamics 1
Supported system 215, 216
Suspended system 215, 217

Table plate 136
Timber 7
Transducer 35
Turbo-generator foundation 135
Transient response 27
Transmissibility 193
Tuning, provision for 217
Tube mill 180, 186

Unbalanced forces 142
Undertuned foundation 138
Upper plate 136

Vibration
absorbers 83, 111
free 8
forced 9
neutralizers 199
dampers 200
Vibrator 33

Yawing 73